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RESEARCH NOTES ON AEROTRIANGULATION

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Note 1: Aerotriangulation With Independent Models By Monomorphic Transformations

Note 2: Purely Analytical Aerotriangulation Using a Stereocomparator and a High Speed Computer

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FOREWORD

This report was prepared by Mr. Gouri B. Das, Visiting Research Associate, under the supervision and guidance of Dr. Sanjib K. Ghosh, Associate Professor in the Department of Geodetic Science, The Ohio State University.

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NOTE I:

AEROTRIANGULATION WITH INDEPENDENT MODELS BY MONOMORPHIC TRANSFORMATIONS

ABSTRACT:

This paper outlines an A.I.M. technique which consists in an initial space resection followed by successive linking of independent models and strip adjustment by monomorphic transformations.

INTRODUCTION:

It is proposed in this brief paper to sketch a complete method of Aerotriangulation with Independent Models using monomorphic transformations of the type:

$$X = \frac{a_1x + a_2y + a_3z + a_4}{d_1x + d_2y + d_3z + 1}$$

$$Y = \frac{b_1x + b_2y + b_3z + b_4}{e_1x + e_2y + e_3z + 1}$$

$$Z = \frac{c_1x + c_2y + c_3z + c_4}{f_1x + f_2y + f_3z + 1}$$

which degenerate to the normal projective transformation when $d_i = e_i = f_i$.

Basically as in other A.I.M. techniques, the initial camera stations are derived by resection and successive independent models are linked to the previous ones by mathematical manipulation. Finally the strip coordinates are transformed to ground coordinates by monomorphic equations.

SPACE RESECTION:

In the initial model five well distributed ground control points with good altimetric variations are required for a strong solution of the projective transformations:

$$X = \frac{a_1x + a_2y + a_3h + a_4}{d_1x + d_2y + d_3h + 1}$$

$$Y = \frac{b_1 x + b_2 y + b_3 h + b_4}{d_1 x + d_2 y + d_3 h + 1}$$

$$H = \frac{c_1 x + c_2 y + c_3 h + c_4}{d_1 x + d_2 y + d_3 h + 1}$$

where (X, Y, H) and (x, y, h) are respectively the known ground and model coordinates of the control points. Solving for a, b, c, d's and substituting the model coordinates of the perspective centers the resection of the initial exposure stations are worked out. If the instruments are not designed to read the coordinate of the perspective center directly, methods can be improvised to derive them by calibration. The projective transformations are used instead of affine ones to take care of any possible second degree model deformation.

LINK ORIENTATION:

The linkage between successive models is achieved with the monomorphic transformation:

$$x' = \frac{a_1 x + a_2 y + a_3 h + a_4}{d_1 x + d_2 y + d_3 h + 1}$$

$$y' = \frac{b_1 x + b_2 y + b_3 h + b_4}{e_1 x + e_2 y + e_3 h + 1}$$

$$h' = \frac{c_1 x + c_2 y + c_3 h + c_4}{f_1 x + f_2 y + f_3 h + 1}$$

Since the first model is absolutely oriented, coordinates of six passpoints and the exposure station common to the next model may be read out as (x', y', h') the corresponding independent coordinates in the next model being (x, y, h). With the help of the six passpoints and the common exposure station, the coefficients a, b, c, d, e, f's are solved and the linkage established. With the link orientation the independent models are absolutely oriented to the reference axes on ground.

It will be ideal if the forward overlap is specified as 80% instead of 60% as normally specified. Although the triangulation will involve about double the number of models the results may be well worth the expenses.

STRIP ADJUSTMENT:

This cantilever extension may be carried on till further ground control points are checked and errors are finally adjusted by monomorphic transformations on seven points. In case, however, there are not enough points in any one model to work out a proper resection or if the instruments are such that the coordinates of the perspective center cannot be easily derived then the first model can be arbitrarily oriented and all other successive models are linked on to it without help of the perspective center as a passpoint and finally the strip as a whole is adjusted in absolute orientation on seven well distributed control points in the strip by a monomorphic transformation.

CONCLUSION:

Monomorphic transformations have been suggested in A.I.M. in lieu of affine transformations in order that model or strip deformations of second or third degree may be rectified at the appropriate stage.

NOTE II:

PURELY ANALYTICAL AEROTRIANGULATION USING A STEREOCOMPARATOR AND A HIGH SPEED COMPUTER

ABSTRACT:

In this short note it is proposed to outline an analytical method of aerotriangulation with photocoordinates measured under stereocomparator (e.g. Zeiss PSK), and orientations carried out analytically by digital computer (e.g. IBM - 360/75).

INTRODUCTION:

This analytical approach is basically rigorous i.e., without any explicit approximations. Relative orientation is effected purely analytically, linkage is established between models through passpoints by affine transformations and finally absolute orientation is obtained by a set of monomorphic transformation removing thereby any possible deformation of the strip as a whole.

RELATIVE ORIENTATION:

Let (x_1,x_2) and (x_3,x_4) be the photocoordinates in two successive photos and (x,y,z) be the ground coordinates of any point appearing in them.

Then in general:

$$x_1 = \frac{a_1x+a_2y+a_3z+a_4}{u_1x+u_2y+u_3z+u_4} \quad \text{(a)} \qquad x_3 = \frac{c_1x+c_2y+c_3z+c_4}{v_1x+v_2y+v_3z+v_4} \quad \text{(c)}$$

$$x_2 = \frac{b_1x+b_2y+b_3z+b_4}{u_1x+u_2y+u_3z+u_4} \quad \text{(b)} \qquad x_4 = \frac{d_1x+d_2y+d_3z+d_4}{v_1x+v_2y+v_3z+v_4} \quad \text{(d)}$$

If, however, z = constant, then

$$x_1 = \frac{a_1x+a_2y+a}{u_1x+u_2y+u} \quad (1) \qquad x_3 = \frac{c_1x+c_2y+c}{v_1x+v_2y+v} \quad (3)$$

$$x_2 = \frac{b_1x+b_2y+b}{u_1x+u_2y+u} \quad (2) \qquad x_4 = \frac{d_1x+d_2y+d}{v_1x+v_2y+v} \quad (4)$$

Solving for x, y from (1) and (2) and substituting in (3) and (4), we obtain the equations for relative orientation as

$$x_3 = \frac{p_1x_1+p_2x_2+p}{r_1x_1+r_2x_2+1} \tag{5}$$

$$x_4 = \frac{q_1x_1+q_2x_2+q}{r_1x_1+r_2x_2+1} \tag{6}$$

If the model is hilly or undulating then the points may be selected under stereoscopic fusion keeping the dot at the same average ground level. Five such points are sufficient but more may be used for obtaining a better solution for the coefficients by the method of least squares.

In (5) and (6) $r_1x_1+r_2x_2+1 = 0$ represents the straight line on the first photo which is the intersection of a plane through the first perspective center parallel to the second photo and $p_1x_1+p_2x_2+p = 0$ and $q_1x_1+q_2x_2+q = 0$ are respectively the straight lines on the first photo corresponding to the axes on the second photo.

LINK ORIENTATION:

The linkage between models is effected by a set of affine transformations using passpoints for deviation of the parameters. Here again more points than necessary may be used for a better solution by the method of least squares.

ABSOLUTE ORIENTATION:

By the above processes of relative and link orientations, a set of models can be assembled into a coherent strip with possibly a second or third degree deformation which can be rectified with the help of known ground points by a suitable monomorphic transformation.

RESECTION AND INTERSECTION:

After adjustment of a strip the resection of individual exposure stations and the orientation elements can be worked out as follows: From equations (a), (b), (c), (d)

$$\left.\begin{aligned} a_1x+a_2y+a_3z+a_4 &= 0 \\ b_1x+b_2y+b_3z+b_4 &= 0 \\ u_1x+u_2y+u_3z+u_4 &= 0 \end{aligned}\right\} \quad \text{and} \quad \left.\begin{aligned} c_1x+c_2y+c_3z+c_4 &= 0 \\ d_1x+d_2y+d_3z+d_4 &= 0 \\ v_1x+v_2y+v_3z+v_4 &= 0 \end{aligned}\right\}$$

respectively, offer the solutions for the coordinates of the two exposure stations. (u_1, u_2, u_3), (v_1, v_2, v_3) are the respective direction ratios of the two optical axes. (a_1, a_2, a_3), $b_1, b_2, b_3)$, (c_1, c_2, c_3), (d_1, d_2, d_3) are the direction ratios of the photocoordinate axes.

Intersection of fresh points are achieved by solving for x, y, z from equations (a), (b), and (c), (a), (b), and (d), (a), (c), and (d), or (b), (c), and (d).

CONCLUSION:

With the development of precise stereocomparator and sophisticated computers it is desirable that rigorous formulas as envisaged in this note may be more helpful in achieving a better result in aerotriangulation.

Iteration is no longer a problem with IBM - type computers and may be used in the method outlined by introducing the differential elevation for a second grinding.

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ADDENDUM 1:

Geometrical Significance of the Parameter:

If the first photograph is supposed to be vertical, then

$$\frac{x_1}{X} = \frac{x_2}{Y} = \frac{f}{H} = s \text{ (constant)}$$

i.e., $x_1 = sX, \; x_2 = sY \; \cdots$

The relative orientation equations are

$$x_3 = \frac{p_1x_1+p_2x_2+p_3}{r_1x_1+r_2x_2+1}$$

$$x_4 = \frac{q_1x_1+q_2x_2+q_3}{r_1x_1+r_2x_2+1}$$

Substituting $x_1 = sX$, $x_2 = sY$, we have

$$x_3 = \frac{p_1X+p_2Y+p}{r_1X+r_2Y+r} = \frac{m_{11}(X-X_o)+m_{12}(Y-Y_o)+m_{13}(Z-Z_o)}{m_{31}(X-X_o)+m_{32}(Y-Y_o)+m_{33}(Z-Z_o)}$$

$$x_4 = \frac{q_1X+q_2Y+q}{r_1X+r_2Y+r} = \frac{m_{21}(X+X_o)+m_{22}(Y-Y_o)+m_{23}(Z-Z_o)}{m_{31}(X-X_o)+m_{32}(Y-Y_o)+m_{33}(Z-Z_o)}$$

where m_{ij} are the usual elements of orientation matrix M of the collinearity condition.

ADDENDUM 2:

An Iterative Solution:

A three-dimensional form of relative orientation may also be derived as follows:

$$\left.\begin{aligned} x_1 &= f_1\ (X,Y,Z) \\ x_2 &= f_2\ (X,Y,Z) \\ x_3 &= f_3\ (X,Y,Z) \end{aligned}\right\}$$ the three collinearity conditions for the photocoordinates x_1, x_2, x_3

Eliminating X Y from the above three equations we have

$$x_3 = \phi(x_1 x_2 Z)$$

In this relationship Z may be approximated by $\kappa(x_3-x_1)$, the usual parallax equation where κ is a constant.

Since the differential heights Z are approximate a second grinding may be necessary in this case after a first adjustment of the strip.

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