EXPECTED VALUE OF A SWEEPSTAKES ENTRY

Wendell A. Johnson
University of Akron
Akron, Ohio

The probability of winning a state lottery game was discussed recently in the March, 1988, issue of the OHIO JOURNAL OF SCHOOL MATHEMATICS. There are other types of games offered to the public which require only that the entrant return a set of materials of some sort to be eligible to win. These sweepstakes games offer the public the chance to win amounts of money varying from $20 to $5,000,000. The probabilities of winning the varying amounts are listed with each sweepstakes game.

The concept of expected value is helpful in determining the worth of a sweepstake entry. This concept concerns the amount that an entrant may be expected to win each time a player enters the game. For example, suppose there are 100 tickets in a raffle and that the winner will receive a $500 prize. On the average, a person having one ticket in such a raffle would be expected to win the contest once in every 100 raffles. Thus, each ticket the player holds in the 100 tries has an average value of $500 divided by 100 tickets, or $5 per ticket. This amount may also be expressed as the product of the probability of winning a prize times the value of the prize. In this example, the amount would be (0.01) times $500, or $5. Of course, no one actually wins $5, but each ticket has that expectation. The next example is a little more complicated since it involves a cost to the player, and there are multiple payoffs.

Suppose that a player pays $3.00 to roll a single die and that the player wins $10.00 if the die shows five spots, $5.00 if three spots are shown, and loses otherwise. In an "ideal" situation the player would be expected to lose four times while winning each of the prizes once in six tries. Thus, the player would have paid $18.00 to play six games and have won $15.00 for a net loss of $3.00. Therefore, the average amount "won" per play would be –$0.50.

The expected value of the dice game may be given by the formula

$$E(x) = \sum_{i=1}^{n} x_i \, p(x_i) - c$$

where x_i is the ith amount that could be won, $p(x_i)$ is the probability of winning the ith amount,
c is the cost of playing each game, and n is the number of different prizes.

The formula shown may be used to determine the expected value of the dice game mentioned above.

The table shown lists the amount that could be won (x_i) and the probabilities $p(x_i)$ of winning each amount.

Table I

x_i	\$5.00	\$10.00
$p(x_i)$	$\frac{1}{6}$	$\frac{1}{6}$

$$E(x) = \frac{1}{6}(\$5) + \frac{1}{6}(\$10) - \$3 = -\$0.50$$

The same formula may be applied to a recent sweepstakes game sponsored by a nationally known magazine. The amounts of the prizes and the approximate probabilities of winning are given by the sponsors of the game. The probabilities and the amounts of the prizes are reproduced in the following table.

Table II

x_i	\$20	\$200	\$5,000	\$10,000	\$25,000
$p(x_i)$	$\frac{1}{3,078}$	$\frac{1}{2,093,333}$	$\frac{1}{10,466,667}$	$\frac{1}{39,250,000}$	$\frac{1}{78,500,000}$

x_i	\$50,000	\$117,000	\$150,000	\$5,000,000
$p(x_i)$	$\frac{1}{157,000,000}$	$\frac{1}{157,000,000}$	$\frac{1}{157,000,000}$	$\frac{1}{157,000,000}$

The measurable cost c of playing the game is \$0.25, the price of a stamp. The expected value of this sweepstakes game is then

$$E(x) = \frac{1}{3{,}078}(\$20) + \frac{1}{2{,}093{,}333}(\$200) + \frac{1}{10{,}466{,}667}(\$5{,}000) +$$

$$\frac{1}{39{,}250{,}000}(\$10{,}000) + \frac{1}{78{,}500{,}000}(\$25{,}000) + \frac{1}{157{,}000{,}000}(\$50{,}000 + \$117{,}000$$

$$+ \$5{,}000{,}000) - \$0.25 = \$0.0415 - \$0.25 = -\$0.2085.$$

It is usually recognized that there is a loss associated with entering a state lotto game, but there is also a loss associated with a sweepstakes game that is not as readily apparent. In the example given above, there is a loss of approximately \$0.21 each time the player enters the sweepstakes.

The concept of expected value is a helpful idea to use in assessing the wisdom of participating in many situations involving prizes and costs. There are several games that may be used in a classroom situation to demonstrate the decision making process and the statistical principles involved. At least one of the games would also be interesting from a sociological standpoint.

The first game is one in which each student is awarded five points per day, and the first student to reach 100 points is the winner of ten bonus points to be added to his/her test and homework scores. If there is more than one winner (as described so far, everybody wins), then the bonus points would be split among the winners. Thus, a change is necessary to provide the possibility for the game to have a single winner. One possibility: if a student correctly guesses three numbers from 1 to 100, the student wins 75 points, which are added to his or her game total. The numbers could be picked from a table of random numbers to assure that each number is equally likely.

The second game involves a cost factor. The structure of the game would be the same as in the previous situation. However, if a student wishes to attempt to guess the three numbers, a charge of two points could be assessed the student.

In the third game, an unfair situation could be created by giving each student a different number of points from 1 to 25. The daily points plus any winnings minus the cost factor would be added to the initial number of points. The winner of the game would be the first person to score 100 points. Again, if there were more than one winner, the bonus points would be divided equally among the winners. This game would seem to be the most interesting of the games since it probably mirrors life most closely. Most people do not have equal assets to begin the game of life, and it might be interesting to observe the strategies developed by the students in an unfair game.

Of course the instructor should feel free to assign any number to the daily points, to the winnings, and to the cost factor. The only real requirement is that the chance of winning is very small and the payoff is large. Some adjustments should be made in each person's total of test points after the game is played so that the unfairness of the game does not affect the student's grade.

In a cryptarithm each letter represents a digit, and different letters represent different digits. Try these created by Miami University elementary education majors:

SAM	PAY	SAD	HOW
+ CAN	+ THE	+ DAD	+ ARE
WIN	MAN	CRY	YOU
Amy Feucht	Stacey Stewart	Janeane Tarala	Kelly Branum