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Report No. 96

ORBIT PERTURBATIONS OF ARTIFICIAL EARTH SATELLITES AS FUNCTIONS OF GRAVITY ANOMALIES AND DIFFERENTIAL CORRECTIONS OF ORBIT AND STATION COORDINATES

by

Karl-Rudolf Koch

Prepared for

Air Force Cambridge Research Laboratories
Office of Aerospace Research
United States Air Force
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Contract No. AF19(629)-5701
Project No. 7600
Task No. 760002, 04
Work Unit No. 76000201, 76000401

Scientific Report No. 17

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FOREWORD

This report was prepared by Karl-Rudolf Koch, Research Associate, Department of Geodetic Science of The Ohio State University, under Air Force Contract No. AF19(628)-5701, OSURF Project No. 2122, Project Supervisor, Dr. Urho A. Uotila, Professor, Department of Geodetic Science. The contract covering this research is administered by the Air Force Cambridge Research Laboratories, Office of Aerospace Research, Laurence G. Hanscom Field, Bedford, Massachusetts, with Mr. Owen W. Williams and Mr. Bela Szabo, Project Scientists.

ABSTRACT

In the first part of this report the orbit perturbations of an artificial earth satellite are given as functions of the gravity anomalies by means of Stokes' formula. To compute the perturbations, the perturbed equations of motion are integrated numerically and the usual integration technique is applied for Stokes' formula by dividing the surface of the earth into surface elements in which the gravity anomaly and Stokes' function are regarded as constants. In a numerical example orbit perturbations are computed for three satellites with different orbital elements from mean anomalies of different sizes of surface elements to determine the influence of the sizes of surface elements on the orbit perturbations.

In the second part it is assumed that the gravity field of the earth is well known from gravity anomalies. To obtain a precise orbit of a satellite by these data, a method using differential corrections is given to compute more accurate orbital elements and more accurate coordinates of the tracking stations, whose observations determine the orbit.

TABLE OF CONTENTS

1. Introduction

If the orbit of a satellite is sufficiently well defined, the satellite may be used as a moving survey point. By observing this satellite the position of a station at the earth's surface can be obtained only by measurements from this station without simultaneous observations with other stations. This method is very helpful for geodetic purposes as well as for navigations of ships and airplanes. For the geodetic determination of station positions a high accuracy is needed which, of course, cannot be better than the accuracy with which the orbit of the observed satellite is known.

To compute an orbit of a satellite and its perturbations, the gravity field of the earth has to be known. Usually the earth's potential is expressed by an expansion into spherical harmonics, the coefficients of which are determined by satellites. Until now a complete set of harmonic coefficients is determined to the 8th degree with additional coefficients up to the 15th degree, Gaposchkin (1966). From a combination of satellite and gravimetric data complete sets up to the 14th and 15th degree are available, Rapp (1967a), Köhnlein (1967). On expressing the gravity field of the earth by gravity anomalies, mean anomalies may be used for $5^\circ \times 5^\circ$ surface elements, which result from gravity measurements and earth models, Uotila (1962, 1964), Kivioja (1963).

When comparing these 2592 mean anomalies for $5^\circ \times 5^\circ$ surface elements with the 250 harmonic coefficients up to the 15th degree of the earth's potential, it is obvious that more detailed information about the earth's gravity field is available from the mean anomalies. However, the more accurate information is coming from the harmonic coefficients determined by satellites.

To get an idea how precise the gravity field of the earth must be known for an accurate orbit determination, a detailed knowledge of the earth's potential is necessary. We, therefore, assume that the gravity field is given by gravity anomalies. Then mean anomalies for different sizes of surface elements are used to compute orbit perturbations. By means of the differences between these results conclusions may be drawn with regard to the size of surface elements needed for the mean anomalies.

In order to integrate the equations of motion of a satellite six constants of integration must be known. These constants, e.g., the six Keplerian elements, are usually not given as accurate as necessary for a precise orbit determination. The coordinates of the tracking stations by which the orbit is determined are usually not known very accurately either. Thus, assuming the gravity anomalies are known at the earth's surface a method is developed to determine the corrections of the orbital elements and of the station coordinates. The same method may be used to compute gravity anomalies if these anomalies are unknown.

2. Orbit Perturbations as Functions of Gravity Anomalies

2.1 Perturbed Equations of Motion

If we regard only short orbits of satellites whose perigees are above 400 km and whose ratio of surface to mass is small, we may neglect the influence of the attraction and radiation pressure of the sun and the moon and the influence of the atmospheric drag. If, furthermore, the mass of the satellite is negligible small in comparison with the mass of the earth, we obtain the equation of motion of a satellite

$$\frac{d^2 \vec{r}}{dt^2} = \text{grad } W , \tag{1}$$

where $\vec{r}$ is the position vector of the satellite, i.e., the vector from the mass center of the earth to the satellite, W is the potential of the earth's gravity field, and t is the time.

By introducing the equatorial coordinate system (x, y, z) of astronomy with the origin in the mass center of the earth, the x-axis toward the vernal equinox, the y-axis 90° eastward in the equator, and the z-axis toward the north pole, we get instead of the vector differential equation (1) the three differential equations of the second order

$$\frac{d^2 x}{dt^2} = \frac{\partial W}{\partial x} , \quad \frac{d^2 y}{dt^2} = \frac{\partial W}{\partial y} , \quad \frac{d^2 z}{dt^2} = \frac{\partial W}{\partial z} . \tag{2}$$

Supposing the gravity potential W is known, Eqs. (2) may be solved by numerical integration, if the constants of integration, the components of the position and velocity vector of the satellite are known. This solution, known as Cowell's method in astronomy, is used for geodetic purposes, for instance by Anderle (1966) and Schneider (1967).

As the components of the position and velocity vector are changing very rapidly during one revolution, it is more convenient for the integration to convert the equation of motion from the rectangular coordinates (x, y, z) to the six parameters (Ω, i, ω, a, e, M) of a Kepler ellipse, the so-called Keplerian elements. Ω is the right ascension of the node, i the inclination, ω the argument of the perigee, a the semimajor axis, e the eccentricity, and M the mean anomaly. As the earth's potential field differs only slightly from a central field, the rates of changes of the Keplerian elements with time are also small except for the mean anomaly M. Thus, these elements are suitable to describe the orbit of a satellite.

Introducing the Keplerian elements into Eqs. (2) we get the perturbed equations of motion, Moulton (1914) p. 404,

$$
\begin{aligned}
\frac{d\Omega}{dt} &= \frac{r \sin(\omega + v)}{n a^2 \sqrt{1 - e^2} \sin i} K_1 , \\
\frac{di}{dt} &= \frac{r \cos(\omega + v)}{n a^2 \sqrt{1 - e^2}} K_1 , \\
\frac{d\omega}{dt} &= \frac{-\sqrt{1 - e^2} \cos v}{n a e} K_3 + \frac{\sqrt{1 - e^2}}{n a e} \left(1 + \frac{r}{a(1 - e^2)} \right) \sin v \, K_2 \\
&\quad - \frac{r \sin(\omega + v) \cot i}{n a^2 \sqrt{1 - e^2}} K_1 , \qquad (3) \\
\frac{da}{dt} &= \frac{2 e \sin v}{n \sqrt{1 - e^2}} K_3 + \frac{2 a \sqrt{1 - e^2}}{n r} K_2 ,
\end{aligned}
$$

$$\frac{de}{dt} = \frac{\sqrt{1-e^2}\sin v}{n a} K_3 + \frac{\sqrt{1-e^2}}{n a^2 e}\left(\frac{a^2(1-e^2)}{r} - r\right) K_2 ,$$

$$\frac{dM}{dt} = n - \frac{1}{n a}\left(\frac{2r}{a} - \frac{1-e^2}{e}\cos v\right)K_3 - \frac{1-e^2}{n a e}\left(1 + \frac{r}{a(1-e^2)}\right)\sin v\, K_2 .$$

K_1, K_2, and K_3 are the components of the perturbing force which results by differentiation of the earth's gravitational field after subtracting the central field. K_1 is the component perpendicular to the orbital plane with the positive direction toward the north pole. K_2 is the component in the orbital plane which acts normal to the position vector, positive in the direction of motion, and K_3 is the component acting along the position vector with the positive direction away from the earth. n is the mean angular velocity given by Kepler's third law

$$n^2 = \frac{kM}{a^3} , \tag{4}$$

where k is the gravitational constant and M the mass of the earth. r is the amount of the position vector which is obtained by

$$r = \frac{a\,(1-e^2)}{1 + e\cos v} , \tag{5}$$

and v is the true anomaly. v can be computed from M by an expansion into a series of e

$$\begin{aligned} v = {} & M + \sin M\left(2e - \frac{e^3}{4} + \ldots\right) + \sin 2M\left(\frac{5e^2}{4} - \cdots\right) \\ & + \sin 3M\left(\frac{13e^3}{12} - \cdots\right) + \cdots . \end{aligned} \tag{6}$$

This series up to the 7th power of e may be found in Brouwer and Clemence (1961) p. 77.

With Eqs. (3) we have preferred the form of the perturbed equations of motion given by Gauss rather than the form developed by Lagrange, since it is easier to differentiate the expression of the earth's potential given in the following chapter with respect to the directions of K_1, K_2, and K_3 than to differentiate with respect to the six orbital elements. These differentiations must be performed in Lagrange's perturbed equations of motion.

It can be shown by examples that the numerical integration of the perturbed equations of motion (3) takes fewer integration steps than the numerical solution of Eqs. (2), if the same integration technique is used and the same accuracy is asked for. Thus, solving Eqs. (3) takes less time and is more accurate than the solution of Eqs. (2).

2.2 Application of Stokes' Formula

As mentioned in the introduction, the earth's gravity field shall be represented by gravity anomalies. The orbit perturbations are therefore sought as functions of the gravity anomalies. This problem was solved by Arnold (1965), who expressed the potential of the earth by Stokes' formula as the solution of the geodetic boundary-value problem. The same method was used by Rapp (1967b) and with some modifications with regard to the computation of the gravity anomalies in comparison with Arnold (1965) by Koch (1967a). The following investigations follow the latter method. For easier references and since more accurate equations are now used to get the normal gravity, all formulas are given here to compute orbit perturbations by gravity anomalies.

The earth's potential W is separated into

$$W = U + T \ , \tag{7}$$

where U is the normal gravity field, i.e., the attracting potential of a level ellipsoid rotating with the same angular velocity as the earth around its minor axis, which is identical with the earth's axis. T is the disturbing potential. If the mass of the earth equals the mass of the ellipsoid and the centers of both masses are identical, the disturbing potential T is given by the generalized formula of Stokes, Heiskanen and Moritz (1967) p. 93, Koch (1967b),

$$T(P) = \frac{r}{4\pi} \iint_q \Delta g \, S(P,Q) \, dq \ , \tag{8}$$

where the influence of the topography on T is neglected, since it is small even at the earth's surface, Koch (1967c). The integration is extended over the surface q of the sphere with the mean radius R of the earth, the midpoint of which lies in the center of the earth's mass. r is the distance of the satellite P from the mass center of the earth, Δg the free-air anomaly, Q the variable point on q and S(P, Q) Stokes' function. It holds

$$S(P,Q) = \tau^2 \left(\frac{2}{D} + 1 - 3D - \tau \cos\psi \left(5 + 3 \ln \frac{D + 1 - \tau \cos\psi}{2} \right) \right) \tag{9}$$

with

$$D^2 = 1 - 2\tau \cos\psi + \tau^2 \quad , \quad \tau = \frac{R}{r} \, ,$$

$$\cos\psi = \sin\varphi \sin\varphi_Q + \cos\varphi \cos\varphi_Q \cos(\lambda_Q - \lambda) \; .$$

The product rD yields the spatial distance between the satellite P and the variable point Q. The angle ψ is the spherical distance between P and Q measured in the center of the earth's mass. φ, λ, and φ_Q, λ_Q are the spherical latitude and longitude of the satellite P and the variable point Q, respectively.

By neglecting terms of the order of f^3, where f means the flattening, we get the potential U of a level ellipsoid from the expansion, Heiskanen and Moritz (1967) p. 73,

$$U = \frac{kM}{r}\left[1 - J_2\left(\frac{a_e}{r}\right)^2 P_2\,(\sin\varphi) - J_4\left(\frac{a_e}{r}\right)^4 P_4\,(\sin\varphi)\right] . \quad (10)$$

P_2 and P_4 are the Legendre polynomials of the second and fourth degree and J_2 and J_4 their coefficients. a_e is the semimajor axis of the ellipsoid. If the potential U is needed at the surface of the ellipsoid, the centrifugal term

$$\frac{1}{2} w^2 r^2 \cos^2 \varphi$$

must be added, where w means the angular velocity of the earth.

According to Stokes' theorem the potential U of a rotating level ellipsoid is determined by the angular velocity w of the earth, the mass and the shape of the ellipsoid. In Eq. (8) it is assumed that the mass of the level ellipsoid equals the mass of the earth and that the disturbing potential T is of a small quantity. For kM, J_2, and a_e in Eq. (10) we, therefore, choose values obtained from observations of earth satellites. Thus, we get a level ellipsoid which approximates the shape and the potential field of the earth very well.

The flattening f of the ellipsoid may be computed by successive approximations from, Heiskanen and Moritz (1967) p. 78,

$$J_2 = \frac{2}{3} f - \frac{1}{3} m - \frac{1}{3} f^2 + \frac{2}{21} fm \tag{11}$$

with

$$m = \frac{w^2 a_e^3}{kM} (1 - f) ,$$

and we get the coefficient J_4 in Eq. (10)

$$J_4 = -\frac{4}{5} f^2 + \frac{4}{7} fm . \tag{12}$$

The normal gravity γ at the surface of the ellipsoid is given by

$$\gamma = \frac{kM}{a_e^2 (1 - f)} \left(1 - \frac{3}{2} m - \frac{3}{7} fm\right) \left[1 + \left(\frac{5}{2} m - f - \frac{17}{14} fm + \frac{15}{4} m^2\right) \sin^2 B + \left(\frac{1}{8} f^2 - \frac{5}{8} fm\right) \sin^2 2B\right] , \tag{13}$$

where B means the geographical latitude of the ellipsoid.

The free-air anomalies are usually computed by the normal gravity γ_I of the international gravity formula and they shall be denoted by Δg_F. These anomalies Δg_F cannot be used in Eq. (8), as we introduce instead of the international ellipsoid an ellipsoid with constants obtained from observations of satellites. If we assume that by introducing the new ellipsoidal parameters, the normal heights of the earth are not changed, we get the anomalies Δg in Eq. (8) with Eq. (13) from

$$\Delta g = \Delta g_F + \gamma_1 - \gamma \, . \tag{14}$$

As Eq. (8), the generalized Stokes' formula, does not contain spherical harmonics of degree zero, constant terms in Eq. (14) may be neglected.

To get the potential P_D of the disturbing force whose components K_1, K_2, K_3 enter Eqs. (3), we subtract the central field kM/r of the earth from Eq. (10) and find

$$P_D = V + T \tag{15}$$

with

$$V = -\frac{kM}{r}\left[J_2\left(\frac{a_e}{r}\right)^2 P_2\,(\sin\varphi) + J_4\left(\frac{a_e}{r}\right)^4 P_4\,(\sin\varphi)\right] .$$

Differentiating P_D we obtain, see Fig. 1,

$$\begin{aligned} K_1 &= \frac{1}{r}\frac{\partial V}{\partial\varphi}\sin\chi + \frac{1}{r}\frac{\partial T}{\partial\varphi}\sin\chi - \frac{1}{r\cos\varphi}\frac{\partial T}{\partial\lambda}\cos\chi \ , \\ K_2 &= \frac{1}{r}\frac{\partial V}{\partial\varphi}\cos\chi + \frac{1}{r}\frac{\partial T}{\partial\varphi}\cos\chi + \frac{1}{r\cos\varphi}\frac{\partial T}{\partial\lambda}\sin\chi \ , \\ K_3 &= \frac{\partial V}{\partial r} + \frac{\partial T}{\partial r} \ , \end{aligned} \tag{16}$$

where χ is the angle between K_2 and the coordinate line λ = const. It holds

$$\begin{aligned} \sin\chi &= \frac{\cos i}{\cos\varphi} \ , \qquad \cos\chi = \frac{\cos(\omega + v)}{\cos\varphi}\sin i \ , \\ \sin\varphi &= \sin(\omega + v)\sin i \ , \end{aligned} \tag{17}$$

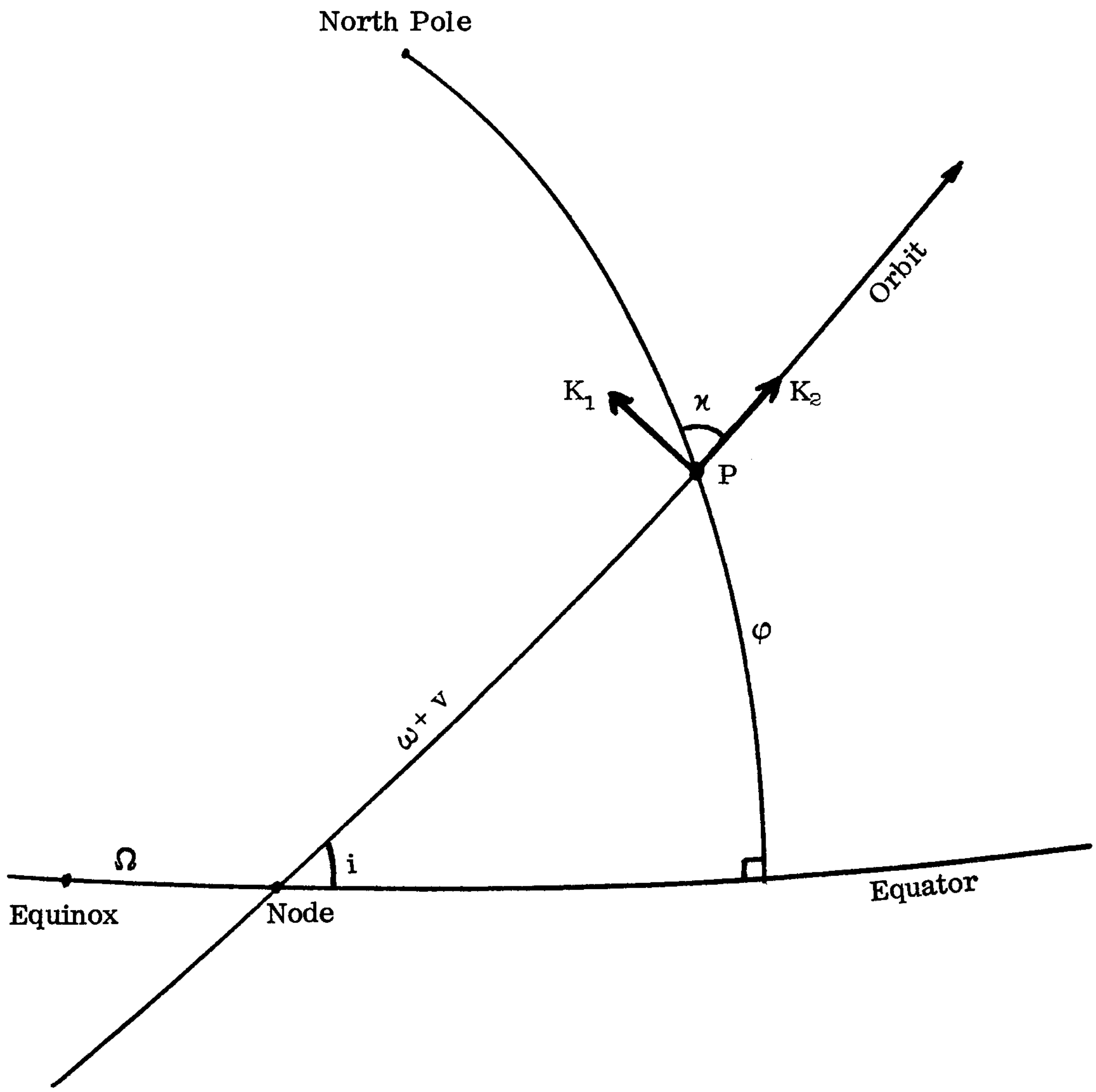

Figure 1

and we find

$$K_1 = -\frac{kM}{r^2} F \cos i + \frac{1}{r}\frac{\partial T}{\partial \varphi}\frac{\cos i}{\cos\varphi} - \frac{1}{r\cos\varphi}\frac{\partial T}{\partial\lambda}\frac{\cos(\omega + v)}{\cos\varphi}\sin i^{(*)},$$

$$K_2 = -\frac{kM}{r^2} F \cos(\omega + v)\sin i + \frac{1}{r}\frac{\partial T}{\partial\varphi}\frac{\cos(\omega + v)}{\cos\varphi}\sin i^{(*)}$$

$$+ \frac{1}{r\cos\varphi}\frac{\partial T}{\partial\lambda}\frac{\cos i}{\cos\varphi}, \tag{18}$$

$$K_3 = \frac{kM}{r^2}\left[\frac{3}{2}J_2\left(\frac{a_e}{r}\right)^2 (3\sin^2(\omega + v)\sin^2 i - 1) + \frac{5}{8}J_4\left(\frac{a_e}{r}\right)^4\right.$$

$$\left.(35\sin^4(\omega + v)\sin^4 i - 30\sin^2(\omega + v)\sin^2 i + 3)\right] + \frac{\partial T}{\partial r},$$

with

$$\cos\varphi = \sqrt{1 - \sin^2(\omega + v)\sin^2 i},$$

$$F = 3J_2\left(\frac{a_e}{r}\right)^2 \sin(\omega + v)\sin i + J_4\left(\frac{a_e}{r}\right)^4\left(\frac{35}{2}\sin^3(\omega + v)\sin^3 i\right.$$

$$\left.- \frac{15}{2}\sin(\omega + v)\sin i\right).$$

The derivatives of the disturbing potential T are given by Heiskanen and Moritz (1967) p. 234,

$$\frac{1}{r}\frac{\partial T}{\partial\varphi} = \frac{1}{4\pi}\iint_q \Delta g \frac{\partial S(P,Q)}{\partial\varphi}\, dq,$$

$$\frac{1}{r\cos\varphi}\frac{\partial T}{\partial\lambda} = \frac{1}{4\pi}\iint_q \Delta g \frac{\partial S(P,Q)}{\cos\varphi\,\partial\lambda}\, dq, \tag{19}$$

$$\frac{\partial T}{\partial r} = \frac{1}{4\pi}\iint_q \Delta g \frac{\partial}{\partial r}[r\, S(P,Q)]\, dq,$$

(*) At the end of the first and second row of Eq. (18) in Koch (1967a) p. 105 sin i instead of cos i must be written.

with

$$\left.\begin{aligned}\frac{\partial S(P,Q)}{\partial\varphi}\\ \frac{\partial S(P,Q)}{\cos\varphi\,\partial\lambda}\end{aligned}\right\} = \tau^2 \begin{Bmatrix}\sin\psi\cos A\\ \sin\psi\sin A\end{Bmatrix}\left[\frac{2}{D^3}+\frac{6}{D}+3\,\frac{D-1+\tau\cos\psi}{D\sin^2\psi} - 8 - 3\,\ln\frac{D+1-\tau\cos\psi}{2}\right],$$

$$\frac{\partial}{\partial r}\,[r\,S(P,Q)] = -2\,S(P,Q) - \tau^2\left[\frac{1-\tau^2}{D^3} - 1 - 3\,\tau\cos\psi\right],$$

$$\sin\psi\cos A = \cos\varphi\sin\varphi_Q - \sin\varphi\cos\varphi_Q\cos(\lambda_Q - \lambda),$$

$$\sin\psi\sin A = \cos\varphi_Q\sin(\lambda_Q - \lambda).$$

A is the azimuth of the variable point Q measured on the sphere q in the point of intersection of the position vector of the satellite and the sphere q.

When computing the latitude λ of the satellite, the rotation of the earth must be considered. If λ_N means the latitude of the ascending node at the time, $t = t_o$, and α the right ascension of the satellite, we obtain

$$\lambda = \lambda_N + \alpha - \Omega - w\,(t - t_o) \tag{20}$$

with

$$\tan(\alpha - \Omega) = \tan(\omega + v)\cos i .$$

Inserting Eqs. (16) to (20) into the perturbed equations of motion (3), we get the derivatives of the orbital elements with respect to the time as functions of the gravity anomalies. The anomalies enter the differential equations (3) by Stokes' formula, thus, a double integral must be computed. Integrating Eqs. (3) a third integral is added. This threefold integration cannot be solved analytically, a solution is only possible under simplifying assumptions, for instance, if Stokes' function is replaced by Stokes' series, as Arnold (1967) has shown. But to avoid errors due to the simplifications and to avoid complicated formulas, as Stokes' series doesn't converge very well, a numerical integration of the differential equations (3) is chosen here.

Terms of the order of the cube of the flattening are neglected when computing the normal potential U from Eq. (10). The gained accuracy is sufficient for the examples computed in the following chapters. In case of a precise orbit determination, however, the error of this simplification has to be investigated as well as the error caused by Stokes' formula, where the integration is extended over the surface of the sphere instead of the surface of the ellipsoid.

The integration of Eqs. (3) gives the orbital elements and to obtain the position of the satellite in the equatorial coordinate system (x, y, z) of astronomy, introduced for Eqs. (2), we use the relations

$$\begin{aligned} x &= r\,(\cos\Omega\cos(\omega+v) - \sin\Omega\sin(\omega+v)\cos i)\ , \\ y &= r\,(\sin\Omega\cos(\omega+v) + \cos\Omega\sin(\omega+v)\cos i)\ , \qquad (21) \\ z &= r\sin(\omega+v)\sin i\ . \end{aligned}$$

2.3 Size of Surface Elements

Many methods, from which a suitable one must be chosen, are known in the literature to integrate the differential equations (3) numerically, cf. Conte (1962), Fehlberg (1966). The usual way to solve Stokes' formula is the division of the earth's surface into surface elements, in which the gravity anomaly and

Stokes' function of the midpoint of the surface element are regarded as constants. This method shall also be used here. With the division of the earth's surface into elements the question of the size of the blocks arises. Of course, the size of the surface elements depends on the desired accuracy of the orbit determination.

To get an idea of the accuracy gained by different sizes of surface elements, examples are computed. It is assumed, that the gravity field of the earth can be represented by the 2592 mean anomalies Δg_F for $5^\circ \times 5^\circ$ blocks, referred to the international ellipsoid and determined by Kivioja (1963) pp. 111-116, from gravity measurements and the hypothesis of isostasy. For the level ellipsoid the following constants are adopted

$$\begin{aligned} kM &= 398603 \times 10^9 \ m^3 \ sec^{-2} \ , \\ J_2 &= 0.0010827 \ , \\ a_e &= 6378160 \ m \ , \\ w &= 0.000072921151 \ sec^{-1} \ , \end{aligned} \tag{22}$$

which except of w were recommended for the reference ellipsoid by the IUGG in 1967 in Lucerne. Although new and probably better constants are available now, Veis (1967), the constants of Eqs. (22) are sufficient for the computation example.

Neglecting constant values in Eq. (14) we get with the international gravity formula and with Eqs. (13) and (22)

$$\Delta g = \Delta g_F - 13.374 \sin^2 B - 0.074 \sin^2 2B \ . \tag{23}$$

It has to be noted that the value of kM in Eqs. (22) includes the mass of the atmosphere, since it is determined by satellites. This fact, however, is without influence on Eq. (23).

By applying Eq. (23) to the midpoints of the surface elements a new set of anomalies Δg for $5° \times 5°$ blocks are computed from Kivioja's anomalies Δg_F . By averaging the results mean anomalies Δg for $10° \times 10°$, $15° \times 15°$, and $20° \times 20°$ blocks are also obtained. The anomalies Δg_F of Kivioja are given in units of 1 mgal. In order to lose no accuracy by applying Eq. (23) and averaging, the anomalies Δg enter the computations in units of 0.01 mgal.

By means of the anomalies Δg the perturbations of the orbits of three satellites with different orbital elements are computed. The satellites are Explorer 9, Satellite 1963 26A and Explorer 19 with the orbital elements at the epoch $t = t_o$

Explorer 9

$$\Omega = 203.^\circ 6802, \quad i = 38.^\circ 828, \quad \omega = 265.^\circ 8568,$$
$$a = 7967500 \text{ m}, \quad e = 0.1062, \quad M = 110.^\circ 1682 , \qquad (24)$$

Satellite 1963 26A

$$\Omega = 140.^\circ 71, \quad i = 49.^\circ 736, \quad \omega = 107.^\circ 94,$$
$$a = 7234481 \text{ m}, \quad e = 0.06166, \quad M = 121.^\circ 032 , \qquad (25)$$

Explorer 19

$$\Omega = 83.^\circ 33, \quad i = 78.^\circ 69, \quad \omega = 160.^\circ 9,$$
$$a = 7854607 \text{ m}, \quad e = 0.1084, \quad M = 104.^\circ 04. \qquad (26)$$

The following approximate altitudes above the earth of the perigee and apogee result from these orbital elements: Explorer 9, 750 km and 2440 km; Satellite 1963 26A, 420 km and 1310 km; Explorer 19, 630 km and 2340 km. It is $\lambda_N = 2.3$ in Eq. (20) at $t = t_o$ for the three satellites. The integration is extended over the time, which corresponds the time u of one, undisturbed revolution

$$u = 2\pi \sqrt{\frac{a^3}{kM}} \quad . \tag{27}$$

At the epochs $t_o + 0.25\,u$, $t_o + 0.6\,u$, $t_o + u$ the positions of the three satellites are computed from Eqs.(21) with using mean anomalies Δg for $5° \times 5°$, $10° \times 10°$, $15° \times 15°$, and $20° \times 20°$ blocks. The results are given in Table 1 for Explorer 9, Table 2 for Satellite 1963 26A and in Table 3 for Explorer 19. The last column of each table contains the distance between the position obtained by using $5° \times 5°$ blocks and the positions computed with the remaining surface elements.

The difference between these positions is a function of the error caused by averaging the anomalies, i.e., the error of representation, and a function of the error of integration, as Stokes' function of the midpoint of each surface element is regarded as constant in each element. To reduce the error of integration, when using $10° \times 10°$ blocks instead of $5° \times 5°$ surface elements, the $10° \times 10°$ blocks are divided into four $5° \times 5°$ blocks in whose midpoints Stokes' function is regarded as constant. Thus, the $5° \times 5°$ surface elements enter the computations with the mean values of the $10° \times 10°$ blocks. The results of these computations are given in Tables 1 to 3 in the rows indicated

Explorer 9

Revolutions	Surface Elements	Coordinates x [km]	y [km]	z [km]	Distance [km]
0.25	5° x 5°	4392.771	-5363.015	5366.638	-
	5° x 5° with 10° x 10° m.v.	.771	.015	.639	0.001
	10° x 10°	.771	.016	.638	.001
	15° x 15°	.771	.011	.638	.004
	10° x 10° with 20° x 20° m.v.	.771	.014	.639	.001
	20° x 20°	.771	.016	.638	.001
0.6	5° x 5°	3141.049	5731.922	-3219.198	-
	5° x 5° with 10° x 10° m.v.	.054	.923	.196	0.005
	10° x 10°	.053	.921	.197	.004
	15° x 15°	.042	.926	.200	.008
	10° x 10° with 20° x 20° m.v.	.047	.927	.196	.006
	20° x 20°	.046	.920	.201	.005
1.0	5° x 5°	-5597.645	-5687.259	2413.801	-
	5° x 5° with 10° x 10° m.v.	.633	.263	.802	0.013
	10° x 10°	.627	.265	.806	.020
	15° x 15°	.673	.255	.769	.043
	10° x 10° with 20° x 20° m.v.	.636	.262	.793	.012
	20° x 20°	.657	.253	.796	.014

Table 1

Satellite 1963 26A

Revolutions	Surface Elements	Coordinates x [km]	y [km]	z [km]	Distance [km]
0.25	5° x 5°	-2021.944	6122.514	-4071.794	-
	5° x 5° with 10° x 10° m.v.	.944	.515	.792	0.002
	10° x 10°	.943	.515	.792	.002
	15° x 15°	.949	.510	.792	.007
	10° x 10° with 20° x 20° m.v.	.945	.513	.781	.013
	20° x 20°	.946	.509	.778	.017
0.6	5° x 5°	-3507.649	-2774.562	5158.903	-
	5° x 5° with 10° x 10° m.v.	.652	.560	.904	0.004
	10° x 10°	.652	.560	.905	.004
	15° x 15°	.610	.587	.895	.047
	10° x 10° with 20° x 20° m.v.	.604	.591	.905	.054
	20° x 20°	.577	.610	.907	.087
1.0	5° x 5°	5834.033	314.627	-4675.826	-
	5° x 5° with 10° x 10° m.v.	.042	.619	.817	0.015
	10° x 10°	.039	.618	.815	.015
	15° x 15°	.000	.694	.884	.095
	10° x 10° with 20° x 20° m.v.	.037	.652	.848	.034
	20° x 20°	3.966	.755	.878	.153

Table 2

Explorer 19

Revolutions	Surface Elements	Coordinates x [km]	y [km]	z [km]	Distance [km]
0.25	5° x 5°	1231.415	8531.352	-1150.433	-
	5° x 5° with 10° x 10° m.v.	.415	.351	.432	0.001
	10° x 10°	.415	.352	.433	.000
	15° x 15°	.417	.352	.432	.002
	10° x 10° with 20° x 20° m.v.	.418	.355	.437	.006
	20° x 20°	.420	.359	.438	.010
0.6	5° x 5°	-1628.362	-2490.817	6630.602	-
	5° x 5° with 10° x 10° m.v.	.362	.821	.599	0.005
	10° x 10°	.363	.816	.603	.002
	15° x 15°	.366	.812	.605	.007
	10° x 10° with 20° x 20° m.v.	.364	.791	.620	.032
	20° x 20°	.367	.766	.639	.063
1.0	5° x 5°	1672.452	625.866	-7937.063	-
	5° x 5° with 10° x 10° m.v.	.462	.873	.064	0.012
	10° x 10°	.463	.854	.066	.017
	15° x 15°	.455	.916	.044	.054
	10° x 10° with 20° x 20° m.v.	.449	.842	.068	.025
	20° x 20°	.417	.805	.064	.070

Table 3

by "5° x 5° with 10° x 10° m.v." The same computations are performed to investigate the error of integration if mean anomalies for 20° x 20° surface elements instead of 10° x 10° blocks are introduced.

The method used for the numerical integration of the differential Eqs. (3) is the method of Adam reformulated and developed for the application with digital computers by Nordsieck (1962). The chosen formulas work with an approximating polynomial of degree five. Such values are selected for the automatic step-size control that an accuracy of 1 m is obtained in the position of the satellite after integrating over one revolution.

The results in Table 1 to 3 confirm the fact that the differences in the positions obtained by mean anomalies of different block sizes are caused by the error of representation and integration. By dividing the surface elements into four blocks, in whose midpoints Stokes' function is regarded as constant, the error of integration can be reduced. As it can be seen from the results, the influence of the integration error while using 10° x 10° instead of 5° x 5° blocks is smaller than the integration error when introducing 20° x 20° instead of 10° x 10° surface elements.

As the orbit perturbations of three satellites only are computed, it is difficult to predict exactly which size of surface elements is necessary to get a certain accuracy of a certain satellite orbit after integrating over a certain time. Moreover, all computations are compared with positions obtained from mean anomalies for 5° x 5° blocks. These positions are also disturbed by the error of integration and representation. However, if we assume that the gravity field, caused by Kivioja's anomalies for 5° x 5° blocks, has some resemblance with the real gravity field, we may draw the conclusion from the results, that mean anomalies of surface elements smaller than 20° x 20° blocks must be used, to get an accuracy better than 10 m of the orbit determination of satellites with perigees in low altitudes, after integrating over more than a quarter of a revolution.

2.4 Check of Computations

To check the formulas given in Chapter 2.1 and 2.2 and the computer program by which the results of Chapter 2.3 are obtained, the anomalies of Kivioja are replaced by anomalies of an earth model. The disturbing potential of this model is caused by point masses. Thus, to check the results of the integration of Eqs. (3) we are able to use Eqs. (2).

The starting point is Eq. (7)

$$W = U + T ,$$

which has to be differentiated with respect to x, y, and z to integrate Eqs. (2). With

$$\sin\varphi = \frac{z}{r} = \frac{z}{\sqrt{x^2 + y^2 + z^2}} \tag{28}$$

we obtain from Eq. (10)

$$\frac{\partial U}{\partial x} = -kM\left[\frac{x}{r^3} + J_2\, a_e^2\left(-\frac{15\,x\,z^2}{2\,r^7} + \frac{3x}{2r^5}\right) + J_4\, a_e^4\left(-\frac{315\,x\,z^4}{8\,r^{11}} + \frac{105\,x\,z^2}{4\,r^9} - \frac{15\,x}{8\,r^7}\right)\right], \tag{29}$$

$$\frac{\partial U}{\partial y} = -kM\left[\frac{y}{r^3} + J_2 a_e^2\left(-\frac{15yz^2}{2r^7} + \frac{3y}{2r^5}\right) + J_4 a_e^4\left(-\frac{315yz^4}{8r^{11}} + \frac{105yz^2}{4r^9} - \frac{15y}{8r^7}\right)\right], \tag{30}$$

$$\frac{\partial U}{\partial z} = -kM\left[\frac{z}{r^3} + J_2 a_e^2\left(-\frac{15z^3}{2r^7} + \frac{9z}{2r^5}\right) + J_4 a_e^4\left(-\frac{315z^5}{8r^{11}} + \frac{175z^3}{4r^9} - \frac{75z}{8r^7}\right)\right]. \tag{31}$$

Three point masses m_1, m_2 and m_3 distributed on the line of intersection of the meridional plane of Greenwich and the equatorial plane with the distances $\bar{x}_1$, $\bar{x}_2$ and $\bar{x}_3$ from the origin of the coordinate system (x, y, z) are causing the disturbing potential T. To fulfill the prerequisites of Stokes' formula (8) the sum of the masses must be equal to zero and their mass center must be identical with the origin of the coordinate system (x, y, z). We get

$$T = T_1 + T_2 + T_3 \tag{32}$$

with

$$T_i = \frac{f m_i}{r_i} = \frac{f m_i}{\sqrt{(x - x_i)^2 + (y - y_i)^2 + z^2}}, \tag{33}$$

$$x_i = \cos(\Omega - \lambda_N + w(t - t_o))\,\bar{x}_i,$$

$$y_i = \sin(\Omega - \lambda_N + w(t - t_o))\,\bar{y}_i,$$

$$i = 1, 2, 3.$$

The derivatives of T_i with respect to x, y, and z are given by

$$\frac{\partial T_i}{\partial x} = -\frac{f m_i (x - x_i)}{r_i^3}, \quad \frac{\partial T_i}{\partial y} = -\frac{f m_i (y - y_i)}{r_i^3}, \quad \frac{\partial T_i}{\partial z} = -\frac{f m_i z}{r_i^3}. \tag{34}$$

At the epoch $t = t_0$ we use the orbital elements (24) of Explorer 9, which has to be transformed into the components of the position vector by Eq. (21) and into the components of the velocity vector by, Kaula (1966) p. 23,

$$\frac{dx}{dt} = \frac{n\,a}{\sqrt{1-e^2}} \, [(-\cos\Omega\cos\omega + \sin\Omega\cos i\sin\omega)\sin v - (\cos\Omega\sin\omega + \sin\Omega\cos i\cos\omega)(e+\cos v)] \, ,$$

$$\frac{dy}{dt} = \frac{n\,a}{\sqrt{1-e^2}} \, [(-\sin\Omega\cos\omega - \cos\Omega\cos i\sin\omega)\sin v - (\sin\Omega\sin\omega - \cos\Omega\cos i\cos\omega)(e+\cos v)] \, , \tag{35}$$

$$\frac{dz}{dt} = \frac{n\,a}{\sqrt{1-e^2}} \, (-\sin i\sin\omega\sin v + \sin i\cos\omega\,(e+\cos v)] \; .$$

Now the equations of motion (2) can be solved by numerical integration and we get with

$$fm_1 = 40 \times 10^9 \text{ m}^3 \text{ sec}^{-2}, \quad fm_2 = -50 \times 10^9 \text{ m}^3 \text{ sec}^{-2},$$

$$fm_3 = 10 \times 10^9 \text{ m}^3 \text{ sec}^{-2},$$

$$\bar{x}_1 = 2 \text{ km}, \quad \bar{x}_2 = 1 \text{ km}, \quad \bar{x}_3 = -3\text{km}, \quad \lambda_N = 2.3,$$

the position of Explorer 9 at the epoch $t = t_0 + u$ with u from Eq. (27)

$$x = -5597476.5 \text{ m}, \quad y = -5687464.7 \text{ m}, \quad z = 2413966.5 \text{ m}. \tag{36}$$

The gravity anomalies at the surface of the model earth are given by the well-known relation

$$\Delta g = -\frac{\partial T}{\partial R} - \frac{2T}{R} \; . \tag{37}$$

It follows from Eq. (33)

$$\frac{\partial T_i}{\partial R} = -\frac{fm_i\,(R - \bar{x}_i\cos\varphi\cos\lambda)}{\sqrt{R^2 + \bar{x}_i^{\,2} - 2R\bar{x}_i\cos\varphi\cos\lambda}^{\,3}} \; , \quad i = 1,\ 2,\ 3 \; . \tag{38}$$

Computing the anomalies Δg by Eqs. (37) and (38) in the midpoints of $5^\circ \times 5^\circ$, $10^\circ \times 10^\circ$, and $15^\circ \times 15^\circ$ surface elements and regarding these anomalies as mean anomalies, we obtain the positions of Explorer 9 after integrating the perturbed equations of motion (3) over the time u of Eq. (27)

$$x = -5597475.6, \quad y = -5687466.0, \quad z = 2413966.4, \tag{39}$$

if $5^\circ \times 5^\circ$ blocks are used,

$$x = -5597474.5, \quad y = -5687466.4, \quad z = 2413967.6, \tag{40}$$

with $10^\circ \times 10^\circ$ surface elements and finally with $15^\circ \times 15^\circ$ blocks

$$x = -5597473.1, \quad y = -5687467.3, \quad z = 2413967.2\,. \tag{41}$$

The distances between the position (36) and the positions (39) to (41) are

1.6 m, 2.8 m, and 4.3 m,

respectively. Using smaller block sizes than $5^\circ \times 5^\circ$ areas, we may assume that the distances between the positions become less than 1 m. It proves that the satellite positions after integrating over one revolution are obtained with an accuracy of 1 m.

The distances between the satellite positions (39), (40), and (41) are smaller than the distances in the Tables 1 to 3 obtained by Kivioja's anomalies for different block sizes. The reason is obvious. As the anomaly field caused by the masses m_1, m_2, and m_3 is smoother than the anomaly field of Kivioja, the differences must be smaller.

3. Differential Corrections of Orbital Elements and of Station Coordinates

3.1 Method of Numerical Determination of the Derivatives

If we assume that the gravity field of the earth is well known by gravity measurements over the whole world, the six orbital elements used as constants of integration and the coordinates of the tracking stations, whose observations determine the orbit, must be given with high accuracy if we want a well defined orbit. An accuracy of the satellite position better than 10 m requires the same accuracy in the coordinates of the tracking stations and a comparable accuracy in the orbital elements, that means the knowledge of seven significant numbers in each orbital element, cf. Eqs. (21). Generally, these requirements will not be fulfilled, but approximate values of the orbital elements and the station coordinate are always known. By means of these approximate values we have to determine more accurate values.

Let us regard the observations O of the satellite in the tracking stations. O may represent the right ascension α or the declination δ of the satellite, the distance of the satellite from the tracking stations, or the results of Doppler measurements. The observation O_t at the time t is a function of the satellite position (31) and, therefore, a function of the six orbital elements E_{t_0} at the time t_0 of the beginning of the integration, i.e., E_{t_0} are the constants of integration. O_t is further a function of the gravity anomalies Δg, of the station coordinates X and of the time difference $t - t_0$, thus,

$$O_t = O_t\ (E_{t_0},\ \Delta g,\ X,\ t - t_0\ )\ . \tag{42}$$

The exact values of E_{to} and X are unknown, but approximate values are given by which the approximate observation $\bar{O}_t$ at the time t is computed. Using the Taylor series we get

$$O_t - \bar{O}_t = \frac{\partial \bar{O}_t}{\partial E_{to}} \, dE_{to} + \frac{\partial \bar{O}_t}{\partial X} \, dX \; , \tag{43}$$

if the approximate values are sufficiently accurate to restrict ourselves to the linear terms of the Taylor series. dE_{to} and dX must be added to the approximate values of the orbital elements and station coordinates to get the sought values.

To obtain the approximate observation $\bar{O}_t$, the perturbed equations of motion (3) must be solved. As proposed in Chapter 2.2, the solution is given by numerical integration. Thus, the derivatives $\partial\bar{O}_t/\partial E_{to}$ and $\partial\bar{O}_t/\partial X$ in Eq. (43) must be computed by numerical integration also. This can easily be done by assuming small changes in the approximate values denoted by bars and by computing the effect of the changes on $\bar{O}_t$, for instance

$$\frac{\partial \bar{O}_t}{\partial \Omega_{to}} = \frac{\bar{O}_t\,(\bar{\Omega}_{to} + \Delta\Omega,\ \bar{i}_{to},\ \bar{\omega}_{to},\ \bar{a}_{to},\ \bar{e}_{to},\ \bar{M}_{to},\ \bar{X})}{\Delta\Omega} - \frac{\bar{O}_t\,(\bar{\Omega}_{to},\ \bar{i}_{to},\ \bar{\omega}_{to},\ \bar{a}_{to},\ \bar{e}_{to},\ \bar{M}_{to},\ \bar{X})}{\Delta\Omega} \; . \tag{44}$$

Inserting these values into Eq. (43), we get a system of linear equations from which the unknowns dE_{to} and dX can be computed. In order to obtain accurate results, the number of observations should be greater than the number of unknowns, so that an adjustment can be performed.

Equation (43) holds in case of given mean gravity anomalies for suitable block sizes. If these anomalies are unknown, we may write

$$O_t - \bar{O}_t = \frac{\partial \bar{O}_t}{\partial E_{to}} \, dE_{to} + \frac{\partial \bar{O}_t}{\partial \Delta g} \, d\Delta g + \frac{\partial \bar{O}_t}{\partial X} \, dX \ , \tag{45}$$

where $d\Delta g$ must be added to the approximate values of the gravity anomalies to get the sought values. When computing unknown gravity anomalies from Eq. (45), the effect of the values dE_{to}, $d\Delta g$, and dX on the difference $O_t - \bar{O}_t$ must be of the same, small magnitude because higher order terms in the Taylor series are neglected. If, for instance, better approximations are known for the gravity anomalies and the station coordinates than for the orbital elements, better approximations have to be computed for the orbital elements in a first step by

$$O_t - \bar{O}_t = \frac{\partial \bar{O}_t}{\partial E_{to}} \, dE_{to} \ . \tag{46}$$

This holds as well, if only orbital elements and station coordinates are determined by Eq. (43).

Equation (46) formulates the differential correction of orbits. This orbit determination is well known in celestial mechanics, cf. Dubyago (1961). The method of the numerical determination of the derivatives, which is used here, is described for instance by Escobal (1965) p. 325. As it is shown, this method can be extended to determine unknown station coordinates and, if necessary, unknown gravity anomalies.

3.2 Numerical Example

To demonstrate the method developed in Chapter 3.1, an example is computed for the differential corrections of orbital elements and station coordinates by Eq. (43).

We assume that the gravity field of the earth can be represented by mean gravity anomalies for 20° x 20° blocks computed from Kivioja's anomalies by averaging the anomalies and applying Eq. (23). The sought values of the orbital elements used as starting values of the integration at the epoch $t = t_o$ are the orbital elements of Explorer 9 given by Eqs. (24). Integrating Eqs. (3) we get the exact values of the right ascension α and the declination δ of Explorer 9 with Eq. (21) from

$$\tan\alpha = \frac{y - y_s}{x - x_s} \ ,$$

$$\tan\delta = \frac{z - z_s}{\sqrt{(x - x_s)^2 + (y - y_s)^2}} \ , \qquad (47)$$

where x_s, y_s, and z_s mean the station coordinates in the equatorial coordinate system (x, y, z) of astronomy. Four different sets of α and δ are computed by Eqs. (47) at the epochs of about

$$t_1 = t_o + 1770 \text{ sec.} \ ,$$

$$t_2 = t_o + 3540 \text{ sec.} \ ,$$

$$t_3 = t_o + 5310 \text{ sec.} \ , \qquad (48)$$

$$t_4 = t_o + 7080 \text{ sec.} \ .$$

These four values for α and δ are regarded as observed values.

The approximate values of the orbital elements of Explorer 9 are:

$$\bar{\Omega} = 203^\circ.6805, \quad \bar{i} = 38^\circ.82805, \quad \bar{\omega} = 265^\circ.8565$$
$$\bar{a} = 7967505 \text{ m}, \quad \bar{e} = 0.1062003, \quad \bar{M} = 110^\circ.1681 . \tag{49}$$

The distance between the positions computed by Eqs. (21) from the values (24) and (49) is 29 m. In the approximate values of the station coordinates (x_s, y_s, z_s) an error of 8 m is assumed in all x_s and an error of 12m in all y_s.

The eight unknowns, i.e., the six unknown orbital elements and the two unknowns in the station coordinates, are now determined with Eq. (43) by comparing the four observations of α and the four observations of δ with the approximate values $\bar{\alpha}$ and $\bar{\delta}$ computed from the approximate orbital elements and station coordinates. The partial derivations in Eq. (43) are obtained by Eq. (44) where the adopted changes in the orbital elements and station coordinates are less than two units of the sixth digit of their values. We obtain

$$\Omega = 203^\circ.6802, \quad i = 38^\circ.82800, \quad \omega = 265^\circ.8567,$$
$$a = 7967500 \text{ m}, \quad e = 0.1062000, \quad M = 110^\circ.1683 . \tag{50}$$

The distance between the positions obtained from the exact values (24) and the computed values (50) is 1.5 m. The errors in the station coordinates after applying Eq. (43) are 2.0 m in x_s and 0.9 in y_s. These results show that in using Eq. (43) the joint determination of orbital elements and station coordinates is possible.

4. Summary and Conclusions

By means of Stokes' formula the orbit perturbations of a satellite can be represented as functions of the gravity anomalies. To compute these perturbations it is proposed to solve the perturbed equations of motion numerically and to apply the usual integration technique for Stokes' formula by dividing the earth's surface into elements in which the gravity anomaly and Stokes' function are regarded as constants. It is shown by numerical examples, that under the made assumptions mean anomalies for surface elements smaller than $20° \times 20°$ blocks are necessary for satellites with perigees in low altitude, if it is asked for an orbit integration extended over more than a quarter of a revolution with an accuracy better than 10 m.

If the gravity field of the earth is sufficiently known from gravity measurements, we need for an exact orbit computation accurate orbital elements, used as constants of integration, and exact coordinates of the tracking stations. A way is given here to obtain these values by using the method of differential corrections of the orbital elements and station coordinates. In case of unknown gravity anomalies, this method can also be used to determine unknown anomalies.

The developed methods of computing orbital perturbations by gravity anomalies and of determining differential corrections of orbital elements and station coordinates use the numerical integration of the perturbed equations of motion. If it is planned to apply these methods to a great amount of computations, integration techniques with accuracies of higher order than Adam's method, which is used here, have to be chosen in order to save computing time.

Another possibility to accelerate the computations is to avoid the series (6) by introducing on the left hand of the last equation of (3) the true anomaly v instead of the mean anomaly M. Furthermore, Arnold (1967) proposed, as already mentioned in Chapter 2.2, to solve the differential equations (3) analytically. An analytical solution of Eqs. (3) could be helpful for easier determination of the partial derivatives in Eqs. (43) and (45).

The gravity anomalies are referred to a level ellipsoid whereas computing the disturbing potential by Stokes' formula the integration is extended over the surface of the sphere with the mean radius of the earth. For accurate orbit determinations the error of this simplification has to be investigated as well as the error produced by neglecting terms of the order of the cube of the flattening in the potential of the level ellipsoid. Moreover, the influence of an error in the product of the earth's mass and the gravitational constant must be regarded and finally the effect of the attraction of the sun and the moon, the air drag and the lunisolar radiation pressure should be investigated although the orbit computations are restricted to short arcs.

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13. ABSTRACT

In the first part of this report the orbit perturbations of an artificial earth satellite are given as functions of the gravity anomalies by means of Stokes' formula. To compute the perturbations, the perturbed equations of motion are integrated numerically and the usual integration technique is applied for Stokes' formula by dividing the surface of the earth into surface elements in which the gravity anomaly and Stokes' function are regarded as constants. In a numerical example orbit perturbations are computed for three satellites with different orbital elements from mean anomalies of different sizes of surface elements to determine the influence of the sizes of surface elements on the orbit perturbations.

In the second part it is assumed that the gravity field of the earth is well known from gravity anomalies. To obtain a precise orbit of a satellite by these data, a method using differential corrections is given to compute more accurate orbital elements and more accurate coordinates of the tracking stations, whose observations determine the orbit.

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14. KEY WORDS	LINK A ROLE	LINK A WT	LINK B ROLE	LINK B WT	LINK C ROLE	LINK C WT
artificial earth satellite						
perturbed equation of motion						
numerical integration						
Stokes' formula						
gravity anomaly						
normal gravity						
differential correction						
station coordinate						

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