

This highly mechanical procedure for factoring the sum and difference of two cubes may be just what some of your "spit-backers" need to succeed. When rational approaches fail (and they often do in strongly algorithmic subjects like algebra) then a slick "plug-and-chug" technique like this can get a kid through a test.

FACTORING THE SUM AND DIFFERENCE OF TWO CUBES

*Barbara Zimmanck Krueger
Ursuline Academy
Cincinnati, Ohio*

Usually in an Algebra II course students learn how to factor the sum and difference of two cubes. A typical approach is just to present the following rules:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

What follows is a detailed example of an alternative approach which I have used successfully with my students.

EXAMPLE 1: Factor $x^3 + 8$

$$x^3 + 8$$

Step 1: List each perfect cube of the binomial as a product of repeated factors

$$x \cdot x \cdot x \quad 2 \cdot 2 \cdot 2$$

Step 2: Set up the binomial times the trinomial with the appropriate signs.

$$(\quad + \quad) (\quad - \quad + \quad)$$

Step 3: Fill in the terms of the binomial, one factor from each group in Step 1.

$$(x + 2) (\quad - \quad + \quad)$$

Step 4 Fill in the terms of the trinomial by grouping the factors in Step 1 into three pairs from left to right.

$$\begin{array}{c} \overbrace{x \cdot x \cdot x} \quad \overbrace{2 \cdot 2 \cdot 2} \\ \swarrow \quad \searrow \quad \searrow \\ (x + 2)(x^2 - 2x + 4) \end{array}$$

The following examples show how well this method works even as the cubed terms become more complex.

EXAMPLE 2: Factor $125x^3 - y^{27}$

$$\begin{array}{c} \underbrace{5x \cdot 5x \cdot 5x} \quad \underbrace{y^9 \cdot y^9 \cdot y^9} \\ \swarrow \quad \searrow \quad \searrow \\ (5x - y^9)(25x^2 + 5xy^9 + y^{18}) \end{array}$$

EXAMPLE 3: Factor $27x^6 + 64y^9z^{12}$

$$\begin{array}{c} \underbrace{3x^2 \cdot 3x^2 \cdot 3x^2} \quad \underbrace{4y^3z^4 \cdot 4y^3z^4 \cdot 4y^3z^4} \\ \swarrow \quad \searrow \quad \searrow \\ (3x^2 + 4y^3z^4)(9x^4 - 12x^2y^3z^4 + 16y^6z^8) \end{array}$$

Readers are invited to describe their method, mechanical or otherwise, for factoring the sum and difference of two cubes.

How many words can you make using the letters from "ARITHMETIC"? We got 145. What have we missed?

1 - letter words: I, a

2 - letter words: ah, am, at, ha, he, hi, it, ma, ti

3 - letter words: ace, act, aim, arc, are, arm, art, ate, car, cat, ear, eat, era, etc, ham, hat, hem, her, him, hit, ice, ire, mar, mat, met, ram, rat, rim, tam, tar, tat, tea, the, tic, tie

4 - letter words: ache, acme, arch, came, care, cart, char, chat, chit, cite, cram, each, emit, etch, hair, hare, harm, hart, hate, hear, heat, heir, itch, item, mace, mart, mate, math, meat, mice, mire, mite, mitt, race, rate, ream, rice, rich, rime, rite, tact, tame, tart, team, tear, teat, them, tier, time, tire, tram, trim

5 - letter words: aimer, attic, caret, chair, charm, chart, cheat, chime, citer, crate, cream, crime, earth, ethic, harem, hater, heart, irate, march, match, mater, matte, merit, mirth, reach, react, remit, tamer, teach, timer, titer, tithe, trace, trait, treat, trite

6 - letter words: hatter, hitter, matter, metric, threat, thrice

7 - letter words: chatter, chimera, ratchet

8 - letter words: thematic