
THE MEANING OF THE MAXWELL FIELD EQUATIONS¹

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ABSTRACT

The objective here is to emphasize the derivation of the electromagnetic field equations from simple, understandable physical considerations, instead of regarding them as arbitrary assumptions. The factors of symmetry, continuity, and propagation determine these equations. These are large-scale attributes of field structure, so we can readily recognize and accept the fact that small-scale structure involves additional factors which do not appear in these equations.

The human mind too often has a tendency to overlook the obvious, to seek complicated and involved explanations where a simple one would suffice. At the moment, "explanations" seem rather out of fashion. Mathematical formulations are accepted as being in themselves ultimate and adequate interpretations of physical phenomena. Efforts to reduce such formulations to factors that our minds can readily comprehend have practically ceased.

The rapid accumulation of vast areas of new knowledge in our times perhaps makes this inevitable. There simply has not been time to digest this accumulation, to seek out and organize its background. Unfortunately some seem to regard such a state of affairs as final, to regard attempts at seeking a fuller understanding and a more coordinated interpretation as improper and futile.

The electromagnetic field, with a mathematical structure which has been well established and used for nearly a century, is still subject to this attitude. An intense aura of mysticism surrounds the very name "electricity," forming a road block to progress. To get past such a blockade, we need to realize that words such as "electricity" and "electromagnetic" are too specialized in their meaning to serve as ultimate concepts; they can only serve to classify phenomena, not to "explain" them. We must look behind such terms, to the processes and activities, the energy patterns which form their background.

To seek a fuller understanding, we need to trace back these more specialized concepts to the definitions used in describing and measuring them. We can then integrate these processes into our more general and comprehensive concepts, to form a coherent picture.

We base our systems of electromagnetic units on the measurement of either "charge" or "current." In either case, the definitions are stated in terms of the appearance of resultant forces, the existence of energy differences in the region considered, which are associated with actual or possible changes in position of the entities specified. To complete the picture we set up mathematical functions distributed in space and time; these potentials of the electromagnetic field permit us to express all such energy differences and changes directly.

Electrostatic "charge" is essentially an external attribute, it serves simply as

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a label which describes the degree of interaction with the surrounding region. The process can be comprehended without requiring us to regard "electricity" as a fundamental concept.

Electromagnetic quantities need not be assigned a significance beyond that contained in their definitions. Nothing in any step of their derivation implies a detailed knowledge of the small scale structure of either the "source" entities or the processes in the surrounding region. The electromagnetic field equations tell us nothing about the structure of the electron, they tell us nothing about the existence of quanta. If we bring out and understand clearly the very few physical factors underlying the structure of these equations, we can better reconcile this structure with formulations containing additional factors, and not waste our efforts looking for things which are not there.

The electromagnetic field equations are essentially an extension or generalization of the inverse square law. To understand these equations we must first be sure we understand the significance of this simpler law. Many physical processes distribute themselves symmetrically about any small "source" region. If this factor alone is sufficient for our description, then the inverse square law is valid; distribution of the physical process reduces to a simple geometrical distribution pattern. To formulate this mathematically, we choose a potential function associated with the process considered, which satisfies the equation of Laplace outside of the source regions.

Such a description is of course limited to stationary or quasi-stationary states, since it is referred to space coordinates alone. Where changes or motions are rapid, it becomes necessary to include time derivatives in our description. The field pattern is modified to take into account the factor of propagation; the potentials become "retarded potentials," they satisfy an equation of propagation, more commonly referred to as a wave equation. Such an equation provides us with a distribution referred to both space *and* time coordinates.

Propagation processes are not limited to the familiar production of electromagnetic waves. Maxwell noted the presence of the velocity c in the structure of the electromagnetic field before such wave processes were realized experimentally. This velocity is basic to such relations as the famed $E=mc^2$, and the Lorentz transformation. It appears as a more fundamental and more general attribute than the field equations themselves (Holm, 1960).

We can readily demonstrate the existence of this velocity even for stationary states. If we consider a steady flow of current through a simple resistance, where there is surely no propagation of electromagnetic waves in the ordinary sense, then the power is equal to current in electromagnetic units, multiplied by potential in electrostatic units, multiplied by the velocity c . The units here are defined directly in terms of force; multiplying by c converts to units of power. These relations are consistent dimensionally, requiring no arbitrary constants. The velocity c exists independently of any changes that we produce, so we are well justified in taking the equations of propagation to be the fundamental equations of the electromagnetic field.

Electromagnetic field processes divide into two categories, which require distinct types of potential for their representation. The field surrounding a stationary charge requires only a scalar potential; the gradient of this potential forms a simple vector, the electric field vector. The motion of a charge introduces another type of field pattern; the potential is itself a vector, associated with the direction of motion. Its space derivative is a "curl," a rotational pattern. Potential differences exist around the "lines of force," rather than along them. The scalar potential commonly used to describe a magnetic field in practical applications provides an incomplete picture. The field pattern is actually a tensor of the second rank, though it is represented adequately for many purposes by a vector along its axis, just as is often done for rotational mechanical quantities.

The two field patterns are basically different in their physical properties. The gradient of the scalar potential is associated with energy of linear motion, while the curl of the vector potential is associated with energy of rotational motion. This distinction between the electric and magnetic fields is far more vital than any formal mathematical analogies between them.

To complete the derivation of the field equations, we require only one further condition. The energy in space associated with our field potentials is conserved if we take these potentials to satisfy an "equation of continuity" analogous to the equation relating charge and current, in regions where no energy transformations are taking place. The usual Maxwell field equations may then be derived from these two conditions, propagation and continuity, using customary definitions (Holm, 1950, 1959).

The procedure suggested is just the inverse of the usual textbook presentation. It serves to emphasize factors of direct, understandable physical significance, and so clarifies both the scope and limitations of the field equations. These equations are concerned with the distribution of energy patterns in space and time. Potentials are assigned to represent these distributions, enabling us to calculate all associated energy differences and energy changes. The potentials describe the transfer of energy from point to point in the field as if it were a continuous process, they do not recognize the existence of quanta. Only energy differences and energy changes play a part in this description, uniformly distributed energy does not appear, small scale structure does not appear. The equations make no further postulates regarding the properties of field energy, they simply recognize the experimental fact that the observed energy differences exist.

REFERENCES

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