

# On the Limits and Possibilities of Causal Explanation with Game Theoretical Models: The Case of Two Party Competition

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Regression-based path- and structural equation-models have two major drawbacks, if they are used for the causal explanation of social phenomena: they are too deterministic and neglect the intentions of the concerned actors as a central driving force of the analysed processes. In order to explain the distribution-effects of two party competition, this article proposes an alternative modelling approach, which is based on the mathematical theory of repeated games. The article explores the limits and possibilities of this approach with regard to the causal explanation of social phenomena and compares the results with the capabilities of the regression approach. It turns out that game theoretical models are especially useful for explaining the non-presence of social phenomena under given causal conditions.

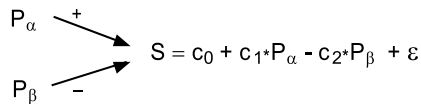
**Key words:** Causal explanation; final causality; determinism; game theory; statistical regression; models of competition for scarce goods.

## 1. INTRODUCTION

Competition is one of the key concepts for understanding the distribution of scarce resources and goods. It is a form of conflict between two or more actors / parties over the sharing of a good, in which they have an antagonistic interest. The competition between industrial firms for market shares, military conflicts between nation states over territories, or party competition for votes in parliamentary elections are just a few examples, which illustrate the importance of the struggle over the distribution of scarce goods (Gabszewicz & Thisse 1999, Barnett et al. 1993).

If quantitative social scientists are asked to explain the outcome of a complex social process like the mentioned two-actor competition, they generally point to regression-based research techniques like *path-* or *structural equation-modelling* (Asher 1991, Kelloway 1998, Pearl 2001: chap. 5, Blalock 1971). The design of such models starts with a network of abstract variables connected by links, which represent the major impacts within the analysed network. If the variables refer to *sequential* points in time, it is generally argued that the network-links stand for *causal* relations, which can be translated into systems of linear regression equations.

**Fig. 1** Example of a causal explanation with a path-model



Legend : S: Share of actor  $\alpha$  in a highly appreciated good.  
 $P_{\alpha}$ ,  $P_{\beta}$ : Power of the actors  $\alpha$  and  $\beta$ .  $\varepsilon$ : Statistical errors.  
 $c_0$ ,  $c_1$ ,  $c_2$ : Regression coefficients.

Fig. 1 gives a very simple example of this design process in order to explain the respective shares  $S$  and  $(1-S)$  of two competing actors  $\alpha$  and  $\beta$  by their power  $P_{\alpha}$  and  $P_{\beta}$ . However, apart from the problematic assumptions of linearity (Abbott 1988), this formalization also illustrates some typical shortcomings of this kind of statistical modelling:

- 1) Path- and structural equation-models are *deterministic*: any deviance of the model from observational data is considered to be due to measurement- or sampling-errors  $\varepsilon$ . Thus, there is no room for *free will* in terms of a choice between alternative options, typically available to social actors (Earman 1986: chap. 12).
- 2) Path- and structural equation-models are mainly driven by the *past* and not by the *intentions* of social actors about their future (Stout 1996: chap. 2–3; Schueler 2003: chap. 2–3). Contrary to the causal explanation of physical phenomena, plans about the future are highly important for social phenomena: the essence of social competition is e.g. the plan to outperform one or several rivals.

**Tab. 1** The general structure of a game with two actors, 2x2 strategies, and payoffs  $a_{ij}$  and  $b_{ij}$

		Actor $\alpha$ :	
		Strategy 1:	Strategy 2:
Actor $\beta$ :	Strategy 3:	$b_{3,1} ; a_{3,1}$	$b_{3,2} ; a_{3,2}$
	Strategy 4:	$b_{4,1} ; a_{4,1}$	$b_{4,2} ; a_{4,2}$

Legend:  $a_{ij}$  = Payoff of actor  $\alpha$  from own strategy  $j$  and strategy  $i$ , played by actor  $\beta$ .  
 $b_{ij}$  = Payoff of actor  $\beta$  from own strategy  $i$  and strategy  $j$ , played by actor  $\alpha$ .

As an alternative to the mentioned type of deterministic models with an over-emphasis on the past, Von Neumann and Morgenstern (1944) developed more than 60 years ago the *mathematical theory of games* (Osborne 2004), where two actors  $\alpha$  and  $\beta$  are assumed to play each against the other. Each of the actors has at least two strategies as alternative options for choice. The behaviour of the actors is insofar driven by *intentions*, as each party attempts to get the highest possible returns from the game, by choosing an appropriate strategy. This choice is strongly influenced by the associated payoff-matrix (see Tab. 1), which shows for each pair of strategies chosen by the two actors the respective returns. Hence, it seems that game theory is able to overcome the mentioned problems of classical path- and structural equation modelling. Consequently, this paper uses game theory in order to demonstrate that it is possible to explain a rather universal social phenomenon like competition by an exemplary causal model, which is not deterministic and not only driven by the past. The case of social competition will however also be useful for a more general discussion about the *explanandum* in social science and the *limits of explainability* of social phenomena.

## 2. COMPETITION AS A REPEATED GAME

Due to the formal structure of mathematical game theory, we are going to analyse a rather simplified type of competition with only two actors  $\alpha$  and  $\beta$ . They are assumed to have different amounts of social power  $P_\alpha$  and  $P_\beta$  in order to increase their respective shares  $S$  and  $1-S$  in a highly appreciated good  $G$ . Thus, between the two parties there is a potential conflict: actor  $\alpha$  uses its power  $P_\alpha$  in order to increase its current share  $S$ . Similarly, actor  $\beta$  makes use of its power  $P_\beta$ , in order to increase its own complementary share  $1-S$  in the good  $G$ , by striving for a part of the share  $S$  of its competitor  $\alpha$ . In an earlier empirical study (Mueller 2006), the author has applied these ideas to the struggle about the distribution of the gross domestic product GDP and the associated personal income distribution. The actors

$\alpha$  and  $\beta$  were in this study the employers on the one hand and the workers and employees on the other.

**Tab. 2** The impact of the strategies selected by the two actors  $\alpha$  and  $\beta$  on the  $\Delta S$  – dynamics

		Actor $\alpha$ :	
		$dS = 0$ :	$dS = +c$ :
Actor $\beta$ :	$dS = 0$ :	$\Delta S = 0$	$\Delta S = +c$
	$dS = -c$ :	$\Delta S = -c$	$\Delta S = 0$

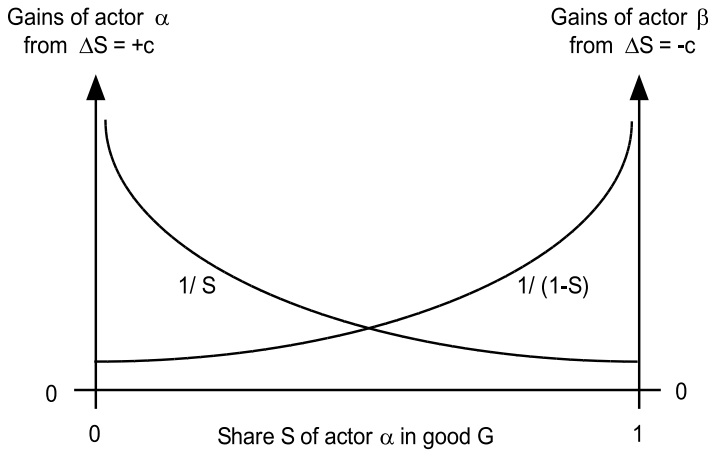
Legend:  $dS=0$ ,  $dS=+c$ ,  $dS=-c$ : Strategies „status quo by withdrawing”, „increase of S”, and „decrease of S”, respectively.  $\Delta S$ : Real change of S, depending on the pair of the selected strategies.

Whether the mentioned potential conflict about the shares S and (1-S) becomes real or not, depends on the strategies used by the two actors. In order to ensure a minimum of alternatives for choice, game theory assumes that each player has always at *least two* strategies. Moreover, to keep the complexity of the model as low as possible, we postulate that each actor has exactly this minimum number of two strategies: actor  $\alpha$  may either attempt to maintain the status quo of the distribution by *withdrawing* ( $dS=0$ ) or *increase* its share by the amount  $dS=+c > 0$  (see Tab. 2). Similarly, the other actor  $\beta$  can either aim at maintaining the status quo of the distribution by *withdrawing* ( $dS=0$ ) or *decrease* the share S by  $dS=-c$  such that its own share 1-S increases correspondingly. The third existing strategy of *increasing* the share S of its competitor by  $dS=+c$  is not in the interest of actor  $\beta$  and belongs only to the strategy-inventory of actor  $\alpha$  (see Tab. 2).

If both actors aim at maintaining the status quo by the strategy  $dS=0$  of withdrawing, the real distribution S of the considered good G will probably not change:  $\Delta S = 0$  and consequently also  $\Delta(1-S) = 0$  (see Tab. 2). If only actor  $\alpha$  attempts to change the situation by increasing its share in the good G, while the competitor  $\beta$  withdraws, actor  $\alpha$  will win the game: according to Tab. 2, the combination of the strategies  $dS=0$  and  $dS=+c$  results in a shift  $\Delta S=+c$  of the distribution of the good G at the expense of the withdrawing actor  $\beta$ . In a similar way, actor  $\beta$  may be the only party, which attempts to change the situation by increasing its share 1-S in the good G, while the other withdraws by using the strategy  $dS=0$ . In this case, actor  $\beta$  will win the game, as the combination of the strategies  $dS=-c$  and  $dS=0$  results in a shift  $\Delta S=-c$  of the distribution of the good G (see Tab. 2), which is consequently changed at the expense of the more passive competitor  $\alpha$ . Finally, both actors may strive for a realization of their interests by playing the strategies  $dS=+c$  and  $dS=-c$ . If the two actors are relatively equal with

regard to their power  $P_\alpha$  and  $P_\beta$ , the result is a costly „battle“ with no real change of  $S$ , neither in favour of the first nor of the second party. Thus, Tab. 2 postulates for this situation that  $\Delta S=0$  and consequently also  $\Delta(1-S)=0$ .

Fig. 2 Gains of the actors  $\alpha$  and  $\beta$ , by share  $S$  in good  $G$



The last-mentioned scenario of a fruitless „battle“ between two ambitious competitors points to the fact that the use of strategies entails costs and promises gains in the case of a victory. Regarding the *costs*, there is only one really critical situation: if  $dS=-c$  and  $dS=+c$ , each actor is harassed by the power of its rival. We assume that this situation entails for actor  $\alpha$  costs  $-P_\beta$  and for actor  $\beta$  an analogous penalty  $-P_\alpha$  (see Tab. 3), where in both cases costs correspond to the power of the *rival*. For all other pairs of strategies, there is no real conflict and consequently *no cost* of conflict (see Tab. 3), since at least one of the two parties withdraws.

Regarding the *gains*, two out of four possible pairs of strategies contribute nothing to the payoffs of the concerned actors, as  $\Delta S=0$ . According to Tab. 2, this situation occurs if both actors use the same strategy, i.e. either withdraw or strive for their interests. In the other two cases, one actor fights and the other withdraws and the result is either a shift by  $\Delta S=+c$  or by  $\Delta S=-c$ . Their significance in terms of psychological gains depends on the value of  $S$  (Samuelson & Nordhaus 1989: 447 ff.): if the share  $S$  of actor  $\alpha$  is relatively *big*,  $\Delta S=+c$  means for the other actor  $\beta$  an important loss, i.e. a *negative* gain. Actor  $\beta$  is in this situation already deprived by its low complementary share  $1-S$  and additionally loses  $c$  units. For actor  $\alpha$ , however, the same change  $\Delta S=+c$  is only a *small* positive gain, as  $\alpha$  gets in an already privileged situation with a high share  $S$  a *relatively* small number of

units, which psychologically do not really matter. By a similar reasoning, it can be shown that for a *small*  $S$ , the shift  $\Delta S=+c$  means a small loss for actor  $\beta$  but a big gain for actor  $\alpha$ . Fig. 2 formalises the mentioned effects on the basis of the hypothesis that the mentioned gains are marginal utilities approximated by the first derivatives of the logarithmic utility functions  $\ln(S)$  and  $\ln(1-S)$  of the shares of the actors  $\alpha$  and  $\beta$ .<sup>1</sup>

**Tab. 3** The payoff-matrix of the competition-game

		Actor $\alpha$ :	
		$dS = 0$ :	$dS = +c$ :
Actor $\beta$ :	$dS = 0$ :	0 ; 0	$-1/(1-S)$ ; $1/S$
	$dS = -c$ :	$1/(1-S)$ ; $-1/S$	$-P_\alpha$ ; $-P_\beta$

Legend:  $dS=0$ ,  $dS=+c$ ,  $dS=-c$ : Strategies. Table entries . ; . : Payoffs of actors  $\beta$  (.) and  $\alpha$  (..).  
 $P_\alpha$  = Power of actor  $\alpha$  = Conflict costs of actor  $\beta$ .  $P_\beta$  = Power of actor  $\beta$  = Conflict costs of actor  $\alpha$ .  
 $S$  = Share of actor  $\alpha$  in good  $G$ .  $1/S$  = Gains of actor  $\alpha$  from change  $\Delta S=+c$ .  $1/(1-S)$  = Gains of actor  $\beta$  from change  $\Delta S=-c$ .

The balance of the costs and the positive or negative gains defines the so called *payoff* of a game, which is a key-concept in mathematical game theory. Hence, we have summarized the previous discussion of this section by adding the different *gains* and *costs*. Their balance is presented in Tab. 3 as a payoff-matrix for the actors  $\alpha$  and  $\beta$ . As we are assuming here a *repeated* game that is played over many rounds, it changes its characteristics over time: some payoffs are depending on the share  $S$  (see Tab. 3), which in turn is changed by the choice of strategies by the actors  $\alpha$  and  $\beta$  (see Tab. 2). The dynamics of  $S$  and the associated payoffs, which are both the result of this feedback-loop, will be analysed in the following section 3.

**3. THE DYNAMICS AND EQUILIBRIA OF THE COMPETITION-GAME**

As mentioned at the end of section 2, the payoffs of the actors  $\alpha$  and  $\beta$  are strongly influenced by their shares  $S$  and  $(1-S)$  in the commonly appreciated good  $G$ . If  $S$  is *very low*, i.e. near zero, the gains of actor  $\alpha$  from striving for an increase in  $S$  are relatively high (see Fig. 2). Consequently, the payoffs of the left-hand-side matrix in Fig. 3 show that the strategy  $dS=+c$  is for actor  $\alpha$  always better than the alternative strategy  $dS=0$ , independently on whether actor  $\beta$  uses the strategy  $dS=0$  or  $dS=-c$ . Hence, the choice of the strategy  $dS=+c$  by actor  $\alpha$  compels the other actor  $\beta$  to use its strategy  $dS=0$  in order to minimize this way its losses. Thus for rational actors there is one deterministic pure strategy Nash-equilibrium<sup>2</sup>:  $dS=0$  and  $dS=+c$ . It entails an increase  $\Delta S=+c$  of the share  $S$  and thus privileges actor

$\alpha$ , which dominates the game. If this *dominance game* (Osborne 2004: 45 ff.) is repeated over a long period of time, *zone I* on the axis *S* becomes instable, since the associated growth process finally drives *S* from zone I to zone III.

A similar situation occurs, if *S* is *very high*. For  $S \approx 1$ , it is actor  $\beta$  instead of actor  $\alpha$ , which is favoured by high gains (see Fig. 2), when attempting to realize its interest in a decrease of *S*. As the matrix on the right-hand-side of Fig. 3 demonstrates, strategy  $dS=-c$  gives actor  $\beta$  always better payoffs than the alternative strategy  $dS=0$  of withdrawal, independently of the strategy chosen by the other actor  $\alpha$ . Thus, if actor  $\beta$  opts for strategy  $dS=-c$ , actor  $\alpha$  has only one reasonable choice, which minimizes its losses in terms of negative payoffs:  $dS=0$  (see Fig. 3). The result is a game dominated by actor  $\beta$  with one Nash-solution:  $dS=-c$  and  $dS=0$ . In a situation of a repeated game, there is a long series of shifts  $\Delta S=-c$ , which create left of  $S=1$  a zone II of instability, since *S* is continuously driven from zone II to zone III.

**Fig. 3** The payoffs of the three sub-games for  $\lambda_- < \lambda_+$ , by share *S* in good *G* <sup>3</sup>

Dominated by actor $\alpha$			„Chicken game“			Dominated by actor $\beta$			
		Actor $\alpha$						Actor $\alpha$	
Actor $\beta$	$dS = 0$	<u><math>dS = +c</math></u>	Actor $\beta$	$dS = 0$	$dS = +c$	Actor $\beta$	<u><math>dS = 0</math></u>	$dS = +c$	
<u><math>dS = 0</math></u>	0 ; 0	- ; ++	$dS = 0$	0 ; 0	- ; +	$dS = 0$	0 ; 0	-- ; +	
$dS = -c$	+ ; --	-- ; -	$dS = -c$	+ ; -	-- ; --	<u><math>dS = -c</math></u>	++ ; -	- ; --	

$\Delta S = +c$		$-c \leq \Delta S \leq +c$		$\Delta S = -c$		
S=0	Zone I of instability	S= $\lambda_- = 1/P_\beta$	Zone III of quasi-stability	S= $\lambda_+ = 1-1/P_\alpha$	Zone II of instability	S=1
	→		↔		←	

Legend: --, -, 0, +, ++: Ordinal payoffs, where -- < - < 0 < + < ++. Rows of matrices: Strategies of actor  $\beta$ . Columns of matrices: Strategies of actor  $\alpha$ . Underlined: Dominant strategies. One and two-headed arrows: Direction of change of *S*, if game is repeatedly played.

If the share *S* is neither very high nor very low, the type of the game may change. This always happens, if *S* reaches a critical value, where the order of two payoffs of Tab. 3 changes. As some payoffs of Tab. 3 are always positive or always negative, there are only two critical comparisons for further analysis:

- 1) The strategy-pair  $dS=-c$  and  $dS=0$  versus the strategy pair  $dS=-c$  and  $dS=+c$  (see Tab. 3). It can be shown<sup>4</sup> that the order of the payoffs of these strategy pairs changes for actor  $\alpha$ , whenever *S* reaches the limit  $\lambda_- = 1/P_\beta$ .

- 2) The strategy-pair  $dS=0$  and  $dS=+c$  versus the strategy pair  $dS=-c$  and  $dS=+c$  (see Tab. 3). It is possible to proof<sup>5</sup> that the order of the payoffs of these strategy pairs changes for actor  $\beta$ , whenever  $S$  reaches the limit  $\lambda_+ = 1 - 1/P_\alpha$ .

If the power-indices  $P_\alpha$  and  $P_\beta$  of both actors are greater than 2, the limits  $\lambda_-$  and  $\lambda_+$  are ordered in the way of Fig. 3:  $\lambda_- < \lambda_+$ .<sup>6</sup> Between these limits  $\lambda_-$  and  $\lambda_+$ , the actors  $\alpha$  and  $\beta$  play a game different from the dominance games discussed up to here. According to the payoff-matrix in the middle of Fig. 3, it is a so-called *chicken game* (Hargreaves Heap & Varoufakis 1995: 35 ff.). If the two actors co-operate, they may decide that both permanently withdraw by playing the strategy  $dS=0$ . For both parties, there are however strong incentives in terms of positive payoffs (see Fig. 3), which may induce them to abandon this agreement and to play independently each of the other. In this new situation there are two deterministic and one probabilistic Nash-solutions. The two deterministic Nash-solutions require pure strategies, where either actor  $\alpha$  or alternatively actor  $\beta$  permanently withdraws by using its strategy  $dS=0$  (Diekmann 2009: 38 ff.). As one of the two actors is in terms of  $\Delta S$  a constant loser, this solution cannot be stable: the permanent loser will soon abandon the strategy  $dS=0$ , in order to challenge the competitor by negative payoffs and to get this way to the other pure strategy Nash-equilibrium, which consequently makes the other actor a constant loser. The dynamics of this competition between the two deterministic pure strategy Nash-equilibria are obviously unpredictable. Similarly, also the results of the probabilistic Nash-solution (Hargreaves Heap & Varoufakis 1995: 197 ff.) are rather unpredictable: in order to maximize their expected payoff, both actors would have to switch between their respective strategies in a probabilistic way by using *mixed strategies* (Osborne 2004: chap. 4.3). With regard to the dynamics of the share  $S$ , this means a random walk between the limits  $\lambda_-$  and  $\lambda_+$ . In sum, zone III on the axis  $S$  between  $\lambda_-$  and  $\lambda_+$  is *quasi-stable* with rather unpredictable choices of strategies. Due to the nature of the chicken game with several Nash-solutions and a considerable moral hazard to abandon the strategies of co-operative withdrawals,  $S$  should permanently *fluctuate* between the mentioned limits  $\lambda_-$  and  $\lambda_+$ .

Obviously, the afore-analysed scenario  $\lambda_- < \lambda_+$  is not the only situation to be studied. If  $\lambda_- = \lambda_+$ , the two limits *coincide* and the zone of stability is just *one point* on the axis  $S$  with the value  $\lambda_- = \lambda_+$ . As this situation occurs only very rarely, it will *not* be discussed any further. A much more frequent scenario is, however,  $\lambda_- > \lambda_+$  (see Fig. 4). It occurs, if at least one of the power-indices  $P_\alpha$  or  $P_\beta$  is rather small. It can be shown that in this case the sub-game between the limits  $\lambda_+$  and  $\lambda_-$  is not a chicken game but a *prisoners' dilemma* (Osborne 2004: chap. 2.2; Axelrod 1984: chap. 1). It has a deterministic pure strategy Nash-solution  $dS=-c$  and  $dS=+c$ , which however yields for both actors negative payoffs (see Tab. 3). Thus it is reasonable,



if both actors  $\alpha$  and  $\beta$  switch to a co-operative solution, where both parties withdraw from „fighting” by using the same strategy  $dS=0$ . As both actors have a better alternative (see Fig. 4), there is a considerable moral hazard of defection, which will occasionally entail a shift  $\Delta S=+c$  or  $\Delta S=-c$ . If both actors protect their willingness to co-operate by the TIT-for-TAT-strategy recommended by Axelrod (1984: chap. 2), the victim of the immediately preceding defection will return to the pure strategy Nash-solution  $dS=-c$  and  $dS=+c$ , which entails  $\Delta S=0$  and stabilizes the share of S for a while somewhere between  $\lambda_-$  and  $\lambda_+$ . However, as the costs of this Nash-solution are for both actors rather high, another change of strategies is likely. In sum, the joint reactions to a prisoners’ dilemma are as *unpredictable* and *non-deterministic* as the reactions in the mentioned chicken game. Consequently, in the long run, it is most likely that S will moderately *fluctuate* between the limits  $\lambda_-$  and  $\lambda_+$  and thus entail *quasi-stability* of the distribution S.

Fig. 4 The prisoners’ dilemma as an alternative to the chicken game of Fig. 3, if  $\lambda_+ < \lambda_-$ .

		Actor $\alpha$	
		$dS = 0$	$dS = +c$
Actor $\beta$	$dS = 0$	0 ; 0	-- ; ++
	$dS = -c$	++ ; --	- ; -

$\lambda_+ =$	$-c \leq \Delta S \leq +c$	$\lambda_- =$
$1-1/P_\alpha$	$\longleftrightarrow$	$1/P_\beta$

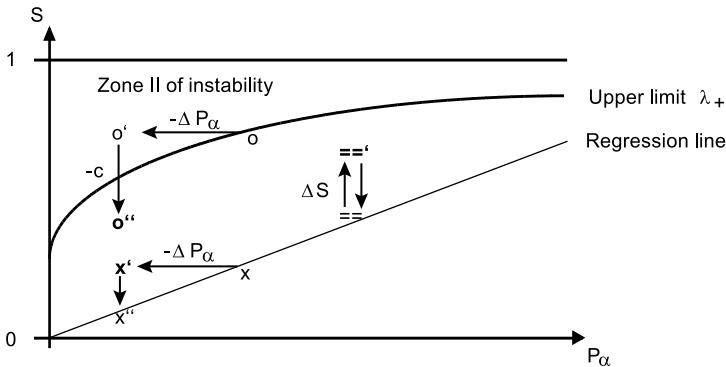
#### 4. CAUSAL EXPLANATIONS: A COMPARISON OF TWO MODELS

This article started with a rather critical assessment of path- and regression-models: we considered them as too *deterministic* and too much driven by the *past*. As an alternative, we developed and analysed in the preceding sections 2 and 3 a repeated game, describing the competition for a scarce good G. For assessing the progress of this new model over the path- and regression-approach and for disclosing its potential drawbacks, we will in the following compare the two models, especially with regard to their *explanatory power*. For this purpose, we are making some virtual experiments, by changing the values of  $P_\alpha$  and S of the actor  $\alpha$  and drawing in Fig. 5 some causal inferences from these changes. (Wilson 1985: pp. 118 ff.; Von Wright 1975: 107 ff.)

A first quasi-experiment is a change of the share S of the actor  $\alpha$  by  $\Delta S$  from the level  $\equiv$  to a higher level  $\equiv'$  (see Fig. 5). Under the assumptions of the *regression*

model, the share  $S$  should immediately return to the original level  $==$ , as the actor  $\alpha$  does not have enough power to defend and maintain the higher level  $=='$ . Hence, in the regression model, the power  $P_\alpha$  *strictly determines* the possibilities of the actor  $\alpha$ . This is much less the case for the *game theoretical* approach to social competition. A shift of  $S$  from the level  $==$  to a higher level  $=='$  does not necessarily entail a return to the original level  $==$ . In the game theoretical model, the increase  $\Delta S$  may e.g. be the result of a temporary deviance of actor  $\alpha$  from a cooperative agreement in a chicken game (see section 3). It is even possible that  $S$  also *increases* in the next following period, because the actors  $\alpha$  and  $\beta$  are playing for a while the equilibrium strategy pair ( $dS=+c, dS=0$ ) of the chicken game, which obviously favours the actor  $\alpha$ . Hence, in the game theoretical approach to social competition, individual actors may not have total freedom as they are depending on others. However, their degree of freedom is *higher* than in the relatively over-determined regression model.

**Fig. 5** Causal inferences from quasi-experimental changes of  $S$  and  $P_\alpha$ : The regression- versus the game theoretical model



Legend: Bold letters  $x', o', =='$  : Causal inferences from game theoretical model. Slim letters  $x'', ==$  : Causal inferences from regression model. For further details, see text.

A second quasi-experiment is a decrease in the power of actor  $\alpha$  by  $-\Delta P_\alpha$  units, where  $S$  is assumed to be constant and relatively *low*. In Fig. 5, this corresponds to a shift from point  $x$  to point  $x'$ , both being *below* the upper limit  $\lambda_+$ . According to the *regression* model, this quasi experiment should immediately entail the causal effect of a decrease of  $S$  from the level of  $x'$  to the lower level of  $x''$ . Thus the change of  $P_\alpha$  is the causal explanation of the drop of  $S$ : the past perfectly determines the future. In the *game theoretical* model there is no such causation, as the change

from point  $x$  to point  $x'$  does not necessarily imply a further shift from point  $x'$  to point  $x''$ . The actors  $\alpha$  and  $\beta$  may e.g. *both* play their co-operative strategies  $dS=0$  such that  $S$  does not change, in spite of a loss of power of actor  $\alpha$ .

The mentioned change of power  $-\Delta P_\alpha$ , however, matters for the upper limit  $\lambda_+$ , which confines by definition the zone of indeterminacy (see Fig. 5). This is important for the third quasi-experiment in Fig. 5. If the afore-mentioned decrease in the power of actor  $\alpha$  by  $-\Delta P_\alpha$  units occurs in a situation, where  $S$  is *near* the upper limit  $\lambda_+$ , a shift from point  $o$  to point  $o'$  has in the *game theoretical* model a real causal effect (see Fig. 5): it destabilizes the share  $S$  of the actor  $\alpha$  and entails a drop in  $S$  by  $-c$  units from the level  $o'$  to the new level  $o''$ . This drop is the result of the intention of the other actor  $\beta$  to increase its share  $1-S$  in the good  $G$  by the use of strategy  $dS=-c$ , as well as the intention of the actor  $\alpha$  to avoid the costs  $-P_\beta$  of a fruitless fight with this competitor  $\beta$ . Thus, by looking at the *intentions* of the two competitors, the game theoretical model is able to give an explanation for the disappearance of the observation  $o'$ , which is based on the idea of *final causality* (Hassing 1997). The shift of observation  $o'$  to a new co-ordinate  $o''$  is however not only an effect of the actors' *intentions* but also a result of structural changes in the immediate *past*, which represent *classical causality*: as mentioned earlier, the loss of power  $\Delta P_\alpha$  has lowered the upper limit  $\lambda_+$  and transformed the original payoff-matrix such that the new situation at point  $o'$  no longer corresponds to a chicken game or prisoners' dilemma but rather to a dominance game (see Fig. 3).

## 5. SOME GENERALISATIONS

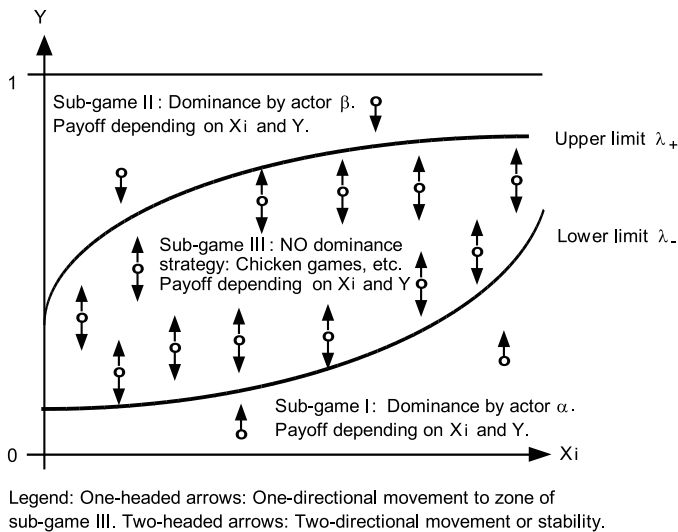
Obviously, not every game theory model has the properties of the competition game described in the previous sections 2 and 3. With regard to generalizations and possible applications of this model to other domains, we briefly summarize the key features of the model of competition between the two actors  $\alpha$  and  $\beta$  in a very general way, thus also using new variables  $Y$  and  $X_i$ . The *dependent variable*  $Y$  corresponds in the previous sections to the share  $S$  and the *independent variables*  $X_i$  were previously the power resources  $P_\alpha$  and  $P_\beta$ .

- 1) Generally, there are two types of mathematical links between the payoff-matrix of the game and the independent and the dependent variables  $X_i$  and  $Y$ , which constitute some kind of a feedback-loop: on the one hand,  $X_i$  and  $Y$  influence the payoff of the game. On the other hand, the strategies chosen on the grounds of this payoff-matrix determine the change of the dependent variable  $Y$ .
- 2) For very *high* values of the dependent variable  $Y$ , the game is dominated by actor  $\beta$ , which is supposed to be interested in a *decrease* of  $Y$ . Consequently, if the game is repeatedly played, this actor is able to *lower* the values of  $Y$  (see Fig. 6).

- 3) In an analogous way, if the values of the dependent variable  $Y$  are *low*, the game is dominated by the other actor  $\alpha$ , which is assumed to be interested in an *increase* of  $Y$ . Consequently, in this situation,  $Y$  is assumed to *grow* due to the repetitive nature of the game (see Fig. 6).
- 4) Because of the mentioned dependency of the payoff on  $X_i$  and  $Y$ , the payoff-matrix is transformed by changes of  $Y$ . Above of a certain limit  $\lambda_-$  and below of an analogous limit  $\lambda_+$ , the game is no longer dominated by one of the two actors. In this zone III, the game becomes either a chicken game, or a prisoners' dilemma, or an other game with stabilizing effects on the dependent variable  $Y$  (see Fig. 6).

As shown by the quasi-experiments of the previous section 4, the most innovative feature of game theoretical models, which fulfil the afore-mentioned properties (1), (2), (3), and (4), is their ability to explain „lacunas“ in the zones above and below the limits  $\lambda_+$  and  $\lambda_-$ : due to the mentioned instabilities, empirical observations should be rare in these zones. Our model of competition assumes that the *size* and the *shape* of these zones are consequences of „classical“ *causation* by past changes in the power-structure. However, the *instability* prevailing in these zones, is considered to be a result of *final causation* of actors pursuing their goals.

**Fig. 6** The general structure of a game theoretical explanation of  $Y$



In sharp contrast to its virtues with regard to the zones of instability, the game theoretical model does *not* give any real explanation of the observed data, which are generally concentrated in zone III between the mentioned limits  $\lambda_+$  and  $\lambda_-$ . Contrary to the classical regression models, game theory models assume that the precise co-ordinates of a data-point in this zone are *indeterminate* and can at best be explained by the history of the selected strategies, but generally not by structural factors. Thus, there is a radical shift in the *explanandum*: instead of giving a positive explanation of the observational data, the mentioned game theoretical models explain their *absence* in certain zones of the data space.

At a first glance, this „agnosticism“ seems to impair the usability of the mentioned game theoretic models for pragmatic purposes like planning or forecasting, etc. A more detailed analysis, however, makes clear that this kind of model still offers a lot of useful facilities:

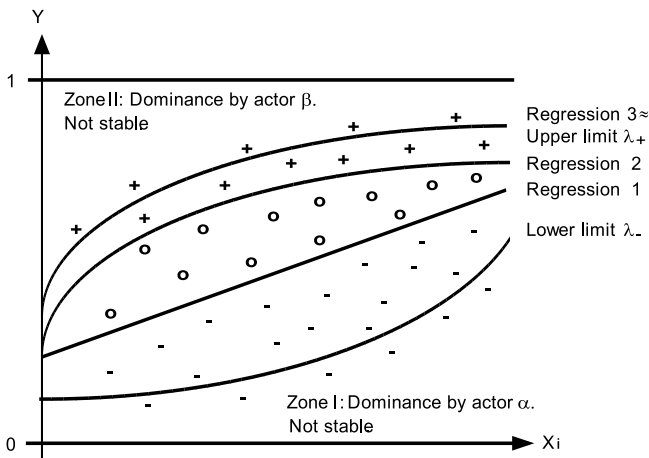
- a) Short- and middle-range *forecasts* are still possible. The upper and lower limits  $\lambda_+$  and  $\lambda_-$  define the range of possible futures. Contrary to conventional forecasts, there are no point-estimates for *one* specific future, but *intervals* of whole sets of possible futures. This is more realistic than the conventional approach. It has however the disadvantage that the mentioned intervals tend to grow and make the forecast less and less precise, if the prognosticated time-horizon increases.
- b) The mentioned game theoretical models can also be instrumental for the *planning* of public and other policies. Among others, they can directly be used for *feasibility* studies: policy goals are only feasible, if they fall into the stable interval between the limits  $\lambda_+$  and  $\lambda_-$ . Sometimes, these boundaries even allow to assess, how much additional resources, e.g. in terms of money or institutional power, are needed, in order to realize the mentioned goals in zone III of Fig. 6.
- c) It is also possible to make *empirical tests* of causal hypotheses derived from the model. As illustrated in Fig. 5, the model allows to make causal inferences about the effects, resulting from changes of its main variables. These effects can subsequently be compared with the observed data, just by using techniques from conventional statistical methodology. Besides, as a special feature already mentioned before, the model allows to deduce hypotheses about *lacunas* in the data space, which afterwards can be compared with the multidimensional, statistical distribution of the corresponding observational data.<sup>7</sup>

## 6. STATISTICAL PROCEDURES FOR ESTIMATING THE LIMITS $\lambda_+$ AND $\lambda_-$

In order to meet the afore-mentioned targets (a) to (c), the limits  $\lambda_+$  and  $\lambda_-$  have to be determined with a sufficient amount of precision. The payoffs of game theoretic models are generally hard to observe and thus cannot be used for the estimation of these boundaries. However, in many situations, the multidimensional distribution

of the dependent variable  $Y$  and its structural determinants  $X_1, \dots, X_n$  is easily available. From Fig. 6 we know, that  $\lambda_+$  and  $\lambda_-$  define the limits between the lacunas of the data-space on the one hand and the zone of relative stability on the other. Thus, Mueller (2003) has proposed to approximate these boundaries between the lacunas and the clouds of the data-space by an iterative statistical procedure. It starts with the explanation of  $Y$  by structural factors  $X_1, \dots, X_n$ , using the conventional OLS-regression method and yielding a first regression line 1 (see Fig. 7). After removing all data-points *below* this regression line, the remaining observations, which have in Fig. 7 the signatures  $o$  and  $+$ , are used for the calculation of another regression line 2. Then, the negative residuals, which are in Fig. 7 denoted by  $o$ , are again removed in order to be able to construct this way a regression line 3. By means of the structural factors  $X_1, \dots, X_n$ , it explains the  $Y$ -values of the remaining data-points with the signatures  $+$ . In Fig. 7, this regression line is already a good approximation of the upper limit  $\lambda_+$ . If the differences between  $\lambda_+$  and the remaining data-points were still too big, the iterative estimation process could be continued, until the *statistical significance* of the coefficient of determination  $r^2$  reaches the best possible value and begins to deteriorate, if the process of discarding negative residuals is continued. By an analogous regression-procedure it is also possible to determine the other, lower limit  $\lambda_-$ .

**Fig. 7** The estimation of the upper limit  $\lambda_+$  by iteratively reweighted least squares regressions



Legend: Regression 1: all data-points. Regression 2: excl. neg. residuals  $-$ . Regression 3: excl. neg. residuals  $-$  and  $o$ .

The proposed method for estimating  $\lambda_+$  corresponds to an *iteratively reweighted least-squares* procedure IRLS (Rubin 1983) with weights  $w=1$  for the positive residuals and  $w=0$  for the negative ones. This implicit choice of the  $w$ -parameters has the disadvantage of systematically reducing the number of data-points, that can be processed by the regression algorithm. Hence, as an alternative, it is proposed to set the weight  $w$  of the negative residuals to a small positive value different from zero, e.g.  $w=0.1$ , which approximately corresponds to the final share of positive residuals in the last step of the iterative regressions. By performing IRLS-regression in this alternative way, the number of data-points remains *constant* and allows to continue the regressions until the estimated parameters describing  $\lambda_+$  converge to stable final values. There is, however, the disadvantage that the mentioned regression coefficients are slightly biased by the influence of the negative residuals with a low weight  $w$  *different* from zero.

## 7. CONCLUSIONS

As shown in the previous section, this article proposes the use of statistical regression in a new way: instead of explaining observed data-points by deterministic causal models, it focuses on the analysis of the lacunas in the data-space by explaining their limits  $\lambda_+$  and  $\lambda_-$ . This corresponds to the idea that there are on the one hand zones of instability (see Fig. 7), where actors are crowded out by the pursuit of antagonistic interests and asymmetries in the power-distribution. Here, the behaviour of actors is relatively predictable. On the other hand there is a zone of indeterminacy confined by the limits  $\lambda_+$  and  $\lambda_-$ . In this zone, the behaviour of actors is *unpredictable*, due to free will, incomplete rationality, or randomly mixed strategies. Sociologist adhering to structural paradigms should accept this gap of knowledge as „insurmountable“: there is enough work to be done in order to explain the limits  $\lambda_+$  and  $\lambda_-$  by structural theories and empirical analyses.

## NOTES

- 1 For actor  $\alpha$ : If utility  $U = \ln(S) \rightarrow dU/dS = 1/S =$  marginal utility of an increase of  $S$  (see Fig. 2). For actor  $\beta$ : If utility  $U = \ln(1-S) \rightarrow dU/dS = -1/(1-S) =$  marginal utility of an increase of  $S \rightarrow +1/(1-S) =$  marginal utility of a decrease of  $S$  (see Fig. 2).
- 2 For a general definition of the Nash-equilibrium, see Osborne (2004: chap. 2.6).
- 3 Figure adapted from an earlier work (Mueller 2006: 2162) of the author.
- 4 If the share of the actor  $\alpha$  in the good  $G$  reaches the critical value  $S=\lambda_-$ , there is by definition an equality of payoffs  $-1/S = -P_\beta$  (see Tab. 3). Thus  $S = -1/ -P_\beta = 1/ P_\beta = \lambda_-$ .
- 5 If the share in the actor  $\alpha$  in the good  $G$  reaches the critical value  $S=\lambda_+$ , there is by definition an equality of payoffs  $-1/(1-S) = -P_\alpha$  (see Tab. 3). Thus  $1-S = -1/ -P_\alpha = 1/ P_\alpha$  and consequently  $S = 1 - 1/ P_\alpha = \lambda_+$ .
- 6 From  $P_\beta > 2$  follows  $\lambda_- = 1/ P_\beta < 1/2$ . Similarly,  $P_\alpha > 2$  implies  $\lambda_+ = 1 - 1/ P_\alpha > 1/2$ . Thus  $\lambda_- < \lambda_+$ .

7 See e.g. the empirical analysis of the limits of social inequality by Mueller (2006) or section 6 of this article.

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