

1991 – THE YEAR OF THE PALINDROME

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The year 1991 is a palindromic year. It is the first palindromic year since 1881 and the only palindromic year of this century. Therefore it seems fitting that we should celebrate this year with our intermediate students by doing some activities with numbers that are palindromes. A palindromic number is one which reads the same backward as forward. Thus 55, 121, 777, 2442, and 15351 are palindromes.

If a number is not a palindrome already we may be able to transform it into a palindrome. Consider the number 57 which is not a palindrome. Reverse the digits and add the two numbers: $57 + 75 = 132$. The sum is still not a palindrome so repeat the process: $132 + 231 = 363$ which is a palindrome. This process will work for most numbers. It has been conjectured that the process will work for all numbers but this has never been proved. There are 249 integers less than 10,000 for which no palindrome can be found in 100 steps or less. (McGinty and Eisenberg, 1978).

Some numbers require only one addition to arrive at a palindrome, for example, 12: $12 + 21 = 33$, and some numbers require many more additions. The number 985 requires eight additions before arriving at the 7-digit palindrome, 1,136,311. Two other numbers which are known to require many additions are 89 and 98. They each require twenty-four additions before a palindrome is reached. The 13-digit number 8,813,200,023,188 is the palindrome which results for both 89 and 98. There are eleven integers under 1,000 that require more than 24 steps before a palindrome is reached.

Bennett and Nelson (1985) introduced palindromic decimals. A palindromic decimal must read the same from left to right as from right to left. For example, 56.65 is a palindromic decimal but 3.43 is not. The process of reversing and adding to obtain palindromes will work for decimals also. Using 3.43 we obtain: $3.43 + 34.3 = 37.73$ which is now a palindrome. We noted above that 89 requires 24 steps before the sum is a palindrome. Inserting a decimal point in the number drastically changes this; 8.9 requires only 4 steps to result in the palindromic decimal, 73.37.

Palindromes and the Standards

Activities with palindromes afford opportunity for implementing the National Council of Teachers of Mathematics *Curriculum and Evaluation Standards*. Calculators would be very useful in forming palindromes, especially when the numbers require more than one addition. Students will get a chance to form conjectures and reason about numbers, look for patterns, and see relationships among numbers as they work with palindromes and then communicate their findings to their classmates. All of these aspects of learning mathematics are important goals in the *Standards*.

Students may come up with conjectures similar to the following: if the sum of the digits is ten, with the exception of the number 55, two additions are required to reach a palindrome. Or they may discover some of the following patterns which were offered by Dockweiler (1985):

1. Two-digit numbers in which the sum of the digits is less than 10 will require only one reversal and addition to obtain a palindrome. The resulting palindrome will be a multiple of eleven.
2. Two-digit numbers in which the sum of the digits is more than 10 may require more than one addition but the resulting palindrome will be a multiple of eleven.
3. The palindrome will be a multiple of eleven, if at any point in the process of obtaining a palindrome a sum with an even number of digits is obtained.
4. If the tens digit of a 3-digit number is less than or equal to 4 and the sum of the hundred and unit digits is less than ten, a palindrome results after one addition.

Class results on palindromic numbers can be displayed by making a bar graph on the chalkboard. An easy and quick way to accomplish this is to use post-it notes. Each student or small group of students has a post-it note on which to write their results and they take turns coming to the board to post their results in the correct place on the graph. Each group of students, for example, could be assigned 8 or 9 of the numbers from 1 to 100 and their task would be to determine how many steps are necessary to obtain palindromes from the numbers. Leave out 89 and 98 at first. Keep these two numbers for the ambitious group who finishes their assignment first. The class graph would then show which numbers require only one step, which numbers require two steps, three steps, and so on.

Activities

Following are some palindromic activities you may wish to try with your students:

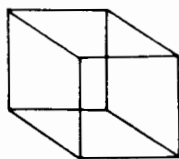
1. When is the next year that will be a palindrome? How old will you be then? When will the second palindromic year occur? Will you live to see it?
2. Is your age a palindrome? If not, make a palindrome from your age. How many additions are necessary?
3. Is your house number a palindrome? If not, try to make a palindrome from your house number. How many additions are necessary? Whose house number requires the most additions? Make a bar graph which shows the class results.
4. Take one page from your city's telephone directory. How many palindromic house numbers do you find? Make a bar graph showing the class results.
5. How many 2-digit palindromes are there? How many 3-digit, 4-digit, etc.? Is there a pattern?
6. Make palindromes from 2-digit numbers which are not already palindromes. How many steps are required? Do the same for 3-digit numbers. Do you see any patterns? Make conjectures about your results.
7. Can you find a number that takes more than five additions to make a palindrome? More than ten additions? Who can find a number that takes more additions than anyone else's number?
8. The following numbers require more than ten steps to form palindromes. See if you can find the palindromes for these numbers: 177, 266, 849, 375, 937, 869, 880.
9. Try inserting decimals into numbers and then converting them to palindromes. Will the number of steps needed to produce a palindromic decimal be less than, equal to, or more than the number of steps required for a whole number? For example, the number 248 requires two additions to make a palindrome. The decimal 2.48 requires only one addition: $2.48 + 84.2 = 86.68$.
10. Write a computer program to find all 3-digit palindromes. Revise the program so that it will find 4-digit palindromes, 5-digit palindromes.

11. Write a computer program which will convert numbers to palindromes. Have the program count the additions and show them on the printout.
12. Make a list of words which are palindromes. Does anyone in the class have a name which is a palindrome? Try to write a sentence which is palindromic. A famous one is: Able was I ere I saw Elba.

References

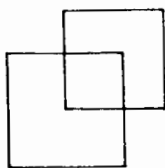
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Magic figures by Alaskan teachers:



Place digits 1-8 on vertices so each face has the same total.

Twyla Mundy



Place digits 0-9 on vertices so each square has the same total.

Judy Jeffrey



Place digits 0-9 on vertices so the five line totals are in arithmetic progression.

Gale Fechik

Answers on Page 30.