Modeling friendship formation, measuring peer effect and optimizing class assignment

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1 Introduction

Peer effects refer to the impact that individuals within the same social circles have on each other. These effects can manifest in various aspects of our lives, including our behavior, attitudes, and performance. Due to its ubiquity and importance, a groundswell of studies have delved into the realm of peer effects (See Guryan et al. (2009), Carrell and Hoekstra (2010), Carrell et al. (2018) and Golsteyn et al. (2021)), exploring how individuals or their specific characteristics influence their peers. These studies predominantly utilize the linear-in-means model, assuming homogeneity in spillover effects among cohorts. Though linear-in-means model is easy to implement and has a clear interpretation, it suffers from two major drawbacks.

Firstly, linear-in-means model assumes a uniform strength of connection between any pair of individuals. Consequently, it does not shed light on network formation and can hardly capture differential peer effects individuals receive from their cohorts. Though under assumptions specified by Manski (1993), linear-in-means model successfully identifies some parameters related to the average peer effect, researchers may be interested in the entire distribution of peer effect, or at least some aspects of the distribution that is not fully pinned down by the average. To uncover the distribution of peer effect, consideration of dynamic network formation is necessary. Linear-

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in-means model ignores intricacies of network development and therefore, struggles to provide insights on distribution of peer effect. Secondly, neglecting the network/friendship development hinders the formulation of effective policies that can address issues arising from peer effects. This deficiency in capturing the nuances of network dynamics poses a challenge when optimizing group or class assignments. Therefore, exploring alternative models that account for the complexities of network formation is imperative for more comprehensive learning about peer effect and more effective design of group assignment policy.

In this paper, we look at one type of the group assignment problems: school principal's class assignment decision. In the context of class assignment, a principal seeks to optimize a certain objective through the peer effect channel. The closest study to our paper is Carrell et al. (2013). The authors conduct an experiment of class assignment aiming to maximize the positive influence of the high ability students on the bottom 33% students' GPA. In their study, the experiment result turns out to be surprising as the designed class assignment policy does not improve the bottom 33% students' performance. The authors reconcile this counterintuitive result by arguing that linear-in-means model does not consider dynamic friendship formation which aligns with aforementioned criticism of linear-in-means model. In contrast, modeling friendship formation is an integral part of our research. The friendship formation model allows us to predict the friendship intensity between different individual students after assigning them into classrooms. Replacing the linear-in-means assumption, we measure peer effect with a more reasonable friendship-weighted assumption. The ultimate goal is to design a class assignment policy that maximizes the peer effect of secondary school classmates who have performed well in elementary school on bottom quartile students' cognitive ability.

Our research will address these questions by (1) predicting friendship formation probability based on predetermined characteristics, (2) measuring peer effects through weighted averages with an instrumental variable approach and (3) simulating counterfactual scenarios to optimize class assignments. The methodologies that we develop in this paper can be easily extended to other group assignment problem when researchers observe either the full network or some aggregated relational data with respect to the network. We will describe in greater detail the 3 components of our research methodologies in later sections of the paper. Next, we summarize in high level some of the important findings we discover in this study.

The findings from the friendship formation prediction reveal a strong inclination for students of the same gender to form connections, with girls exhibiting more influence on boys than vice versa. Notably, a significant number of girls predominantly receive peer influence from other girls. Comparative testing against the linear-in-means model underscores the importance students place on their friends' gender in our model, as evidenced by significantly lower mean square errors out of sample. This result can be perceived as a special case for the well known homophily effect in network formation literature. Our result emphasize that in the context of Chinese secondary school education, gender homophily effect is a dominant factor in friendship formation and should be considered when it comes to using class assignment policy as a tool to optimize peer effect. The other major finding is that students who performed well academically in their elementary school tend to be considered friends by more classmates. Coupled with the gender homophily effect, this finding points at the potential benefit of dividing students into gendered classrooms.

The instrumental variable (IV) estimation outcomes highlight a meaningful positive impact of peers' sixth-grade rank on current cognitive scores, indicating that a 10% increase in friends' average class rank results in a 0.1253 point increase in cognitive scores which are measured on a scale from -2 to 3.

Additionally, when classes are divided by gender, the counterfactual analysis demonstrates that, post-assignment to all-boy classes, average boy students' realized peer effect increase by 2.19%. Similarly, after assignment to all-girl classes, average girl students see an enhanced realized peer effect improving by 2.12%. These results underscore the nuanced dynamics of peer influence in the context of gender-based class assignments, shedding light on potential optimal class assignment policy for students' cognitive outcomes.

Our research makes several noteworthy contributions to the understanding of peer effects and class assignments. Firstly, it leverages neural network method to approximate the functional relationship between students' own characteristics and their friendship preferences, providing valuable insights into the intricate processes of how students form connections. This approach allows for an examination of friendship dynamics beyond what traditional linear models can offer. Secondly, replacing the linear-in-means assumption with a more reasonable friendship weighted assumption, we conduct a comprehensive analysis of heterogeneous peer effects based on students' characteristics. This allows us to discern the varying impacts of peer influence across different demographic and behavioral dimensions, contributing to a deeper understanding of peer effect distribution. Lastly, through counterfactual analysis, we go beyond describing observed patterns and actively identify optimal class assignment strategies that maximize students' cognitive scores. This practical dimension of our research holds implications for educational policies and practices, offering actionable insights for educators and policymakers seeking to enhance student outcomes through thoughtful class assignments.

2 Data description

We use **China Education Panel Survey (CEPS)** data to conduct this research. The data is open to public and ensures that our project is replicable.

CEPS surveyed 7649 students (and their family members and teachers) from 179 classes. If a class is selected to participate in CEPS, **all** students from the class were surveyed. The survey was conducted in two rounds, once in 7th grade, once in 8th-grade. Both rounds of surveys asked all participants (student, family member and teacher) their demographic information (such as age, gender, occupation, income, education level and studying hours per week). In total, more than 2000 variables are collected. The richness of CEPS helps extend our research into many future research directions. In our paper, we investigate a very specific channel of peer effect. Future research can leverage the model that we propose in this paper and look into other mechanisms of the peer effect.

Students and family members were asked about the students' 6th grade information. This

C20. Please write down NAMES of 5 of your best friends truthfully in the first column of the table below, and then fill in the blanks with number that best describes their conditions. (Please write down as many best friends as you have if there are less than 5.)

Names of 5 of your best friends	Sex 1. Male 2. Female	Location of Hukou 1. In the local county/district 2. Not in the local county/district	Does he/she attend the same school with you? 1. Yes. 2. No.	Is he/she in the same class with you? 1. Yes. 2. No.
1___________	[___]	[___]	[___]	[___]
2___________	[___]	[___]	[___]	[___]
3___________	[___]	[___]	[___]	[___]
4___________	[___]	[___]	[___]	[___]
5___________	[___]	[___]	[___]	[___]

C21. How many of your best friends mentioned above fit in the following descriptions?

	None of them	One or two of them	Most of them
Doing well in academic performance	1	2	3
Studying hard	1	2	3
Expecting to go to college	1	2	3

Figure 1: The friendship data is aggregated relational. The identification of network formation parameter with such data is studied by Breza et al. (2020). We focus on friendship formation prediction instead of inference on the preference parameters.

feature of CEPS is extremely useful for the class assignment policy design question that we aim to answer in our project. We exclusively use students' 6th grade information to predict friendship formation as it aligns with the principal's class assignment problem in which principal has to divide students into classrooms upon their school entry. The only information that the principal has at the time of making class assignment decision is students' 6th grade information.

Students were asked to provide aggregated relational data of their friends. The linkage data, though surveyed in section C20 as depicted in Figure 1, is redacted from CEPS data. Instead, aggregated relational data of friendship network (See section C21 from Figure 1) is provided. Theretofore, we have information on "out of the five best friends of a student, how many of them _______________?" where _______________ can be filled by any question from section C21. We use the aggregated relational data to uncover friendship formation pattern.

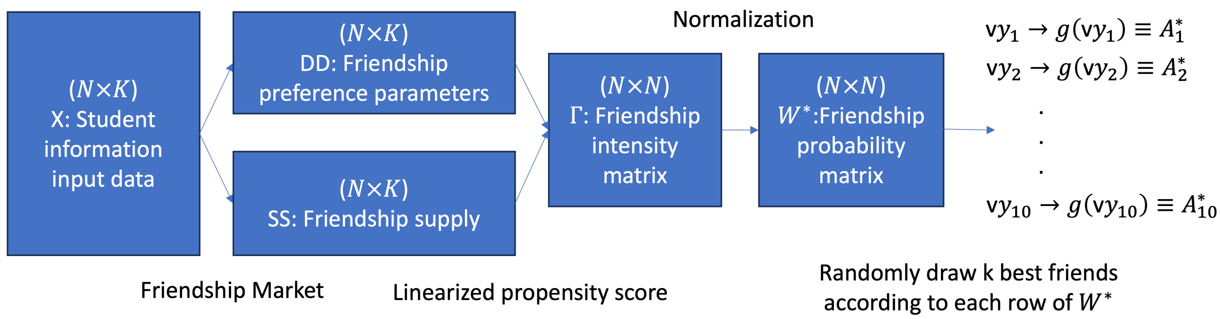

Figure 2: Illustration of the neural network algorithm for our friendship formation model

In order to study the causal impact of friends' predetermined characteristics on students' academic outcome, we exploit the random class assignment. From the total sample of 7649 students in CEPS, 5860 students are randomly assigned to 139 classes conditional on school choice. Leverage on the random assignment data structure, we construct valid instruments to address endogenous network formation issue.

3 Model setup

We solve the class assignment optimization problem with the following three steps: (1) predicting friendship formation probability based on predetermined characteristics, (2) measuring peer effects through weighted averages with an instrumental variable approach and (3) simulating counterfactual scenarios to optimize class assignments.

3.1 Friendship formation neural network

We build a novel neural network architecture to predict friendship formation with aggregate relational data. The neural network model can be divided into four stages: (1) friendship market, (2) linearized propensity score, (3) softmax normalization and (4) simulation-based estimator. As compared to more conventional neural network architectures, our model is micro-founded and interpretable (can be explained as a discrete choice model in economics). The overall architecture is summarized in Figure 2. Next, we will describe the four stages separately.

	gender	did well in 6th grade
Bill	1	1
Om	1	1
Jack	1	0
Haoran	0	1
Lei	0	0

Table 1: $SS = X$ contains students' predetermined characteristics: gender and whether the student did well in 6th grade

	gender	did well in 6th grade
Bill	1	0.5
Om	1	0.5
Jack	0.5	-0.5
Haoran	-1	0.5
Lei	-0.5	-0.5

Table 2: Preference parameter DD indicates students' preference for making friends with classmate with certain characteristics

Stage I: Friendship market

In our model, we assume that students have preference for friends and those preference parameters DD ($N \times K$) are functions of their observed characteristics. We refer to these functions as f. In short, $DD_i = f(X_i)$ where DD_i is the preference parameters of student i, it is a K-dimensional vector. DD_{ik} indicates how much student i prefers to have a friend with characteristic k. We use example 3.1 to illustrate the concept of preference parameters. We take an agnostic view of the function that maps student input information, X to preference parameters DD. Hence, we use feedforward neural network to approximate this function.

Example 3.1. Use two predictors: gender and whether a student did well in 6th grade to predict friendship formation. We have the data matrix (Table 1) for a classroom consisting of five students.

Since both input variables are binary, there are four types of students. (1) boy who did well (2) boy who did not do well (3) girl who did well (4) girl who did not do well. Assume that the four types of students have the following preferences:

- Boys who did well in elementary school prefers to make friends with boys over girls and they prefer to make friends with those who did well in elementary school.
- Boys who did not do well in elementary school prefers to make friends with boys over girls and they prefer to make friends with those who did not do well in elementary school.
- Girls who did well in elementary school prefers to make friends with girls over boys and

they prefer to make friends with those who did well in elementary school.

- Girls who did not do well in elementary school prefers to make friends with girls over boys and they prefer to make friends with those who did not do well in elementary school.

A possible DD matrix to describe such preference pattern is Table 2. Note that DD_i is a function of X_i and since Bill and Om share the same X values, their preference parameters are identical.

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Stage II and III: Linearized propensity score and normalization

With those preference parameters and observed characteristics of all students in a classroom, we are able to calculate the deterministic part of the utility of student i picking student j as best friend (i.e. linearized propensity score for student i consider student j to be his best friend), $\Upsilon_{ij} = \sum_{k=1}^{K} DD_{ik} SS_{jk} = (DD \cdot SS')_{ij}$, where $SS = X$. Utility for student i to consider student j as best friend is subject to a type I extreme value distribution, $U_{ij} = \Upsilon_{ij} + \xi_{ij}$, where $\xi_{ij} \overset{iid}{\sim}$ Gumbel distribution. In our model, student cannot make friend with himself. Therefore, we set U_{ii} as negative infinity. Since student i picks his best friend based on who gives him the highest utility, the probability of student i picking student j as best friend is given by softmax function

$$W^*_{ij} = \frac{\exp(\Upsilon_{ij})}{\sum_{j'}^{N} \exp(\Upsilon_{ij'})},$$

where $\Upsilon_{ii} = -\infty$ and N is the number of students in the classroom where student i is from. W^* denotes the probability all students considering each of their classmate as best friend. We continue example 3.1 to demonstrate how Stage II and III work.

Example 3.2. We compute the outer product of DD and SS from example 3.1 to compute the linearized propensity score Υ_{ij}.

$$\begin{pmatrix} 1 & 0.5 \\ 1 & 0.5 \\ 0.5 & -0.5 \\ -1 & 0.5 \\ -0.5 & 0.5 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1.5 & 1.5 & 1 & 0.5 & 0 \\ 1.5 & 1.5 & 1 & 0.5 & 0 \\ 0 & 0 & 0.5 & -0.5 & 0 \\ -0.5 & -0.5 & -1 & 0.5 & 0 \\ 0 & 0 & -0.5 & 0.5 & 0 \end{pmatrix} = \Upsilon$$

We then set the diagonal entries of Υ to be negative infinity such that the probability of a student making friend with himself is 0. After that, we apply softmax function to each row of Υ.

$$\begin{pmatrix} 1.5 & 1.5 & 1 & 0.5 & 0 \\ 1.5 & 1.5 & 1 & 0.5 & 0 \\ 0 & 0 & 0.5 & -0.5 & 0 \\ -0.5 & -0.5 & -1 & 0.5 & 0 \\ 0 & 0 & -0.5 & 0.5 & 0 \end{pmatrix} \overset{\Upsilon_{ii}=-\infty}{\Longrightarrow} \begin{pmatrix} -\infty & 1.5 & 1 & 0.5 & 0 \\ 1.5 & -\infty & 1 & 0.5 & 0 \\ 0 & 0 & -\infty & -0.5 & 0 \\ -0.5 & -0.5 & -1 & -\infty & 0 \\ 0 & 0 & -0.5 & 0.5 & -\infty \end{pmatrix}$$

$$\overset{\text{rowwise softmax}}{\Longrightarrow} \begin{pmatrix} 0 & 0.455 & 0.276 & 0.167 & 0.102 \\ 0.455 & 0 & 0.276 & 0.167 & 0.102 \\ 0.277 & 0.277 & 0 & 0.169 & 0.277 \\ 0.235 & 0.235 & 0.143 & 0 & 0.387 \\ 0.235 & 0.235 & 0.143 & 0.387 & 0 \end{pmatrix} = W^*$$

$\triangle$

Stage IV: Simulation-based estimation

CEPS provides aggregated relational data. We design Stage IV in accordance this data feature. If researchers have network linkage data, this stage can be simplified.

Given a $N \times N$ matrix W^* whose rows are multinomial distributions. W^*_{ij} is the probability of students i considering student j as a friend. The diagonal of W^* is 0. Each row of W^* sums to 1.

If we knew the linkage data, we could do a classical maximum likelihood estimation. In the case of aggregate relational data, we can randomly draw friends for all students based on W^* **without replacement**. Algorithm 1 describes a typical drawing without replacement procedure.

Algorithm 1 An algorithm for drawing best friends of student i

```
procedure DF(B_i, W_i^*)
    W_i ← W_i^*
    V_i ← 0_N            ▷ V_i is a placeholder for whom are considered by student i as best friends
    for b ← 1 to B_i do
        Sample bf from W_i                    ▷ Sample b^th best friend for student i
        V_i ← V_i + bf
        W_i ← W_i − bf ⊙ W_i                  ▷ ⊙ indicates elementwise multiplication
        W_i ← W_i / (1 − bf · W_i)            ▷ · refers to matrix product or dot product for vector multiplication
    end for
    return V_i
end procedure
```

V_i is an N-dimensional vector, the entry for the drawn friends for student i will take on a value of 1, the rest of the entries are 0. We have **both** aggregate relational data on how many friends of student i engages in different behaviors such as study hard, have a relationship, skip classes and etc, A^* **and** whether each classmate engages in different behaviors such as study hard, have a relationship, skip classes and etc, Y^*. Therefore, we can measure how well W^* predicts friendship formation by the following loss function

$$\sum_{i}^{N}\sum_{q}^{Q}(A^*_{iq} - g(V_i \cdot Y^*_q, B_i))^2$$

where q is the question number in section C21 from Figure 1 and function g is defined by Table 3.

So far, we describe the loss function for one class. CEPS provides information about multiple classes, so we should sum up the loss function over all classes. As a result, we obtain loss function

$$\min_{f}\sum_{c}^{C}\sum_{i \in I_c}\sum_{q}^{Q}(A^*_{ciq} - g(V_{ci} \cdot Y^*_{cq}, B_{ci}))^2.$$

where c is the index for a classroom. I_c is the set of students who are in classroom c.

	$V_i \cdot Y_q^* = 0$	$V_i \cdot Y_q^* = 1$	$V_i \cdot Y_q^* = 2$	$V_i \cdot Y_q^* = 3$	$V_i \cdot Y_q^* = 4$	$V_i \cdot Y_q^* = 5$
$B_i = 1$	$\{1\}$	$\{2,3\}$	$\times$	$\times$	$\times$	$\times$
$B_i = 2$	$\{1\}$	$\{2\}$	$\{2,3\}$	$\times$	$\times$	$\times$
$B_i = 3$	$\{1\}$	$\{2\}$	$\{2,3\}$	$\{3\}$	$\times$	$\times$
$B_i = 4$	$\{1\}$	$\{2\}$	$\{2\}$	$\{3\}$	$\{3\}$	$\times$
$B_i = 5$	$\{1\}$	$\{2\}$	$\{2\}$	$\{3\}$	$\{3\}$	$\{3\}$

Table 3: Students' responses to friendship questionnaire may cause ambiguity. Given B_i and $V_i \cdot Y_q^*$, we cannot uniquely pin down students' responses to the friendship questionnaire. We randomly break tie in those ambiguous cases. Comparing our predicted responses and the true responses from the student generate the ultimate loss function which completes the neural network architecture construction. Over 90% of the students report 5 friends, so we do not anticipate much impact from the random tie-breaker.

Algorithm 1 corresponds to a micro-founded model where students continuously pick their best friends. Right before taking the survey, student i decides that he will write down B_i best friends' names (we do not claim to provide micro-foundation for this behavior). At the moment of taking the survey, the student stochastically pick his best friend and write down that best friend's name on section C20 of Figure 1. After writing the first name, the student then randomly draw another best friend, if it repeats with previous best friend choice, then he discard that draw and draw until he gets a new best friend. Student i stops this iterative procedure when he writes down B_i best friends' names.

3.2 Instrumental variable peer effect measurement

The conventional econometric model for peer effect measurement is the linear-in-means model

$$y_{ics} = \beta \underbrace{\frac{\sum_{j=1,j\neq i}^{N_{cs}} z_{jcs}}{N_{cs}-1}}_{\text{Exclusive means}} + X_{ics}\gamma + \underbrace{\theta_s}_{FE} + \underbrace{\mu_{cs}}_{RE} + \epsilon_{ics}$$

where y_{ics} is student i's 8th grade cognitive ability in class c school s; z_{jcs} is classmate's 6th grade class rank in class c school s; X_{ics} is additional controls for student i such as gender, age, ethnic group and parents' education; θ_s is school fixed effects and μ_{cs} is class random effects. We can rewrite linear-in-means model as the $y_{cs} = \beta W_{cs} z_{cs} + X_{cs}\gamma + \theta_s + \mu_{cs} + \epsilon_{ics}$ where

$$W_{cs} = \begin{pmatrix} 0 & \frac{1}{N_{cs}-1} & \frac{1}{N_{cs}-1} & \cdots & \cdots & \frac{1}{N_{cs}-1} \\ \frac{1}{N_{cs}-1} & 0 & \frac{1}{N_{cs}-1} & \cdots & \cdots & \frac{1}{N_{cs}-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{1}{N_{cs}-1} & \frac{1}{N_{cs}-1} & \frac{1}{N_{cs}-1} & \cdots & \cdots & 0 \end{pmatrix}.$$

Linear-in-means model implicitly assumes that all students exert the same amount of peer influence to all their classmates as evidenced by W matrix. In our model, we assume that momentarily, a student receives peer effect exclusively from his chosen best friend. Therefore, the peer effect a student receives from one of his classmate is proportional to the probability of him considering that classmate to be his best friend. We replace the linear-in-means assumption with a friendship-weighted assumption (i.e. replace W with W^* obtained from the neural network model as shown in Eq(2)).

However, we face endogeneity issue with W^* matrix. We illustrate the issue with endogeneity with example 3.3.

Example 3.3. Students may choose to make friends with those of similar IQ as themselves. IQ also has an impact on students' 8th grade cognitive ability test score. Then, if we run regression based on Eq(2), we may overestimate peer effect β.

$\triangle$

We resort to classical instrumental variable approach to address the endogeneity. The instrument that we use is the average 6th grade class rank for classmates that are randomly assigned to the students. A valid instrument needs to fulfill two properties: exclusion constraint and relevance constraint. We continue example 3.3 to illustrate why both properties are satisfied.

Example 3.4. We continue with example 3.3 where IQ is an endogenous variable.

Exclusion constraint requires the instrument (average 6th grade class rank for all classmates) to be uncorrelated with the unobserved confounder IQ. This requirement is trivially satisfied since all classroom assignment is random conditional on school choice which is a covariate included in Eq(2).

Relevance constraint requires the instrument and the endogenous variable to be correlated. Both average 6th grade class rank ($W_{cs}z_{cs}$) and friends' weighted average 6th grade class rank ($W^*_{cs}z_{cs}$) are dependent on all classmates' 6th grade class rank (z_{cs}). Therefore, the relevance constraint is satisfied. $\triangle$

$$\underbrace{\sum_{j=1}^{N_{cs}} W^*_{ij} z_{jcs}}_{\text{Friends' average 6th grade ranking}} = \pi_1 \underbrace{\frac{\sum_{j=1, j\neq i}^{N_{cs}} z_{jcs}}{N_{cs}-1}}_{\text{Classmates' average 6th grade ranking}} + X_{ics}\delta + \phi_s + \eta_{ics} \tag{1}$$

$$\underbrace{y_{ics}}_{\text{Student i's 8th grade cognitive test score}} = \beta \sum_{j=1}^{N_{cs}} W^*_{ij} z_{jcs} + X_{ics}\gamma + \underbrace{\theta_s}_{FE} + \underbrace{\mu_{cs}}_{RE} + \epsilon_{ics} \tag{2}$$

We estimate Eq(2) and Eq(1) with two-stage least square estimator.

3.3 Counterfactual analysis of class assignment

With the friendship formation neural network model and instrumental variable estimate, we are able to simulate different class assignment policy and predict all students' peer effect. There are two possible scenarios to select the best class assignment policy.

3.3.1 Finite number of possibilities

If a principal is contemplating a finite number of possible class assignment policies, then he can use our model to predict peer effect for all students and choose the one among the finitely many candidate policies that optimizes the objective function. We adopt this approach to optimize class assignment policy.

3.3.2 Combinatorial optimization

Class assignment is a combinatorial optimization problem. In principle, there are finitely many possible class assignment policies and therefore, we can use the approach described in section 3.3.1

to pick the best class assignment policy. However, the approach is computationally infeasible. We use example 3.5 to illustrate this problem.

Example 3.5. A principal needs to divide 100 students into two classrooms, each consisting of 50 students. There are $\frac{100!}{50!}$ ways to assign students into classrooms. In the context of Chinese secondary school education, most principals face a much larger student population than 100 and need to divide the students into more than 2 classrooms. Therefore, the brute force enumeration method does not work. $\triangle$

We resort to heuristic optimization methods such as genetic algorithm and simulated annealing.

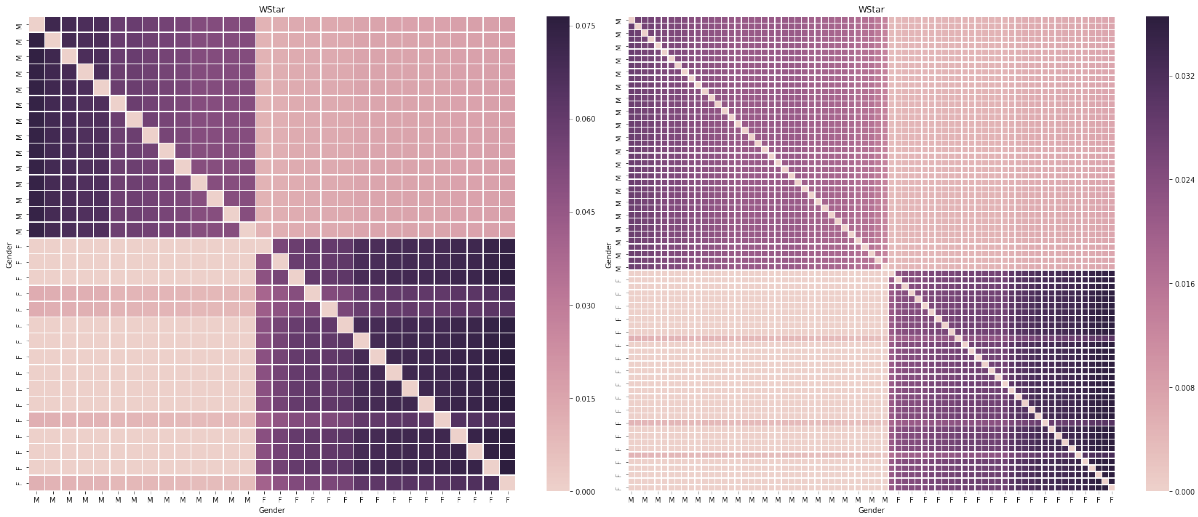
4 Results

4.1 Friendship formation pattern

Our friendship formation neural network model finds that gender is the single dominant factor in terms of predicting friendship formation. Boys almost exclusively make friends with boys and girls almost exclusively make friends with girls.

Figure 3 plot the heat of matrix W^* for two classrooms. In Figure 3, darker color indicates higher probability of student i considering student j as his best friend. (i, j) are the row and column number of the heat map (matrix) respectively. The students are group into two gender groups first, and within each group, boys are ordered from the best student to the to the worst student in terms of 6th grade class rank; girls are ordered from the worst student to the best student in terms of 6th grade class rank. Therefore, the best students are placed at the two ends of the heat map and the worst students are placed in the middle.

Two patterns emerge. (1) As mentioned previously, gender is the most powerful predictor for friendship formation. There is a very clear discontinuity in the color patterns in both Figures 3a and 3b. The discontinuity happens exactly as we move from the worst boy student to the worst girl student (both in rows and in columns). This indicates that the probability for a boy to consider



(a) Heat map of W^* for classroom 1 (b) Heat map of W^* for classroom 235

Figure 3: Heat maps for two classrooms. Classroom 1 is the first class in our dataset, classroom 235 is the biggest class in our dataset.

a girl to be his best friend is much lower than than him considering a boy as his best friend and vice versa. This aligns with the cultural context in Chinese secondary school education where teenage romantic relationship is high stigmatized and discouraged by parents and educators. (2) Students who did well in elementary school are more popular than those who did not do well. This is evidenced by darker color of the columns at left and right sides of the heat map. This observation suggests a gender-based classroom assignment policy. We compare this policy with the existing random class assignment policy in section 4.3.

We also compare W^* with W in terms of predicting friendship formation. In CEPS data, there are 1789 students that are not randomly assignment to 40 classrooms. We use them as test data. Note that there is a data shift as the students in the test set are no longer randomly assigned to classrooms. Nevertheless, using these 1789 students as test data is still fitting for our problem because the ultimate goal for our research is to devise optimal class assignment, which would be a nonrandom class assignment policy.

Figure 4 shows that our model is able to predict the gender pattern in friendship formation much better than the linear-in-means model. However, for the other questions in Figure 1, our model can only perform comparably as the linear-in-means. One possible explanation is that while

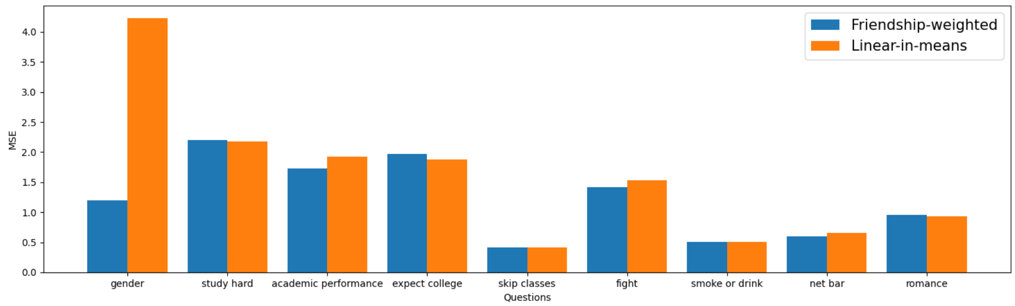


Figure 4: Our model is able to predict friendship formation more accurately than the classical linear-in-means model commonly adopted in the peer effect literature.

A^* is student i's perception of his own best friends, Y^* is classmates' self-perception. These two perceptions may not align with each other for question like "How many of your best friends are in a romantic relationship?". Student i might consider his best student j to not be in a relationship whereas student j is secretly in a romantic relationship since teenage romance is highly stigmatized in China. Nevertheless, the large gain in prediction accuracy for gender pattern in friendship formation is a strong argument for researchers to consider friendship formation when it comes to measuring peer effect.

4.2 Positive peer effect from "good students"

We estimate Eq(1) and confirm the presence of positive peer effect from "good stundet" (students who did well in elementary school). In particular, if student i's best friends' weighted average 6th grade class quantile improves by 10%, his 8th grade cognitive ability test score improves by 0.1253 on a scale of 5.

4.3 Some discussions on "optimal" class assignment

The positive peer effect from good student informs the principals that in order to maximize peer effect, he should place those students who are more likely to make friends with a particular good student in the same classroom as that good student. In other words, the principal should design class assignment policy that maximizes the probability of other students considering "good

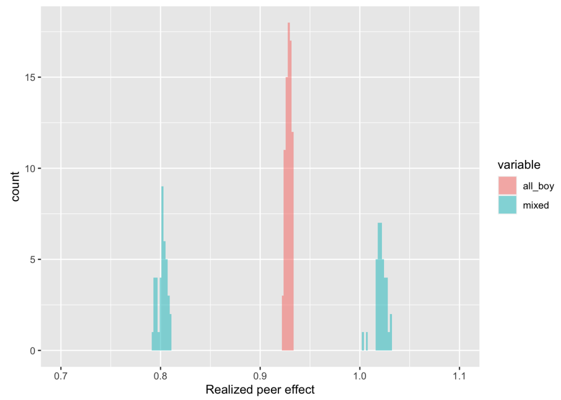
Table 2: The Peer Effects of Sixth Grade Rank on Cognitive Test Scores

	(1)	(2)	(3)	(4)
Rank	1.090***	1.102***	1.105***	1.091***
	(0.042)	(0.038)	(0.039)	(0.038)
Peer's rank	2.029***	1.093***	1.127**	
	(0.136)	(0.247)	(0.465)	
Rank of predicted peers				1.253**
				(0.515)
School FE	NO	YES	YES	YES
Class random effects	NO	NO	YES	YES
Observations	5860	5860	5860	5860
Adjusted R^2	0.255	0.393		
- Log Likelihood			5,798.824	5,799.724

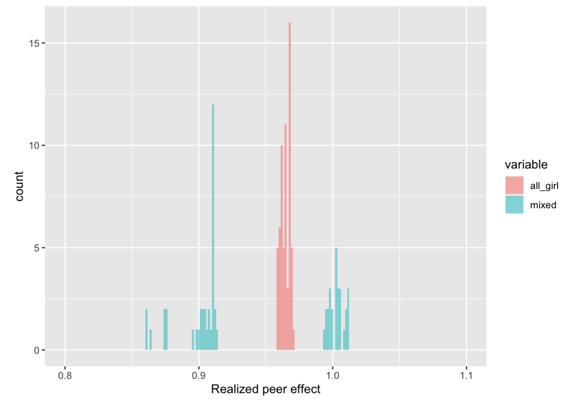
Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 4: Holding everything else fixed, when a student's friends' 6th grade class rank improves by 10%, the student's cognitive ability test score in 8th grade improves by 0.1253 point (on a scale of 5 total points).



(a) After being assigned to all-boy classes, average boy students' realized peer effect improves from 0.9086 to 0.9285.



(b) After being assigned to all-girl classes, average girl students' realized peer effect improves from 0.9446 to 0.9646.

students" as best friends.

4.3.1 Gender-based class assignment

There are two major findings from friendship formation model (1) gender is a dominant factor in predicting friendship (2) good students are more influential than bad students. These two observations suggest a gender-based class assignment can potentially maximize peer effect from good students. We run our simulation two classrooms from the same school (classroom 234 and 235). We divide the students from these two classrooms into all-boy classroom and all-girl classroom, predict their friendship formation $\hat{W}^*$, calculate their realized peer effect $\hat{\beta}\hat{W}^* z$.

The results align with our expectation. In Figure 5a, we plot histogram of the realized peer effect for boys. The blue bars indicate the realized peer effect for boys when they are randomly assigned to classrooms; the pink bars indicate the realized peer effect for all boys when they are assigned to a all-boy classroom. Their average peer effect improves from 0.9086 to 0.9285 when they are moved to a all-boy classroom. We plot the same histogram for girls in Figure 5b. Girls' average peer effect improves from 0.9446 to 0.9646 when they are moved to a all-girl classroom.

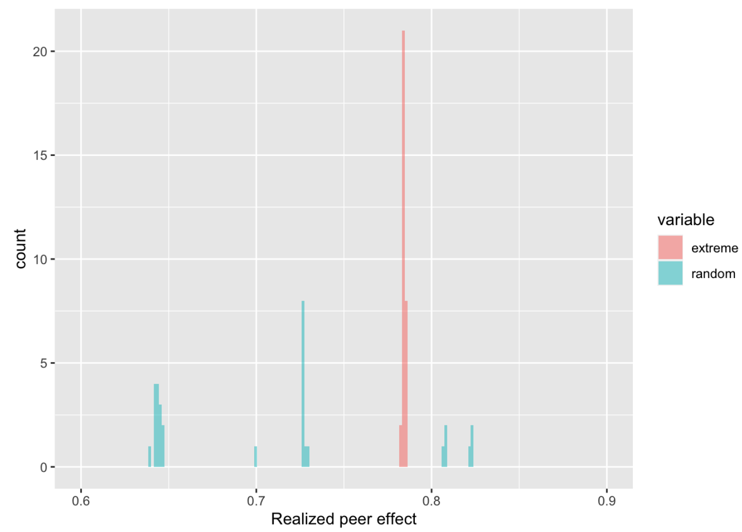


Figure 6: We divide students into three groups. Top quartile, middle 50%, bottom quartile in terms of 6th grade classroom quantile and put the top and bottom quartile students into one class. The simulation setup is similar to the experimental setup in Carrell et al. (2013). Bottom quartile students' average realized peer effect improves from 0.7055 to 0.7842.

4.3.2 Extreme mixing class assignment

We mix the the top and bottom quartile students in terms of their 6th grade class rank into one classroom. The model predicts that the realized peer effect for the bottom quartile students improves from from 0.7055 to 0.7842. We plot the realized peer effect for bottom quartile students in Figure 6. Though such a classroom assignment policy does hurt top quartile students as their realized peer effect drop, this type of extreme mixing class assignment policy might be appealing to principals who are particularly concerned about educational outcome equity. Alternatively, this policy can be used as an initial point for heuristic optimization. For example, we can use genetic algorithm and swap students until the class assignment policy no longer hurts good students in hope that the resulting policy can still benefit bottom quartile students substantially.

5 Conclusion

This paper builds a friendship formation prediction model and estimates peer effect using a modified version of the linear-in-means model. From the results of the friendship formation prediction and peer effect parameter estimates, we are able to simulate counterfactual peer effect for all students. We compare some class assignment policies to random assignment and find out that gender-based class assignment helps improve average peer effect and extreme mixing class assignment helps with bottom quartile students.

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