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On Electromotive Forces

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1. There is no part of electrical theory more fundamental than that which treats of electromotive forces, and no part which students of physics and applied electricity appear to have more difficulty in comprehending. The subject is full of pitfalls and its literature is replete with errors, sometimes made by distinguished men of science. The present article attempts to define and classify the different kinds of electromotive force in a brief and incomplete way, to make some of them clearer by means of examples and analogies, and to rectify some errors which are often committed. Much which the article contains is old and well known; some will be new to at least most of its readers.

2. If a linear force or an angular force (torque) F drives a body with linear or angular velocity v through a linear or angular distance d in its own direction, it does an amount of work $W = F\,d$; and the rate P at which it does work, or the power it expends, is $P = F\,v$.

If now we start with work or power as the fundamental quantity instead of force, we may define force by either of the relations

$$F = W/d \text{ or } F = P/v \qquad (1)$$

which are essentially equivalent. That is, the force exerted by an agent may be defined as the work it does on the body per unit displacement, or the power it expends per unit velocity.

3. Forces in general may be divided into two groups: (1) forces which are brought into existence by the transformation of energy of some other type into mechanical energy, as the force exerted on the crank shaft of a gas engine or the shaft of an electric motor; and (2) other forces, which may transfer mechanical energy (like the force exerted by the shaft connecting a motor with a direct driven machine, or like the force exerted by a belt) or dissipate mechanical energy (like the force of friction), but do not produce it.

An agent which originates a force of the first kind, or which produces mechanical energy, is called a prime mover. It is, ultimately, only through the action of a prime mover and what may be called its intrinsic force that forces of the second type can be produced.

4. Electromotive forces, like forces, may be classified under two heads, intrinsic electromotive forces and non-intrinsic electromotive forces.

An intrinsic electromotive force is one by which electrical energy is produced by transformation from energy of some other type; and the agent which is capable of effecting the transformation, or has an intrinsic electromotive force, is called an electric generator. Common examples of generators are the dynamo, the voltaic cell, and the termocouple, which transform into electrical energy mechanical, chemical and thermal energy, respectively. A generator is the electric analogue of a prime mover.

Non-intrinsic electromotive forces, on the other hand, never generate electrical energy, but are themselves always produced, ultimately, by intrinsic electromotive forces of generators in some part of the field.

5. Non-intrinsic electromotive forces may be grouped under three heads: Field electromotive forces, resistance electromotive forces, and inertia or self-inductance electromotive forces. The last term is usually used through a misunderstanding.

Field electromotive forces, by which electrical energy may be *transferred* from one part of the field to another, are of two kinds, potential differences and certain electromotive forces of induction, or combinations of these.

6. This article will be restricted to stationary or quasi-stationary systems, that is to circuits in which the electric current across every section may be assumed to have the same magnitude—circuits in which the currents are constant, or which, if the currents vary, have lengths small in comparison with the wave lengths of any electrical oscillations in them. It will be conducive to clearness to treat first potential differences, which may be produced by some other types of electromotive force, and then to proceed with the others.

7. In an electrostatic field, or field due to free electric charges at rest, like the field between the plates of a charged condenser fixed in position, or between two wires carrying a steady current, potential differences are often the only electromotive forces present, the existence of these electromotive forces being due always and solely to the presence of free electric charges which have been separated from neutral matter by some means not necessary to discuss at present.

The potential difference from a point A to a point B, or the electromotive force-due-to-the-presence-of-free-charges-at-rest from A to B, is defined as the work per unit charge done by the field (i. e. at the expense of the energy of the field) when an infinitesimal charge moves from A to B, and is independent of the particular path chosen between A and B. If dW denotes the work done by the field, and dq the charge carried from A to B, the potential difference V is

$$V = dW/dq \qquad (2)$$

A charged electric condenser is analogous to a stretched spring, and the potential difference is analogous to the stress which stretches the spring. The potential difference between two points of a wire carrying an electric current is analogous to the pressure difference between two points in a pipe carrying a liquid.

8. The potential at a point is simply an abbreviation for the potential difference from the point to a point taken as an origin, usually a point of the earth where the experiments are in progress, or the conducting enclosure forming the experimental or observational room.

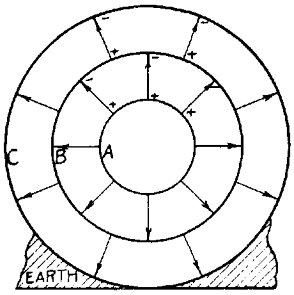

Fig. 1

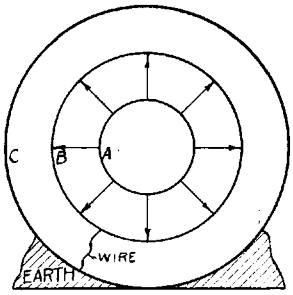

Fig. 2

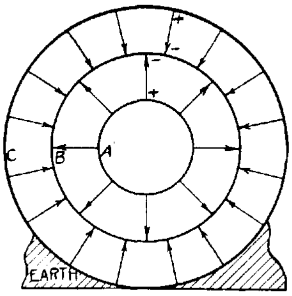

Fig. 3

Although potential difference is analogous to force, a special case of which is pressure difference, it is quite misleading to consider potential as analogous to pressure. For the pressure at a point in a body determines largely the physical condition at the point; while the potential at a point of an electrical field has no necessary connection with either the magnitude or the sign of the electric intensity (potential gradient) or the electric density, by which the electrical state at the point is determined.

Thus in Figs. 1—3 the electrical state of the conductor A, that of the inner surface of B, and that of every point of the medium between A and B are entirely independent of the external field, or field connecting B with C or the earth. The potential of B may be annulled, and that of A lowered just so much, by destroying the field between B and C by connecting a wire between them, as in Fig. 2; or the potentials of both A and B may even be reversed in sign, as in Fig. 3, without producing any change whatever in the steady state within the outer surface of B.

Failure to appreciate this point has caused a great deal of confusion in electrical teaching and electrical literature.

9. If the region between the plates of a condenser is a so-called vacuum, containing aether only and no gross matter, the potential difference acts on the aether only, and the condenser takes a certain charge. If gross matter is present also, the matter is strained together with the aether, the positive constituents of the atoms being (elastically) displaced in one direction (the direction of the electric field, or the direction of the potential difference), the negative constituents in the other, and the condenser takes a greater charge.

This characteristic of acting on both aether and matter is common to all field electromotive forces.

10. The electromotive force, or intrinsic electromotive force, E, of a generator is defined as the rate P at which the generator produces electrical energy (by transformation from energy of some other type) divided by the current I traversing it. That is,

$$E = P/I \qquad (3)$$

A generator is always more or less completely reversible. For example, a dynamo may act as a motor producing mechanical energy from electrical energy, and a storage battery on charge stores up chemical energy at the expense of electrical energy. In these cases the power is *negative,* and the electromotive force has the direction opposite to the current and is negative.

If in (3) the numerator and denominator are both multiplied by the infinitesimal time dt, the former becomes the infinitesimal amount dW of electrical energy produced in the time dt and the latter the infinitesimal charge dq passing through the generator (and across every other section of the circuit if closed) in the same time. Thus equation (3) may be written also

$$E = dW/dq \qquad (4)$$

whose *form* is identical with that of (1), and which gives another (and equivalent) definition of an intrinsic electromotive force, but one which is not in general so useful to the engineer as that given by equation (2). This latter is the exceedingly useful power equation.

11. A generator does not generate electricity, but puts electricity into motion or maintains it in motion. It *separates* the two kinds of electricity, its intrinsic electromotive force driving positive electricity (when it is mobile) in one direction (that of the conventional direction of the electric current) around the circuit, and negative electricity in the other.

Moreover, an intrinsic electromotive force acts on electricity only; it never acts upon the aether, as does a potential difference or other field electromotive force. This characteristic, together with its existence independently and as an ultimate cause of all other electromotive forces, distinguishes an intrinsic electromotive force from all other types.

12. A mechanical analogue of the resistance electromotive force is the frictional force acting upon water flowing through small pipe. Consider a small pipe ACB, Fig. 4, with the water flowing steadily in the direction ACB with the velocity v under the action of a force F due to the pressure difference from A to B (itself produced, ultimately, at least, by a pump somewhere else in the circuit) and a frictional force G. This frictional force G is proportional to the velocity in magniture and opposite in direction. Hence we may write

$$G= -f\ v$$

where f is a constant for the given pipe ACB traversed by water at the given temperature. Since the motion is uniform, the total force on the water is zero. Therefore $F+G=0$, or

$$F= -G=fv \qquad (5)$$

13. The electrical analogue of this system is a homogeneous wire ACB traversed by a steady current I in the direction AB under a potential difference V directed from A to B. If R is the resistance of the conductor AB, we have

$$V=R\ I \qquad (6)$$

V is thus the analogue of F, I the analogue of v, and R the analogue of f, the frictional constant. The electricity is again the analogue of the water.

Equation (6) may be written $V-R\ I=0$. That is, the total electromotive force acting upon the electricity in the wire is zero, being made up of two oppositely directed parts equal in magnitude, viz. the potential difference V driving the electricity in the direction of the current (or the opposite direction if the electricity is negative), and the resistance electromotive force $-R\ I$, exactly analogous to the frictional force $G= -f\ v$ in the case of the water pipe, opposing the motion. The motion is uniform, or the electric current steady, because the two electromotive forces are exactly balanced.

The resistance electromotive force acts on electricity only, like an intrinsic electromotive force, but is not reversible with the direction of the current and always transforms electrical energy into heat, never heat into electrical energy. In the case under consideration the potential difference V transfers electrical energy at the rate VI to the conductor ACB, and the resistance electromotive force *dissipates* this energy at the same rate $RI^2=VI$.

14. If the velocity of the water in the pipe AB is changing, and if its mass is denoted by m, we have instead of equation (5) the relation

$$F=fv+m\ dv/dt \qquad (7)$$

A force fv is required to overcome the friction and a force equal to the product of the mass and acceleration to produce the acceleration.

Similarly, in the electrical case, if the current is changing, we have in place of (6) the relation

$$V=RI+L\ dI/dt \qquad (8)$$

where L denotes the inductance of the conductor AB and a field of constant permeability is assumed. An electromotive force RI is necessary to drive the electricity against the resistance electromotive force $-RI$, and an electromotive force $L\ dI/dt$ is necessary to produce the electrical acceleration dI/dt.

Equation (7) may be written

$$F-m\ dv/dt=fv \qquad (9)$$

or, the applied force minus the force which produces the change of momentum is equal to the force which overcomes friction. The second term $-m\ dv/dt$, the negative of which is equal to the force which produces the change of momentum, or (by the third law of motion) the reaction on the body which applies the force, is often called the force of inertia. It is in no sense a force on the body which is being accelerated.

In the same way (8) may be written

$$V-L\ dI/dt=RI \qquad (10)$$

or, the impressed electromotive force minus the electromotive force necessary to produce the electrical acceleration, or to change the electrical momentum LI, is the electromotive force which overcomes the resistance electromotive force. The term $-LdI/dt$, equal to the negative value of the electromotive force which produces the electrical acceleration, is often called the electromotive force of self-induction, but attention should be called to the fact that it is in no sense an electromotive force acting on the electricity with which we are concerned.

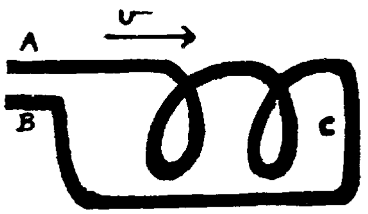

Fig. 4

15. While the water in the pipe AB is flowing with steady velocity v, suppose the ends AB suddenly connected by means of a frictionless pipe of negligible length. Then the only force acting on the water in the (now) closed circuit ACBA will be the force of friction $-fv$, which will gradually bring the water to rest, the acceleration dv/dt and the rate of change of momentum mdv/dt remaining negative until v becomes zero. During

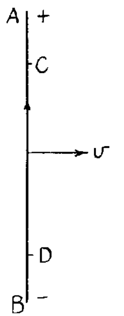

Fig. 5

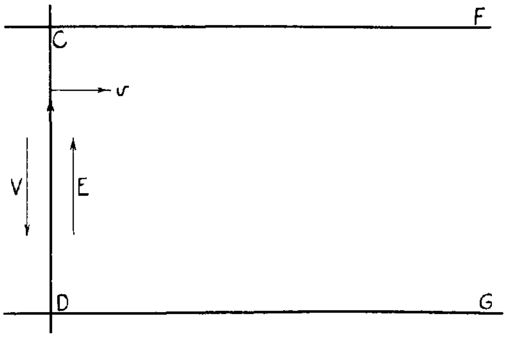

Fig. 6

this time the water exerts around the circuit on the pipe the force of reaction $fv = -m\,dv/dt$, but the only force upon the water is $-fv$.

In the same way, suppose the ends AB of the electrical system ACB to be suddenly connected by means of a short wire of negligible resistance. Then the only electromotive force acting upon the electricity with which we are concerned will be the electromotive force of resistance $-RI$, which will gradually reduce the current to zero. The electricity like the water comes to rest when all the kinetic energy has been dissipated by friction in heat.

16. For simplicity suppose the friction in the water system ACBA (after short-circuiting at AB) to be zero; and while the water is moving with constant speed v suppose a stopcock in the circuit to be quickly turned so as to prevent the water from flowing. The water, as its momentum decreases, will produce a force $-m\,dv/dt$ on the cock; and the cock will produce a force $m\,dv/dt$ on the water to bring it to rest.

If the electrical circuit ACBA, supposed free from resistance for simplicity, is suddenly interrupted, the electricity flows on, accumulates at the gap, and produces between the terminals of the gap a potential difference $-L\,dI/dt$ (when reckoned through the gap in the direction in which the current is flowing), which is equal and opposite to the electromotive force $L\,dI/dt$ which brings the electricity to rest.

17. If the magnetic flux N through any circuit A at rest is altered by moving an adjacent coil B traversed by an electric current, or by changing the current in B, there is a field electromotive force of induction produced around the circuit A equal to $-dN/dt$.

If the circuit A is simply a line in the insulating medium, the electromotive force produces in the medium along A a displacement current, both aether and matter being acted upon, the positive portions of the matter being (elastically) displaced in the direction of the electromotive force, the negative portions in the opposite direction. If the circuit is conducting, a conduction current results.

18. This induction electromotive force does not generate electrical energy; it simply transfers it, as does a potential difference, from one part of the field or from one circuit to another. Thus the electrical power supplied to A and transferred thereto by the agency of the coil B is not generated in A; but it is generated somewhere in B's circuit by a generator with intrinsic electromotive force, transferred as electrical power to the coil B along the wires by the potential difference, and then transferred through the insulator to the circuit A.

19. On the other hand, if the coil B remains fixed, with constant current, and we move the circuit A, we shall have, for the same value of dN/dt, an electromotive force around A of exactly the same magnitude and direction as before, but this time the electrical energy will be generated on the spot and the electromotive force will be intrinsic. Moreover, only the moving matter, exclusive of the aether, will be directly acted upon—for the reason that the aether permeating the matter does not share the latter's motion. In the former case the lines of magnetic induction moved through both the aether and the matter; in this case only the matter moves across the lines, while the aether remains fixed like the lines.

20. Consider a straight wire AB, Fig. 5, moving with velocity v across a uniform magnetic field whose induction B is directed into the paper. The wire is built of positive and negative electricity, part of which is mobile with respect to the rest. A positive particle moving to the right constitutes an electric current directed to the right, and will therefore, in accordance with Ampère's law, be acted upon with an upward force. A negative particle moving to the right constitutes an electric current directed to the left, and therefore will be acted upon with a downward force. Thus the result of the motion is to separate positive and negative electricity. The upper end of the wire becomes positively charged, and the lower end negatively charged. Thus an electric field is produced in and about the wire,

(Continued on page 68)

ON ELECTROMOTIVE FORCES

(Continued from Page 54)

and the lines of electrical intensity stretch from the upper half to the lower half of the wire. The separation of the electricities ceases, and the charges become static relatively to the wire, when the wire has become charged in such a way that the downward potential difference V along any length such as CD of the wire is just equal to the upward intrinsic electromotive force E produced in this same length by the motion, which is equal in magnitude to the rate dN/dt at which it cuts across magnetic flux. The total electromotive force in the length CD is then E—V=O.

21. If the wire, Fig. 6, is laid perpendicularly on two parallel conducting rails CF and DG distant CD apart and moved as before, the rails will become charged to the potential difference V everywhere. If now the ends F and G are brought into contact, the charges accumulated on the rails will be partly annulled, the potential difference V from C to D will become less than E, and the potential difference from CF to DG becomes steadily less as we pass from CD to FG, where it vanishes; and as long as the motion continues in the steady magnetic field there is a permanent current around the circuit in the direction DCFGD.

22. In the case represented the intrinsic electromotive force produced a separation of charges and therefore potential differences; and the potential differences maintained the current against the electromotive force of resistance in the part CFGD of the circuit, while the electromotive force E—V maintained the current against the resistance in the part CD of the circuit.

A current, either steady or varying, may, however, flow in a circuit without any accumulation of charges, and therefore without any potential difference. Thus consider a homogenous circular wire ACB with its plane perpendicular to the axis of a coaxial vertical symmetrically magnetized magnet, with its positive pole uppermost and in or near the plane of ACB. If the wire is pushed downward, an intrinsic electromotive force will be produced around the circuit in the clockwise direction (to one looking down upon the circle ACB), but from symmetry there will evidently be no accumulation of electricity of one sign in excess of that of the other anywhere, hence there will be no potential differences.

This is roughly similar to what happens in a short-circuited dynamo armature.

Similarly, if we take a cylindrical coil of wire with or without a cylindrical iron core, and alter the current in the coil, a coaxial circular homogenous wire will have induced around it an electromotive force equal to the negative of the rate of change of magnetic flux through it; but again there will be no accumulation of electricity and no potential differences.

This is approximately similar to what happens in the secondary of a transformer when short-ciruited.

A rough mechanical analogue of the first case is a horizontal circular symmetrical water trough with the inner or outer surface moveable about its axis. When the moveable surface is rotated the water will be set into motion in the same direction as the surface, but there will be no pressure differences along any circle drawn in the water coaxial with the trough.

23. As another instructive example, consider a parallel plate condenser CD with a dielectric whose specific inductive capacity, or dielectric constant, is K. Suppose the condenser to move with velocity v perpendicular to the lines of induction in a uniform magnetic field with its armatures C and D parallel to the lines, and suppose the condenser to be short-circuited by a wire ABE at rest, its ends A and E making sliding contact with the armatures C and D.

In this case there is an intrinsic electromotive force E acting from D to C (the directions of CD and v relative to the magnetic induction being assumed the same as in Figs. 5 and 6) on the electricity of which the dielectric is composed equal in magnitude to dN/dt, the rate at which any line from C to D moving in the field with the condenser cuts across magnetic flux. There is no intrinsic electromotive force or other electromotive force in the wire ABE, and thus no potential difference between C and D.

If the electromotive force from D to C acted on the aether as well as the matter, the armature D of the condenser would have ac harge equal to the product of E by K and by the capacity C the condenser would have with aether alone as dielectric, that is, to CKE. As the electromotive force acts on the matter only, however, since the aether remains at rest, the charge taken by D (equal and opposite to that taken by C) is (K—1) CE, since K—1 is the dielectric constant of the matter alone, that of the aether being unity.

If now the short-circuiting wire ABE is removed, and afterward the condenser brought to rest, the intrinsic electromotive force will disappear, the charges will become free, and there will result a potential difference V from D to C such that KCV=(K—1)CE, or

$$V=(K-1)/K.E \qquad (11)$$

since V acts on the whole dielectric, aether and matter, and the charges of the plates remain unaltered.