# THE MOTION OF A BALL ON A BOWLING ALLEY.

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A number of years ago the writer had occasion to work out the theory of the motion of a homogeneous sphere on a plane, with friction. Some of the results invited experimental work. Access to a bowling alley, normally idle during the summer vacation period, recently made this possible.

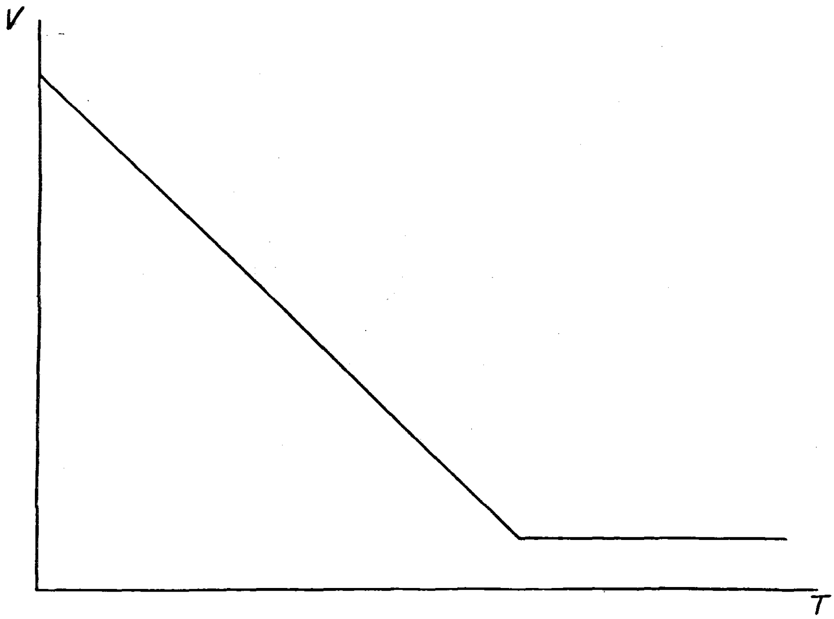

FIG. 1. Theoretical Velocity-Time Relation.

What may be termed the "simple theory" for present purposes, shows that a sphere, projected as described, subject to constant frictional force, ultimately ceases sliding and thereafter executes a motion of pure roll. The transition from sliding to rolling occurs, if the initial projection was without rotation, at the instant that the velocity of the center of the ball becomes five-sevenths of the initial velocity. This is entirely independent of the value of the coefficient of friction.

Furthermore, the time elapsed and the distance travelled when this stage is reached may be expressed by the relations:

$$t = \frac{2v_o}{7a} \quad \text{and} \quad s = \frac{12v_o^2}{49a}$$

respectively, where $v_o$ represents the initial velocity and $a$ the acceleration of the ball.

The velocity—time relation should be straight line up to the point where pure roll sets in. Rolling friction being not measurably different from zero, the velocity is constant from this point on. A discontinuity in the transition from sliding to rolling motion is implied by our assumption of constancy of the coefficient of friction. (See Fig. 1).

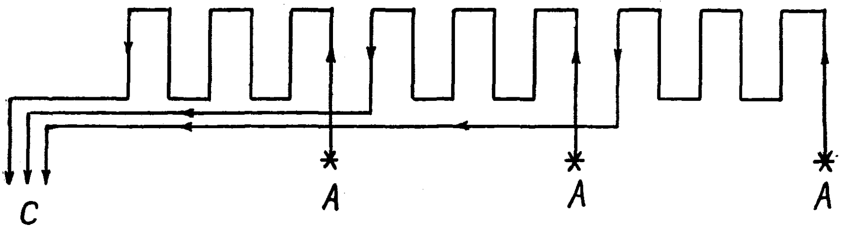

FIG. 2. Arrangement of Arcs (A) and Camera (C) and Course of Light Beams.

These conclusions were subjected to test as follows: A beam of light was reflected back and forth across the alley at a level just below the top of the ball, thence into a camera which carried a moving film, the speed of the film being about ten cms. per second. Every passage of the ball through the beam produced a break in the otherwise continuous line traced by the stationary spot of light on the moving film. More than thirty reflections were involved in the optical system. Since the loss at each reflection was such as to limit us to twelve reflections, the optical system was divided into three parts, with an arc for each part as shown. (See Fig. 2).

The use of sixty-cycle alternating current on these arcs produced one hundred twenty extinctions per second, thus providing a time scale on the film. (See Fig. 3). The section of film shown here illustrates the way the observations were recorded. The interval between successive dots represents one one hundred twentieth of a second, while that between the successive larger extinctions of the line represents a fifty-centimeter travel of the ball, fifty centimeters being the distance between the mirrors along the alley.

That initial velocities might be reproducible, the ball was launched by a catapult rather than by hand. This consisted of a pendulum swinging on ball bearings, holding the ball in a three-point suspension. (See Figs. 4 and 5). Delivery of the ball was effected by retarding the pendulum with a spring as it reached its lowest point. A slow-motion adjustment of the length of the pendulum made it possible to deliver the ball tangent to the alley, without audible impact.

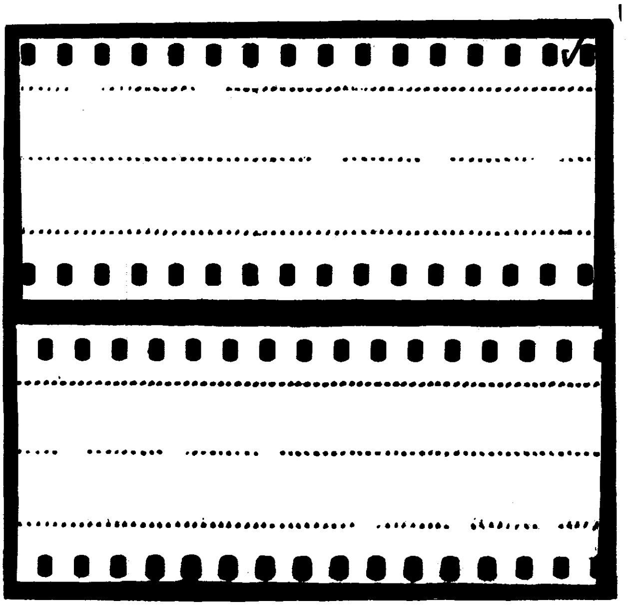

FIG. 3. Sections of Film, bearing typical records.

Some five hundred records were taken, about equally divided between ten different velocities. The velocity-time relation for seven of these is shown graphically herewith. (See Fig. 6). The two higher velocities placed the transition point beyond the range of our optical system, and hence are not included here, and the lowest velocity is excluded because it reached the transition point before yielding sufficient information concerning the acceleration while sliding.

Qualitatively these curves are not far different from that of the simple theory. Quantitatively there is considerable difference. The passage from sliding to rolling is gradual instead of abrupt, which was really to be expected. This "smoothing out" of the knee of the curve indicates a departure

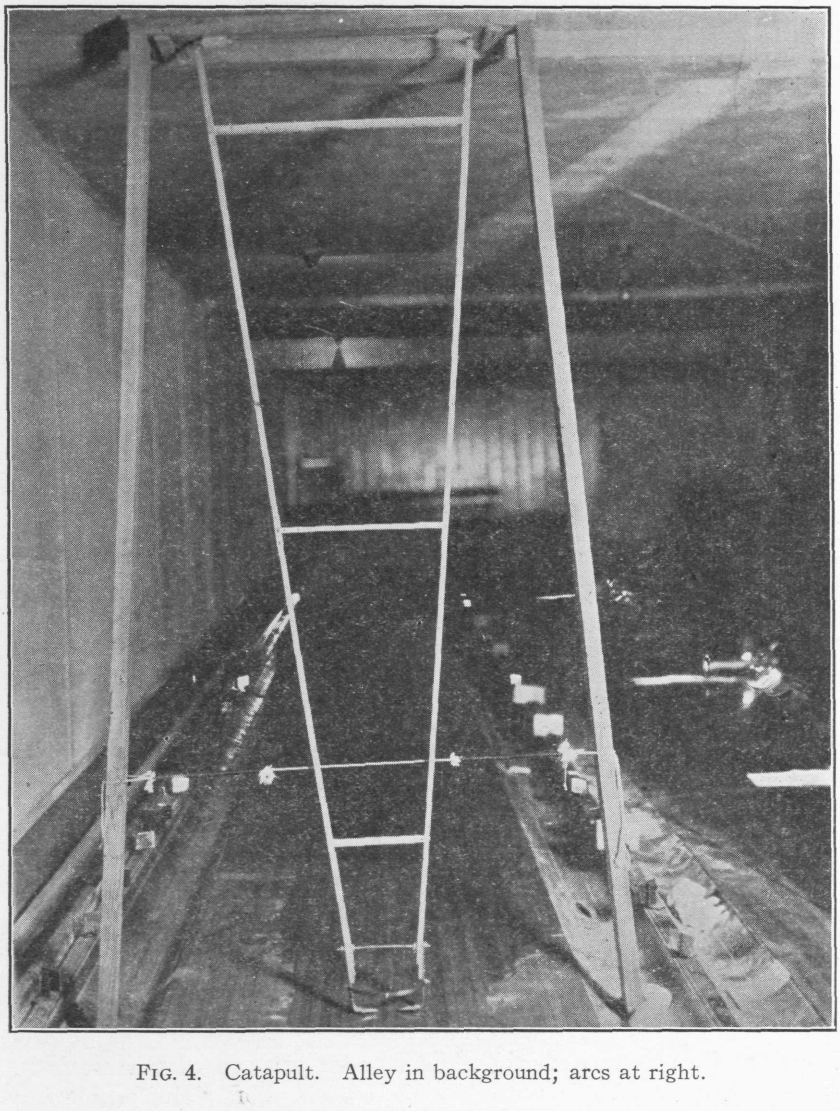

FIG. 4. Catapult. Alley in background; arcs at right.

of the frictional force from constancy. The departure begins to be evident when the relative velocity of the sliding surfaces becomes less than about sixty cms. per second. But for the law of this departure, we have insufficient information. The establishment of a sufficient number of points on the knee of

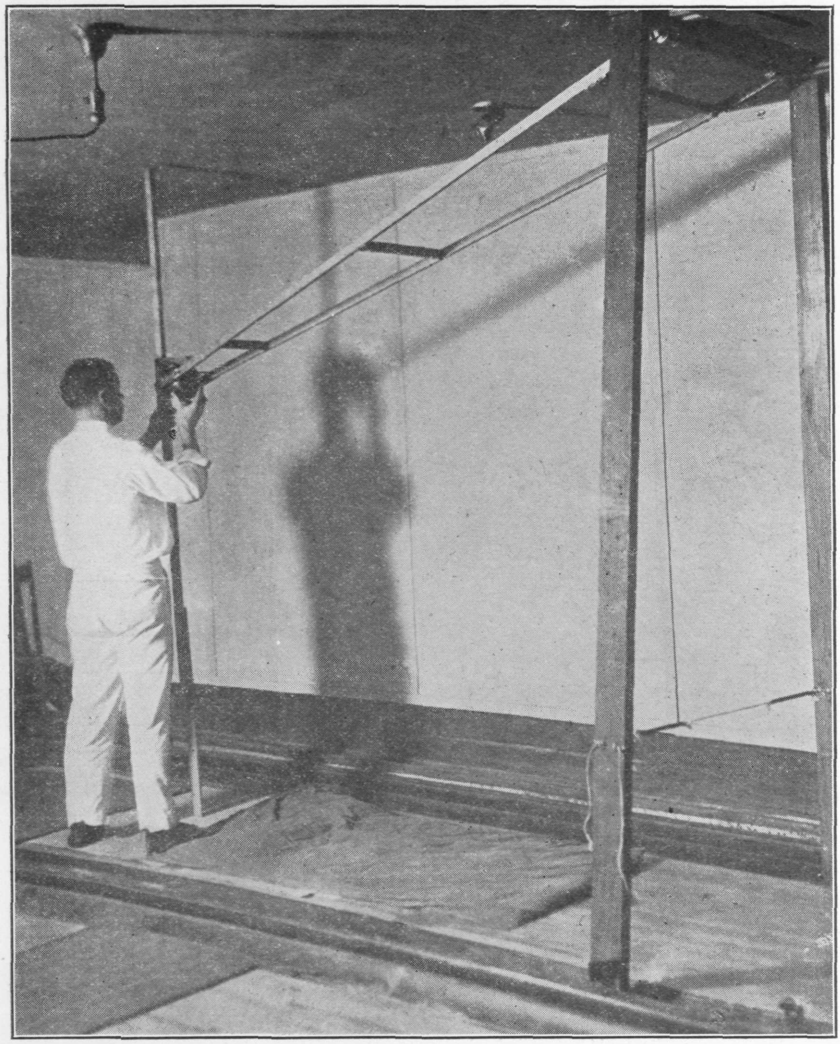

FIG. 5. Method of Launching Ball by Catapult.

the curve would require observations at intervals considerably smaller than the diameter of the ball. This would require an experimental method entirely different from the one used.

The first point of each curve represents the average velocity over the first fifty cms. of travel; therefore, presumably an approximation to the instantaneous velocity twenty-five cms. from the starting point. The actual initial velocity we can only infer by producing the curve back to the velocity axis, as shown by the dotted line. This procedure may not be justified. There is some evidence that the friction at the outset is less than that along the larger part of the course. By dusting the alley with lycopodium powder, we found that if the ball was launched by hand, rather than by the catapult, its motion was one of bouncing rather than sliding for nearly if not quite the full length of the alley. The bouncing could not be entirely eliminated by even the most careful adjustment of the catapult, though it could be confined to a region of less than twenty-five cms. at the beginning of the course. That

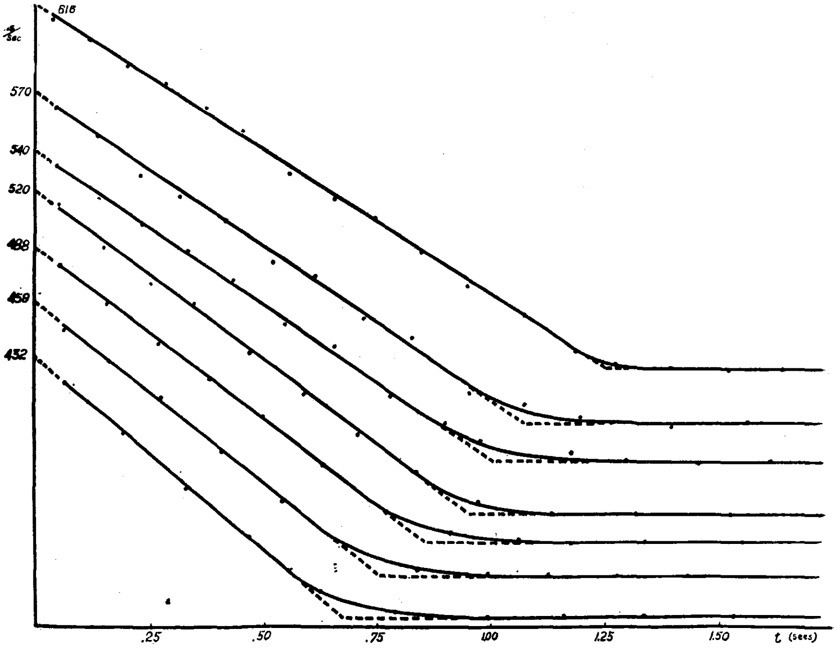

FIG. 6. Velocity-Time Grafs, for different initial velocities.

the average effective friction was the same for the bouncing motion as for the sliding would be a decidedly questionable assumption. There is reason to believe that it is less. In that case the worst possible situation would be zero friction, or the initial velocity approximately equal to the velocity at twenty-five cms. The effect of such an assumption we shall observe in a moment.

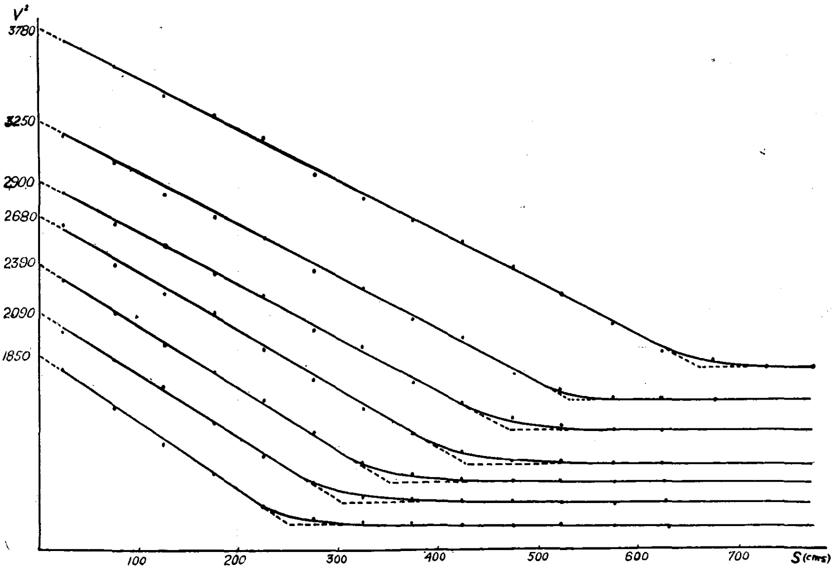

FIG. 7. Velocity-Distance Grafs, for different initial velocities.

The observations give the *final* velocities quite definitely. The simple theory indicates that this final velocity should be five-sevenths of the initial, regardless of the friction. The observations, using the extrapolated value of the initial velocity, place the final velocity consistently three percent lower than the theoretical value. But using the average velocity over the first half-meter as the initial velocity as suggested above, the observed final velocity is higher than the theoretical by two percent. This suggests that the initial velocity is less than the extrapolated value but greater than the average velocity over the first half-meter of travel, a conclusion not at all startling. In other words the final velocity constitutes the most reliable basis for estimating the initial.

The departure from constancy on the part of the coefficient of friction in the region of transition from sliding to rolling renders any test of the other two relations rather inconclusive. Such a test might be made by noting the point of intersection of the dotted extensions of the two straight portions of the curve, a familiar though usually questionable expedient. In this case it yields results which average nearly seven percent higher than the theoretical, basing the calculations on the initial velocity given by extrapolation. Using the smaller initial velocity increases this discrepancy to twelve percent. So a definitely longer time is required to reach the rolling state than the simple theory suggests, even when we attempt to eliminate the effect of decrease of friction in the region of the point of transition.

The final relation, that for the distance travelled when the transition point is reached, may be checked by a velocity-distance graph. (See Fig. 7). Plotting the *square* of the velocity against the distance rectifies the first part of the curve and gives a set which closely resembles the previous one for the velocity-time relation. The slope of these lines represents twice the accelerations. Measurement of these slopes checks within less than one percent the values of the accelerations taken from the previous curves. But the distance travelled by the ball when the transition point is reached fails to check the theoretical value by about the same margin as did the time. The discrepancy is either five or ten percent, depending upon the choice of initial velocities.

We have seen that the value of the coefficient of friction appears to gradually decrease at velocities less than sixty cms. per second. How it decreases we cannot say, except that it appears to vary continuously from that for sliding to that for rolling friction. From sixty up to the maximum of about six hundred cms. per sec., shown in the preceding curve, there is no clear evidence of variation of the coefficient of friction with velocity. The value is not quite the same for different curves, possibly due to the fact that the catapult was moved sideways several times to avoid wearing channels in the alley. In the two high-initial-velocity curves not shown, up to seven hundred cms. per sec., there was unmistakable indication of a larger slope at the beginning than subsequently. The information on the departure from constancy of slope at the beginning is even more meager than for that at the end. At present all that can be said is that for the velocity range of sixty to six hundred cms. per sec., the coefficient of friction between the composition known as "mineralite" and varnished wood, parallel to the grain is .18, though there is clear evidence that it changes both above and below this range.

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