considered, each faculty member may feel unhappy with the new salary.

The reader is encouraged to study actual budget data to see how frequently this apparent anomaly noted in Tables I and II actually occurs.

MAKING TRIGONOMETRIC RATIOS MEANINGFUL

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High school students working with trigonometric ratios often do not have a complete understanding of the meaning of the functions, not relating them to similar right triangles and the resulting equality of ratios of the corresponding sides. One way to make the trigonometric functions more "real" to students and to integrate use of both the calculator and geometric constructions is to have students construct similar right triangles, take measurements, calculate ratios and compare them. The results, presented in tabular form, make the trigonometric functions appear to be a logical consequence. A suggested sequence of activities is outlined below.

Begin by giving students two activity sheets each containing four line segments of different lengths and orientations. The line segments on one page may all be labeled AC and those on the second $A'C'$. Instruct students to construct right triangles ABC and $A'B'C'$ for each of the given line segments for specified degree measures of angles A and A' (where the measure of angle A is equal to the measure of angle A') and with the right angle at C. A protractor could be used to draw angles A and A' or, if their degree measures are carefully chosen, angles A and A' may

be constructed with straight edge and compass as should the right angle C (see Figures 1 and 2).

Once the triangles are completed, students are asked to measure the length of legs AC and BC and hypotenuse AB and legs A'C', B'C', and hypotenuse A'B'. Provide students with two tally sheets, one for each set of right triangles on which they can record their results and calculate the indicated ratios (see Figure 3). A sample of a completed table is shown in Figure 4.

Questioning may be used to direct students' attention to the relationships which the completed tables demonstrate, focusing in on the calculated ratios across all right triangles with the same acute angle. A discussion may follow regarding the relationship among the four right triangles of each set and the patterns which appear to emerge in the tables. Comparing the results of each set of right triangles, within each set, the tables clearly show the three calculated ratios to be very nearly equal for those triangles with the same acute angle irrespective of the lengths of the sides. Students may further be encouraged to verbalize the reason for such a relationship, relating back to similar triangles and the equality of corresponding ratios. A formal proof may be presented at this point. Following such a sequence of activities, students would readily agree that it seems reasonable to apply specific names to such ratios and thus the terms sine, cosine, and tangent may be introduced and formal definitions of these functions can be made.

This approach is totally appropriate for a tenth grade geometry class which is seeing formal definitions of trigonometric functions for the first time and can have more meaning than the simple statement of such ratios. Having the advantage of using a calculator should convince students that the ratios will come out the same regardless of the lengths of the sides of the triangles or the measures of the angles (let them choose their own lengths and angle measure). Furthermore, the activity allows integration of constructions and application of the problem solving heuristic of looking for a pattern.

Triangles Set I

Directions: For each of the four line segments given below, construct a right angle at C. Complete right triangle ABC such that the measure of angle A = 40°.

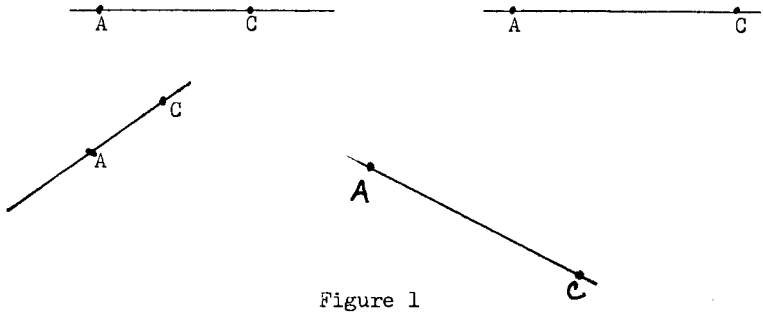

Figure 1

Triangle Set II

Directions: For each of the four line segments given below, construct a right angle at C. Complete right triangle ABC such that the measure of angle A = 25°.

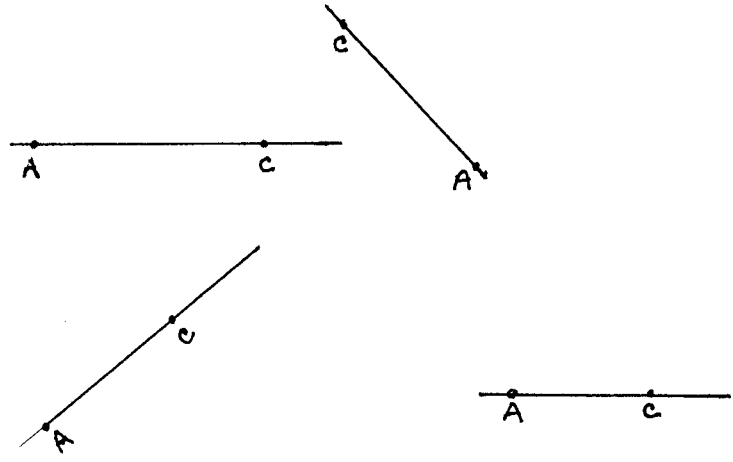

Figure 2

Summary Table: Right Triangle Measurements

	Triangle I	Triangle II	Triangle III	Triangle IV	
measure of angle A					
length of leg AC					
length of leg BC					
length of hypotenuse AB					
ratio of AC to AB					
ratio of BC to AB					
ratio of AC to BC					

Figure 3

Summary Table: Right Triangle Measurements

	Triangle I	Triangle II	Triangle III	Triangle IV	
measure of angle A	40°	40°	40°	40°	
length of leg AC	7.8	5.3	6.4	3.9	
length of leg BC	6.2	4.4	5.4	3.2	
length of hypotenuse AB	10.3	6.9	8.2	5.1	
ratio of AC to AB	.76	.77	.78	.76	
ratio of BC to AB	.60	.64	.66	.63	
ratio of AC to BC	1.26	1.20	1.19	1.22	

Figure 4