REFERENCES

Baron, M.D. "John Napier." Dictionary of Scientific Biography, vol. 9, Scribners, 1974, 609-613.

Carslaw, H.S. "The Discovery of Logarithms by Napier." Math. Gazette, 8 (1915), 76-84, 115-119.

Goldstine, H. A History of Numerical Analysis from the 16th through the 19th Century. Springer-Verlag, 1977, Chapter 1.

Gridgeman, N.T. "John Napier and the History of Logarithms." Scripta Mathematica, 29 (1973), 49-65.

Knott, C.G., ed. Napier Tercentenary Memorial Volume. Royal Society of Edinburgh, 1915.

Struik, D.J. A Source Book in Mathematics, 1200-1800. Harvard U. Press, 1969, 11-21.

LOSING AT LOTTO

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Many states have instituted a game called Lotto in an attempt to raise funds for various state projects. The common procedure for those games is that the player selects r numbers (usually 6) from a set of n numbers (varying from 30 to 44, depending upon the state). At the end of a given time period (often one week), the state lottery commission randomly selects a winning set of r numbers from the total set of n numbers.

Usually a player wins a cash prize if he/she has selected 4, 5, or 6 of the winning numbers that the lottery commission has identified. In some states a free play is awarded to the player who has matched three of the winning numbers.

A common impression of those who play Lotto is that not only is it very difficult to win anything, but it is very difficult to even come close to winning. Is this impression confirmed by actual probability calculations?

Consider the game in which six numbers are selected from the set of n numbers. What is the probability of a player selecting exactly w winning numbers from the set of n numbers? There are exactly

$$\binom{n}{6} \text{ or } \frac{n!}{6!(n-6)!}$$

ways in which the six numbers can be selected. The number of ways of selecting w winning numbers (out of 6) and (6-w) "losing" numbers (out of (n-6)) is

$$\binom{6}{w} \binom{n-6}{6-w}$$

The probability of selecting exactly w winning numbers is then

$$\frac{\binom{6}{w} \binom{n-6}{6-w}}{\binom{n}{6}}$$

Now let us calculate the values of this probability for several combinations of n and w. Table I reflects these calculations.

Table I

n	w	Problem selecting with winning numbers	Cumulative probabilities
30	0	.22667846	.22667846
	1	.42949602	.65617448
	2	.26843501	.92460949
	3	.06817397	.99278346
	4	.00697234	.99975580
	5	.00024252	.99999832
	6	.00000168	1.00000000
36	0	.30484518	.30484518
	1	.43897706	.74382224
	2	.21104666	.95486890
	3	.04168823	.99655713
	4	.00334995	.99990708

	5	.00009241	.99999949
	6	.00000051	1.00000000
40	0	.35038323	.35038323
	1	.43495850	.78534173
	2	.18123271	.96657444
	3	.03117982	.99775426
	4	.00219233	.99994659
	5	.00005315	.99999974
	6	.00000026	1.00000000
44	0	.39108382	.39108382
	1	.42663689	.81772071
	2	.15685180	.97457251
	3	.02390123	.99847374
	4	.00149383	.99996757
	5	.00003230	.99999987
	6	.00000014	1.00000000

It is interesting to note the increasing probability of "coming up totally empty" as n increases. Since, in many games, the selection of three winning numbers provides a free play, consider the cases where w = 0, 1, or 2 as "total losses."

What is the probability of a total loss for different values of n ? Table II reports these probabilities.

Table II

n	Probability of Total Loss
30	92.5%
36	95.5%
40	96.7%
44	97.5%

Playing Lotto may be entertaining and provide the state with revenue, but it certainly doesn't appear to be a safe investment strategy!