The Middle Path

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Both traditional and integrated high school mathematics curricula have their strengths and weaknesses, and the author describes a middle path which is a hybrid of the two, offering the best of both systems. Each branch of math is studied in each year, but all the algebra is grouped together, then the geometry, then probability and statistics. The author and fellow teachers wrote their school's own electronic texts. Designed for classroom interactive whiteboards, the texts are available to students electronically and online. Issues in writing, implementing, and evaluating the texts are discussed.

Background

In the debate between the traditional and the integrated mathematics curricula, we owe it to colleagues and students to make a good faith effort to discern the strengths and weaknesses of each path (Reys and Reys, 2009). At Notre Dame Academy, an all girl college-prep school in Toledo, we used a traditional curriculum until 15 years ago, when we made the transition to integrated mathematics using the McDougal Littell series of texts. We made that decision partly in deference to the fact that most of the developed world uses some form of integrated mathematics, and we in the U.S. often fall short in competition (Program for International Student Assessment, 2007). In our experience with integrated math, we came to appreciate the fact that students studied from each branch of mathematics each year and they could readily see the connections between the branches. In retrospect, however, we discovered the value in the continuous exposure to one branch of math at a time which is the hallmark of the traditional program; students' progress in the various branches seemed more efficient and less fraught with peril, as they knew their bearings more readily due to the focused nature of traditional math.

Our Integrated Curriculum

As a department, we had committed ourselves to the integrated approach, but teachers' dedication to it wavered as we discovered what we missed about the traditional curriculum. While the department as a whole had made the decision to switch to integrated math, this was very much driven by one or two individual teachers, including the department chair. In their defense, professional development and support was regularly given, and this included informal mentoring by teachers at a nearby school which had implemented the same integrated text. But student and teacher frustration grew. It's difficult assigning blame for this, as we'll probably never know to what extent our teachers (including new members who weren't involved in the original decision to switch) truly committed themselves to integrated math.

Integrated Math Challenges

The biggest complaint teachers had about integrated math was that classes switched from topic to topic so often that progress in any one topic was hindered. Some teachers began rearranging the order of chapters and sections in our books in an attempt to cover the branches in a more organized fashion. This seemed like a good response to students' frustration at switching from branch to branch so often, but at least one colleague was accused of "disintegrating" the text.

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Our response to the issues surrounding that charge gradually led us to an epiphany: There is a middle path, a mathematics curriculum that offers us the best of both worlds. We decided to embark on it, constructing our ideal curriculum. In the first three courses of our new program, typically freshman through junior year, we include each major branch of mathematics each year, but we do so in an organized, continuous manner. All the algebra in a given year is taught together, followed by all the geometry, then all the probability and statistics. (In the first two years, the algebra unit is preceded by a number and measurement unit). We were led in our organization by the curriculum guidelines for the state of Ohio, which we modified slightly (The Ohio Department of Education, 2008). A brief outline of the chapters in each unit (given in boldface) for each course is given in Table 1.

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Table 1 Outline of Chapters

	Unit 1 Number / Measure	Unit 2 Algebra	Unit 3 Geometry	Unit 4 Probability Statistics
Course 1	**1. Sets of Numbers** **2. Operations** *order of operations, rules of exponents* **3. Other Concepts** *radicals, scientific notation, absolute value*	**1. Expressions with Variables** **2. Polynomials** **3. Functions and Transformations** *linear, quad, absolute value, square root* **4. Writing Linear Functions** **5. Solving Linear Equations** **6. Solving Quadratic Equations** *real roots only* **7. Systems of Linear Equations** *no matrices*	**1. 2-D Figures** **2. Coordinate Geo.** *proofs, transformations* **3. Similar Polygons** **4. Triangle Trigonometry** *no inverses* **5. 3-D Figures** *definitions and measures only*	**1. Counting** *no permulations or combinations* **2. Basic Probability** *with geometric probability* **3. Single Variable Statistics** *displays, measures, using samples*
Course 2	**1. Exponents of Form 1/n** **2. Complex Numbers** **3. Matrices** **4. Numerical Analysis** *Error in Units*	**1. Functions and Trans-formations** *with reciprocals* **2. Linear Systems** *with matrices* **3. Quadratic Functions** *with complex roots* **4. Rational Equations** **5. Variation**	**1. Reasoning** **2. Lines and Angles** **3. Triangles** **4. Quadrilaterals** **5. Circles** *arcs, sectors* **6. Transformations** **7. 3-D Figures** *loci, nets* **8. Triangle Trigonometry** *with inverses*	**1. Advanced Counting** *with permutations and combinations* **2. Probability Equation** *with adv. counting* **3. Single Variable Statistics** *normal distributions* **4. Two-Variable Sta-tistics** *displays, regression*
Course 3		**1. Polynomial Functions** **2. Exponent and Log Functions** **3. Other Functions** *root, piecewise, rational* **4. Trigonometric Functions** **5. Sequences and Series**	**1. Polygons** **2. Circles** *with angles and polygons* **3. Vectors and Trig** **4. 3-D Figures** *polyhedra* **5. Indirect Proofs**	**1. Advanced Probability** *conditional probability, expected value, simulations* **2. Advanced Statistics** *standard deviation, z scores, more normal distributions*

Two Simple Questions

We could find no satisfactory textbook to address our needs; integrated texts were either integrated within chapters (our choice 15 years ago) or else chapters were organized somewhat by branch but covered in what seemed a random order, with algebra chapters interspersed among the other chapters, in a way that rendered a simple reordering during the course year difficult. Looking for texts was frustrating, as we didn't want to forego the strengths of either integrated or traditional mathematics. During this process of trying to reconcile our ideal curriculum with available textbooks, we came back again to again to two simple questions: *Why do we let a committee in Chicago (or California, or Texas, or some other place) decide how we organize our courses?* And *how do students and teachers make use of a mathematics textbook, anyhow?*

For that second question, we decided students use it for the homework problems; it's a rare student who ever read our old texts. For teachers, the text is also used as a source of homework problems and for short and long term lesson planning. That simple realization opened the door for us, because we could certainly write our own homework problems, we knew how to teach individual topics well, and we believed that our curriculum plan – the middle path between traditional and integrated – was superior to others.

Creating our own Curriculum

Moodle and Smart Notebook

Ultimately, we decided to write our own texts. Fortuitously, our school began acquiring both interactive whiteboards and the ability to host classroom websites at the same time that we were considering the curricular issues. (At Notre Dame, our interactive whiteboards are *Smart Boards*, and our websites are hosted on *Moodle*.) Our new texts are written on *Smart Board Notebook* software. The chapters in each unit are given in Table 1, and each chapter contains anywhere from 2 to 8 sections, for a total of 45 sections per year. (Notre Dame is on a block schedule, and 45 sections per year works out well.) Each section is an individual *Notebook* document averaging about 16 pages of text, examples, activities, and homework. Following each section is a separate answer key document for the homework problems. Each year's curriculum is organized by file folders on our school's server, and teachers also usually carry the text on a flash drive. We use the sections in class on the *Smart Boards*, and the text is also available to students through our school's computer server, as an electronic copy for their own use, and through the *Moodle* website.

Lesson Details

The individual sections vary from each other, much like individual sections in a traditional textbook. Overall, though, there's a fairly consistent typeset and layout, with an informal tone, similar to how a teacher might lead a discussion with a class. At Notre Dame, we believe that whenever possible students should discover the relevant mathematics, so many pages consist of activities and experiments for them to try individually, in small groups, or as a class. (For instance, students might be asked to graph a family of functions to make a general discovery, to construct some triangles in order to formulate a geometric conjecture, or to use a tree diagram to solve a counting problem in order to discover a quicker formula. If there's a good classroom activity for a certain topic, we've tried to include it right in our text.) There are definition pages when appropriate, and pages with examples problems, both solved right away and with blank spots for someone (hopefully a student, perhaps the teacher) to fill in on the whiteboard

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during class. Usually after the activities and examples, there is a page or two summarizing what's been discovered. We've tried to mix up the presentation a bit, partly because a successful block-long class consists of a variety of activities. The last few pages of each document contain homework problems. Many of these provide routine practice, some stretch students by presenting new scenarios (often foreshadowing sections to come), and, as discussed below, some help us achieve some of the strengths of the integrated philosophy by their open-ended nature.

Our Third Year

We're beginning our third and final year of phasing in this change; Math III is new this year, so now all our Math I, II, and III courses, both regular and honors, are included in the project. We have no plans to change our fourth-year courses, which by their nature (precalculus, statistics, and AP Calculus) are more aligned with published texts with which we're satisfied.

Writing Duties

As current department chair, I'm the principal author of this new text, but I've had much help. The outline of the texts -- manifest in the chapter and section titles within each unit -- is a collective effort. Starting with a rough outline I generated, we consulted the national and state mathematics content standards (there's no use reinventing the wheel), examined a variety of traditional and integrated texts, and, perhaps most importantly, thought about our own experiences in learning mathematics and teaching the girls at Notre Dame.

After completing the outlines, we began writing the individual sections. For Math I, I wrote about half the sections, and three other teachers wrote the rest. This sharing of duties proved somewhat problematic. While the documents written by other teachers were very good in their own way, I found myself editing them both stylistically and for content, in order to insure cohesiveness of the text. (In retrospect, this isn't surprising, and had I not been lead author my contributions would need similar editing.) For Math II and Math III, I've written the vast majority of the sections. The help I've received from other teachers in the preparation of the texts for these two courses has been in the writing of homework problems, the answer keys, and in the composition of a smaller number of mostly self-contained sections.

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Curriculum Details

Beyond Traditional Mathematics

This isn't just traditional mathematics on a different scale. We thought carefully about the ordering of these branches so as to allow for the emphasis on connections that is a strength of the integrated curriculum. Certainly following Number/Measurement by Algebra was a natural choice, since a lot of algebra is a generalization of arithmetic, and students can quickly discover algebraic rules by considering their experience with numbers. By covering Geometry next, we were able to both review some algebra through applications and emphasize the relationships between the two branches. For instance, in sections about area, perimeter, and volume, we could include problems with figures of variable length sides, (this also allows us to pave the way for optimization problems in later courses), and in sections about transformations of plane figures and coordinate proofs, students can use their algebra skills in new contexts. Finally, concluding with Probability/ Statistics allows us to naturally incorporate algebraic topics such as function notation and regressions, while the inclusion of geometric probability problems also serves to tie those two branches together.

Rich Homework Tasks

To further emphasize the connection between branches, we consciously include homework problems that by their open-ended nature force students to think of a variety of approaches. For instance, when studying inverse functions, students are asked to investigate the relationship between inverses and transformations. (Is flipping over the line $y = x$ the same as a combination of a horizontal and vertical flip over the axes? If a vertically shifted function is inverted, how is the result related to the inverse of the original?) They're asked to draw sketches and then to formulate examples algebraically. Some problems ask students to explain a certain result in a different way than the text, and this encourages them to consider other branches of math. Students are asked to explain when a statement like $(a+b)n = an + bn$ is true, and when it's false, in as many ways as possible, and this can be done algebraically and numerically. When studying logs, students are asked how the order of operations ought to be modified to include them. Examples like these continue to grow, because we teachers enjoy formulating the questions!

A traditional geometry teacher may claim that her students also make connections, because they know some algebra and so can use variables in geometric study. But we're not making connections between a branch of math studied last year and a branch studied this year; we're making cumulative connections between branches studied each year, every year.

Beyond an Integrated Approach

This approach is not simply integrated mathematics on a different scale. After all, we embarked on this experiment because of our frustration with the problems inherent in the usual manifestation of integrated mathematics. Covering the mathematics in this organized and prescribed order serves to ground students' studies and alleviate their major complaints about integrated mathematics. While the built-in connections and open-ended problems we assign serve to help students see connections between the branches, we believe that our students are also more able to make connections within each branch, which is often more important. To take an example from algebra, when solving linear equations is soon followed by solving quadratics, it's difficult to avoid making the observation that instead of "separating variables and constants" which is used to solve a linear equation, one's approach with quadratics is to "move everything to one side" (except when able to use square roots, of course). When the two ideas were separated by months of time, plus by a fair amount of geometry and probability to boot, we're less convinced the contrast between methods was appreciated. They were both remembered as valid approaches, but exactly when one used each approach was more nebulous in student thinking. Instead, in covering mathematics branch by branch, students are able to discern the large branch structures on which to place the various skills needed to succeed in mathematics.

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Evaluating Materials

Perhaps the most important contribution of my fellow teachers continues to be the evaluations we conduct on the individual texts. We have seven math teachers at Notre Dame, and we've arranged schedules so there are at least three different teachers for each course. I distribute an evaluation form after each unit, soliciting teachers' opinions on what worked well, what didn't work, what needs added or changed, and so on. Implementing the suggestions is a work in progress. These substantive changes take place during the summer months, for the benefit of the next year's classes. (Corrections in examples and homework answers, mainly an issue only for the first year a text is implemented, are accomplished immediately by updating our saved documents and websites.)

Just as important as the yearly evaluations is the one we're implementing on the three-year series. A few topics have been moved from one year to another, partly due to time constraints. But more crucial are evaluations about the increasing sophistication of topic coverage. A certain amount of review work is probably always going to be necessary in a mathematics class (an examination of a traditional Algebra II text will confirm that), and we think a quick review is a good thing. Usually at least some students in class will benefit, and it helps build confidence in the rest of students. But new material must soon follow. We had to be sure we were covering the units in a more advanced way each year. This is where the national and state standards and the collective eyes of the math teachers in our department were essential. Some of this progression is evident in the table of chapters given above: from Math I to Math II, studying quadratics progresses from real roots to complex roots, and we factor those with leading coefficients of one to the more general case; while triangle trigonometry begins in Math I, inverse trig functions don't appear until Math II, and the laws of sine and cosine and vectors follow the next year; fractional exponents progress from the form 1/n in Math II to the form m/n in Math III; in Math I the only geometry proofs are coordinate ones, Math II contains synthetic direct proofs, and indirect proofs don't appear until Math III; finally, counting problems in Math I don't include permutation or combinations, while this content appears in Math II. There are other examples, as this spiraling approach, a strength of integrated mathematics, permeates the curriculum.

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Curriculum Challenges

Naturally, we knew we were in for some growing pains. We learned the hard way to make certain, within the first two class days, that students knew how to access the text from home and school computers. We still field the occasional, though rare, complaint from parents about not having a text. (Our response is that the text is electronic, both online and available for copying.) An issue we're currently weighing is whether or not to provide a hardcopy of the homework assignments. (As it now stands, the homework is written on the last few pages of each section in the text as student access it. Some students print out the homework pages, others just treat it like they would any other textbook and do their work on their own paper or in notebooks, while the problems are displayed on their computer monitors.) Other areas of concern are more systemic: In the event of time constraints, probability and statistics would regularly get short-changed; to prevent that, we remind each other to make small cuts as needed during the year, instead of waiting until the end. I'm currently department chair, and while I like to think this has been a bottom-up change, I can't be sure to what extent other teachers have deferred to me. (I maintain we can better judge this experiment after this final year of phase-in, and I believe a new department chair should probably complete the final assessment of this project.) Finally, my own personal biases about mathematics education have certainly crept into the texts. For instance, the early and regular exposure to function transformation is my doing.

Benefits of the Curriculum

We're realizing many unexpected benefits to this approach. Naturally, we've written the text with our school in mind. Many homework problems and examples make use of our Toledo setting, all-girl student population, sports teams, and rivalries, and this helps generate interest. The ability to customize our text is proving especially important for our first year course, because we have the fairly unusual situation of students coming from about 50 feeder schools. We can tweak our curriculum from year to year, fine tuning it based on our experience, and have these improvements appear in our very text. If another teacher discovers a good activity for a particular topic, we can include it in the text for all to benefit; this kind of collaboration doesn't occur accidentally, but it's built into how we use and evaluate our customized text.

Another benefit of having an electronic text is the ease of connecting to other files and internet sites. We've found that students will more readily consult the internet when asked. They're already sitting at the computer, and so we've written homework problems that require them to do a little research. Examples include finding out about the history of irrational or imaginary numbers, the history of the game of chess (related to exponential functions), researching the formula for solving cubic equations (extending the quadratic formula), finding out about the Richter scale (another application of logarithms). A quick internet search is all this takes (initially) and so these problems are usually enjoyable for students. Additionally, they provide the class with topics for good conversation the next time everyone meets. The text also contains computer links to *Geometer's Sketchpad* in many of the geometry sections, other math or science sites on the internet, and, occasionally, entertaining sound effects and images. (We'd like to write in more of those.)

We've discovered that when we ask students to read a section ahead of class, they're much more likely to do so in this format than they did when we had our old textbooks. This may be due to the electronic nature of the text, or to the fact that the text covers exactly what we want it to, so students know that everything is written there for a reason, and they're going to be responsible for it all, which is something they couldn't have said about our old textbooks. Finally, students genuinely like not having to lug heavy textbooks home and back again each class day, so this generates a certain amount of good will.

I'm looking forward to surveying the standardized test results from the first group of students to make it all the way through the curriculum, but that will have to wait for a year or two. In the meantime, I remain guardedly pleased about this experiment. I'm certain we've made some mistakes in the implementation, and we'll be kept busy for the next few years fine-tuning this approach and writing more homework problems. But I'm convinced we're on the right path, taking the strengths of both traditional and integrated mathematics. Ω

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Bibliography

Program for International Student Assessment (PISA). *Highlights from PISA 2006*. 2007. Retrieved from http://nces.ed.gov/surveys/pisa/

Reys, Robert & Reys, Rustin. (2009, April). Two High School Curricular Paths-Which One to Take? *Mathematics Teacher*, 568-570.

The Ohio Department of Education. *Mathematics Academic Content Standards.* 2008. Retrieved from http://www.ode.state.oh.us.

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