SIMULATING PROBABILITY EXPERIMENTS --A PROBLEM-SOLVING STRATEGY

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1. Introduction

Simulation is one of the more powerful and more useful of the problem-solving strategies that students can readily acquire. The methods of simulation will be illustrated by a natural probability experiment that is closely related to the conditional probabilities discussed in recent articles of the Mathematics Teacher (see, for instance, Flusser--December 1984; Johnson, Koppelman, Stueben--May 1985; Bemis, Gorfin--March 1986).

The focus of these articles was the (theoretical) probabilities of variants of Bertrand's box paradox and arguments supporting various methods of determining such conditional probabilities. We consider a natural probability experiment that may be considered as a variant of the box paradox. However, we deal with empirical probabilities rather than theoretical probabilities.

The strategy of simulation may be more meaningful to students when they devise and carry out their own plans for solution, when the probability experiment is one to which they can easily relate and that seems natural to them in an everyday sense, and when the (theoretical) probability is not obvious to them, even though the experiment is quite simple.

2. The Experiment

Consider a population consisting of all two-child families, where it is assumed that the gender of any child is as likely to be male as female. Suppose that a family is randomly selected from this population. Also, suppose you know the following fact, and only the following fact, about this family: "There is a female child in the family." Under these circumstances, what is the probability that one child in this family is male?

We have found that most students cannot determine this probability in a reasonable amount of time and when pressed to guess the probability, they usually respond "1/2," reasoning that the "other child" is either male or female.

3. The Simulation

The experiment can be simulated so that numerous samples can be obtained and an empirical probability determined from the samples. For instance, the results obtained from tossing a pair of coins can be identified with a randomly selected family from the population of all two-child families. One could take head on each coin to represent female child and ignore the tosses resulting in tail-tail. By repeating the coin tossing a large number of times and determining the relative frequency of tosses resulting in a tail showing, one could obtain easily an empirical probability from the samples.

The simulation by coins requires considerable time just in the physical acts of mixing and tossing the coins. These physical motions can be avoided, however, by a simulation using random digits, obtained from a table, a calculator, or a computer.

The random digits can be grouped in consecutive pairs, representing the two children in a family in our population. If--say--even digits represent females, and odd-odd pairs are ignored, then one can determine quickly the relative frequency of pairs resulting in an odd digit showing. Thus, an empirical probability is obtained easily and quickly.

We recorded 110 usable pairs of digits generated by a hand-held calculator. The pairs having both digits odd were ignored. The pairs having exactly one even digit numbered 69; thus, the empirical probability of a family of two-children having one child of each gender, given there is a female child in the family, is 69/110 = .63, based on 110 randomly selected samples.

4. Conclusion

Although a computer program could be written easily to simulate the coin tossing or the generation of pairs of random digits and to record the data, the lack of availability of a computer and the time required to write a program may be deterrents to using a computer based problem-solving strategy. However, calculators with random number generation capabilities and tables of random numbers are both readily available and hand tabulation is quite efficient for basic simulations of the type described here.

Of course, the theoretical probability is easily seen to be 2/3 (two of the three equally likely outcomes female-female, female-male, and male-female are favorable), but the point of this little exercise is not to have the students be shown convincing arguments of theoretical solutions, but rather to equip students with additional problem-solving strategies to be used as tools and to give them experience in using the tools.

Finally, this particular example has possibly unexpected utilitarian value. At the next social event attended by your students, they can apply their new knowledge of conditional probabilities to the Jones family--a two child family, one child being named Jane--by guessing that Jane has a brother. The odds your students are correct are two to one!

ERRATA

The Editor regrets that in the listing of those who have served as president of OCTM, two errors were made. The president in 1980 - 1982 was Joseph Kern and not William Kern, and the president in 1986- 1988 is William Speer and not William Steer.