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Methods of Physical Geodesy

by

W. A. Heiskanen
and
H. Moritz

Prepared for
Air Force Cambridge Research Laboratories
Office of Aerospace Research
United States Air Force
Bedford, Massachusetts

Contract No. AF 19(628)-1628
Project No. 7600
Task No. 760002

The Ohio State University
Research Foundation
Columbus, Ohio 43212

June 1964

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INSTITUTE OF GEODESY, PHOTOGRAMMETRY AND CARTOGRAPHY

W. A. Heiskanen, Director

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METHODS OF PHYSICAL GEODESY

by

W. A. Heiskanen and H. Moritz

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FOREWORD

This report was prepared by Dr. W. A. Heiskanen, Director, and Dr. H. Moritz, Research Associate, of the Institute of Geodesy, Photogrammetry and Cartography of the Ohio State University, under Air Force Contract No. AF 19(628)-1628, OSURF Project No. 1474, under the supervision of Dr. Weikko A. Heiskanen, Director of the Institute. The contract covering this research is administered by the Air Force Cambridge Research Laboratories, Office of Aerospace Research, Laurence G. Hanscom Field, Bedford, Massachusetts, with Mr. Owen Williams and Mr. Bela Szabo, Project and Task scientists.

ABSTRACT

This report gives a reivew of the progress of physical geodesy as of 1963. The latest developments are given briefly in the Final Report (No. 31) on this effort.

Section 1 is the historical introduction; section 2, the gravity measurements and gravity material; section 3 deals with the gravimetric and astronomic measurements as well as astrogeodetic methods. Also the obtained accuracy of the geodetic field base lines, calibration base lines, and standard base lines are explained with several tables.

The geopotential includes the theory of the gravimetric, astrogeodetic, and combined methods. Section 5, Constants of the Earth, includes the equatorial radius, equatorial gravity, and harmonic coefficients of the earth's gravitational field. The flattening and normal figure of the earth and perturbing forces at Satellite altitudes is also discussed.

Methods of Physical Geodesy

by

W. A. Heiskanen and H. Moritz

1. Historical Introduction

The fundament of physical geodesy was established already in the middle of the 1800's by the quartet of Cambridge, England: Green, Pratt, Airy, and Stokes. Green and Stokes set up the mathematical fundament of physical geodesy; Pratt and Airy were the fathers of isostasy which plays an important role in physical geodesy. Stokes' formula tells in mathematical form that the shape of the earth can be determined if the gravity anomalies are available around the earth.

This formula had only academic interest until needed gravity observations were available. That happened to some extent when in the 1920's Vening Meinesz explored part of the oceans gravimetrically. This information, in addition to the quite extensive gravity material measured in America, Europe, and in India, and in smaller amounts also in other places, gave hints that the Stokes' formula can be applied on a world-wide scale.

The purpose of physical geodesy was spelled out in 1951 in Heiskanen's paper, "On the World Geodetic System, pp. 22-23. The gravimetric method, either alone or together with astronomical determinations can give the following results:

"1. A <u>general World Geodetic System</u> and the data to convert the existing geodetic systems (North American, European, Indian, etc.) to this system.

"2. The geodetic coordinates, in the World Geodetic System, of any needed point in the world where astronomical observations exist or which is plotted on a local map with a reliable grid.

"3. The distances and directions along the reference ellipsoid, between any required points in the world.

"4. The conversion of these distances and directions from the ellipsoid to the geoid.

"5. The control of maps on 1:100 000 and smaller scale.

"6. The real shape of the geoid.

"7. The reduction of triangulation base lines from the geoid to the reference ellipsoid.

"8. The corrections of the closure errors of the triangles caused by the deflections of the vertical.

"9. The corrections to the used reference ellipsoid.

"10. The correction to the used gravity formula.

"In addition, new knowledge about the structure of the earth's interior close to the earth's surface can be obtained."

Later on have come other geodetic problems which also can be solved gravimetrically.

1. The continuation of the gravity anomaly field and the geopotential to different altitudes, which are now of basic significance. This problem has been studied quite extensively in Columbus during the last years.

2. The determination of the gravity field at the satellite altitudes, and the derivation of the geodetic and gravimetric constants for the earth. These studies, which are intended to link together physical and satellite geodesy, have been started quite recently and will continue.

The undulations of the geoid were computed gravimetrically first on world wide scale by R. A. Hirvonen [1934]. He computed the geoid at 62 points between the latitudes 10° and 60°. This study was methodically right, but the scarce material did not permit getting reliable values. L. Tanni [1948] computed the geoid on a basis of much larger material at 218 points in such areas where a reasonable amount of observations were available. He applied isostatic gravity anomalies. His geoid covers all of Europe to 60° latitude; almost the whole United States, and India, Indonesia, and Russia between 30° and 60° longitude; and a belt, 15° broad, over the Atlantic.

Encouraged by the results of Hirvonen and Tanni we started, in Columbus, Ohio [1951], to collect and analyze material from around the world and at the same time study and develop the theoretical background for the computations of the geoid. First result was the "Columbus Geoid" presented in 1956 in the annual meeting of the American Geophysical Union and published in 1957 [Heiskanen, 1957]. Some details on this computation will be given in Section 4.1.

2. Gravity Measurements. The Gravity Material

The basis of physical geodesy is the gravity anomalies which should cover the whole surface of the earth, and should all be computed in the same system.

We have two kinds of gravity observations: absolute measurements and relative measurements. In the absolute measurements the gravity g itself is measured. In the relative observations one measures only the ratio g/g_0 between the gravity g at the field point and gravity g_0 at the base station. Then $g = (g/g_0) \cdot g_0$.

The absolute gravity observations we need, theoretically speaking, only at one point. Gravity at all the other points of the earth's surface can be obtained relative observations. For practical reasons, however, (and because of national vanity), it has been measured at a dozen stations.

In geodetic computations we operate with the gravity anomalies which are different. Therefore, we need not know the absolute gravity very accurately for geodetic purposes ([Heiskanen and Vening Meinesz, 1958], p. 76). Most important is that all gravity observations are in the same system.

This is the reason why absolute gravity is known only with the accuracy of several milligals. During the last years, however, the interest in accurate absolute gravity measurements has increased considerably, because we can determine the product GM (gravitational constant x earth's mass) from absolute gravity with high accuracy, which is needed for celestial geodesy. A score of different methos has been invented: some of them are in test phase; some others have already been used. The modified Cambridge pendulum, the Gulf Oil pendulum, the method of a falling body, and the method of very long pendulum seem to guarantee a relative accuracy of g essentially better than 1:100 000. If an accuracy of 0.1 mgal can be obtained, as several scientists believe, this corresponds to the relative accuracy of 1:10 000 000.

New methods of relative gravity measurements (shipborne and airborne gravimeters) will gradually fill the empty or unsufficiently covered areas. In the meantime, we have to "guess" values in this area, i.e., to extrapolate or to predict gravity anomalies. For this purpose we may use isostasy (zero isostatic anomaly [Heiskanen and Vening Meinesz, 1958], p. 72), isostatic-topographic methods (Kivioja, work in progress), statistical methods ([Hirvonen, 1956], [Kaula, 1959 a,b], and [Rapp, 1962], [Moritz, 1962 c]), or a combination of them. Another possibility is a low order development in spherical harmonics, as proposed in [Molodenskii et al., 1962], p. 164); see also Section 4.1.

Isostatic and statistical methods are best combined, but even then the accuracy is not very high. The weakest point of all these methods since correlation influences the accuracy of gravimetric computations to the same degree as the standard error of gravity itself. On the other hand, a reliable low order development in spherical harmonics is not yet at hand. The present status of observed gravity anomalies can be seen in the following map prepared on the basis of the material collected at the Ohio State University during 12 years, which is taken from [Uotila, 1959] (Figure 1).

3. Geodetic and Astronomic Measurements. The Astro-Geodetic Method.

Some decades ago the accuracy of 1:200 000 in triangulation was sufficient for most purposes. Now much higher precision is needed. Fortunately, also much better measuring methods exist. We need to know the equatorial earth radius at least with a standard error of 6 m, which means that the length measurements have at least an accuracy of 1:100 000. Therefore, the highest possible precision in the triangulation and/or in the electronic measuration methods is necessary. The progress of last decades shows that this accuracy not only can be but has been already obtained. The triangulation of the Baltic Geodetic Commission around the Baltic Sea, accomplished in 1924-48, gave a standard error of 2.46 m for an arc of 3000 km, i.e., a relative accuracy of 1:1 200 000 [Ölander, 1948]. To get this accuracy, the precision of the <u>field</u> base lines must be higher. Also, that has been accomplished. As Table 1 indicates, the mean accuracy of the 15 Finnish base lines is 1:4 800 000 [Heiskanen, 1960, a].

5°x5° MEAN FREE AIR ANOMALIES

DECEMBER 31, 1959

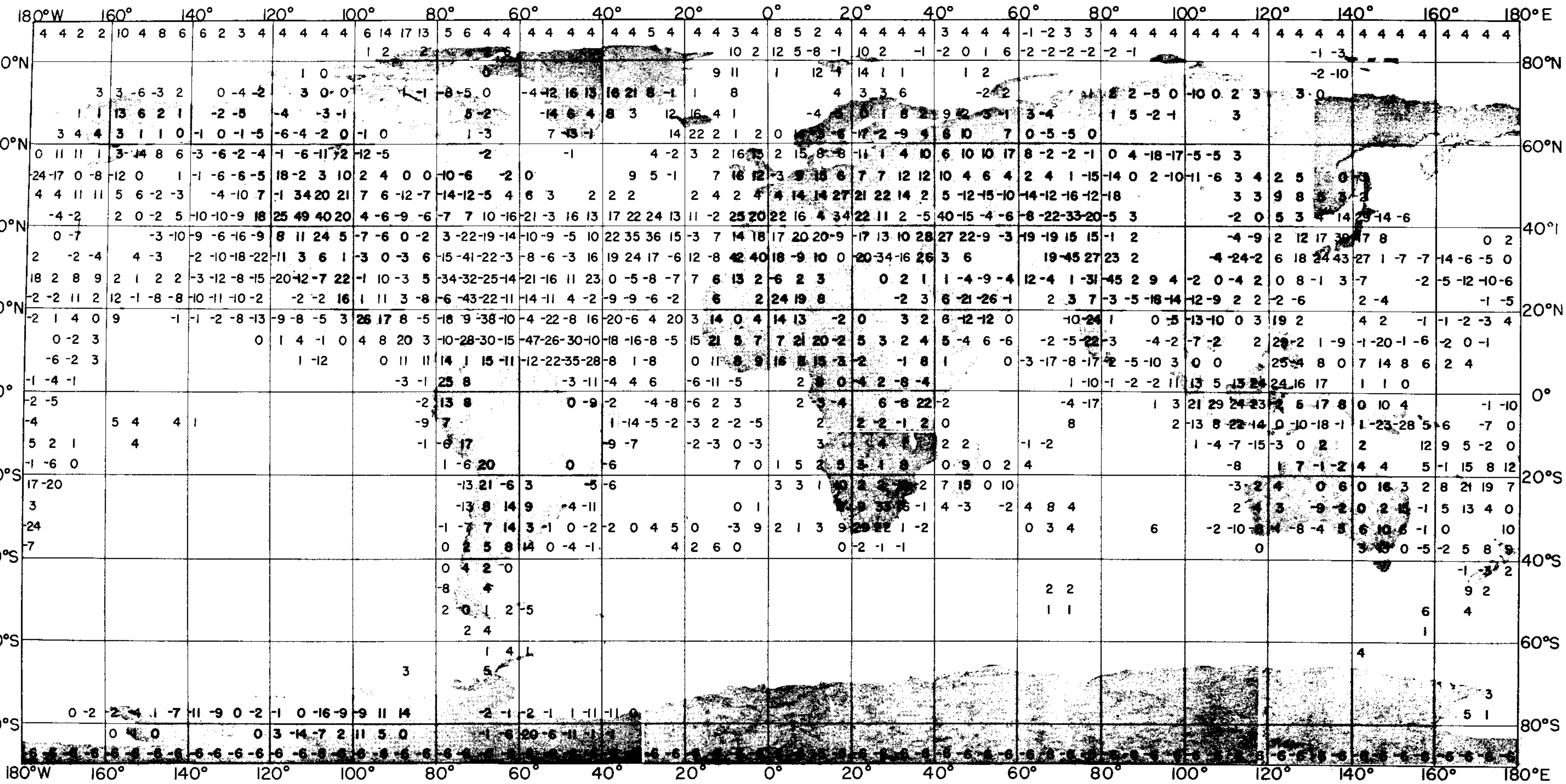


Fig. 1

Table 1

BASE LINES OF THE FINNISH GEODETIC INSTITUTE

Year	Base Line	Length m	Standard Error mm
1923	Saltvik	2615.3592	±1.0
1923 and 1929	Hanko	5882.8506	±1.0
1925	Lappträsk	4032.3154	±1.9
1928	Jääski	3239.6002	±1.1
1928	Maaninka	4605.1316	±1.3
1930	Ilmajoki	6193.3767	±0.8
1934	Viljakkala	3645.2047	±1.0
	Otava	3983.5766	±0.9
1935	Soanlahti	4128.1932	±1.0
1939	Oulainen	4415.6034	±0.8
	Kuhmo	3455.5015	±0.7
1949	Rovaniemi	3839.6920	±2.0
1952	Kuusamo	4990.4223	±0.8
1958	Laanila	3742.2676	±0.8
1961	Vihti	6049.9046	±0.5

To obtain this accuracy in the field base lines, we must have standard base lines whose precision is still higher. Such standard base lines based on the light interference method and measured by the Väisälä comparator have been established and measured in Nummela, Finland, 1947; Buenos Aires, Argentina, 1953; Loenermark, Netherlands, 1957; Munich, Germany, 1958; and Lisbon, Portugal, 1962. The accuracy of these base lines can be seen in Table 2.

Table 2

STANDARD BASE LINES MEASURED WITH VÄISÄLÄ LIGHT INTERFERENCE COMPARATOR

	Year	Length m	Standard Error mm
Finland: Nummela	1947	864.12189	±0.05
Finland: Nummela	1952	864.12155	±0.06
Finland: Nummela	1955	864.12149	±0.08
Finland: Nummela	1958	864.12137	±0.06
Finland: Nummela	1961	864.12146	±0.06
Argentina: Boenos Aires	1953	480.00178	±0.05
Netherlands: Leonarmark	1957	576.09226	±0.05
Germany: Munich	1958	864.06427 *	±0.09
Germany: Munich	1961	864.06427	±0.13
Portugal: Lisbon	1962		

*Measured in two parts

Side by side with the standard base line we have the calibration base line, or lines, for testing the length of the invar wires to be used in the measurement of field base lines. Table 3 gives the results of the calibration of eight invar wires September 5-9, September 20-22, and October 2-4, 1961, at the Nummela standard base line in Finland.

Between these calibrations the Vihti field base line of 6 km was measured. The table shows that the average change of the wires during a period of one month was 18 x 10^{-3} mm, or 1: 1 250 000.

Table 3

CALIBRATIONS OF INVAR WIRES FOR THE VIHTI BASE LINE

Wire No.	1961, Sept. 5-9 24 m	Sept 20-22 24 m	Oct. 2-4 24 m
635	±1.350 mm	±1.357 mm	±1.289 mm*
637	±0.606	±0.592	±0.608
1042	±1.043	±1.053	±1.048
1044	±0.477	±0.487	±0.462
1045	±1.086	±1.063	±1.064
1115	-0.725	-0.733	-0.756
1117	-0.454	-0.457	-0.468
1118	-0.948	-0.938	-0.946

*Accident between second and third calibrations

From these values one can interpolate the real length of the invar wires, during the field base line measurement, with the accuracy of 1:4 000 000.

Also, the modern angle measurements of the triangulation can give a precision not much less than $0''.2$ corresponding to the linear accuracy 1:1 000 000. In addition, the modern geodimeters and tellurometers--when calibrated with care--give at least an accuracy of 1:1 000 000 as several test measurements indicate.

In order not to lose accuracy in the reduction of the base lines and of the observed angles (and distances) to the ellipsoid, we must know the geoid undulations.

As to the precision of the astronomic latitude φ' and longitude λ' of the astrogeodetic points of the triangulation, a standard error of $0''.2$, corresponding to a linear accuracy of 1:1 000 000, can be obtained. φ' and λ', however, refer to the geoid, but we need the geodetic latitude φ and longitude λ which refer to the ellipsoid. φ and λ are obtained from the equations

$$\varphi = \varphi' - \xi$$

$$\lambda = \lambda' - \eta_g \cos \varphi$$

where ξ and η are the gravimetrically computed-absolute-vertical deflections. Consequently, the precision of the important astrogeodetic method depends a great deal on the accuracy of the gravimetric deflections of the vertical; the better the gravity station net the more accurately they can be obtained.

All these methods together are necessary to determine the equatorial radius of the earth (Sec. 5.1), and (in combination with height measurements and geoid undulations) to fix the position of points in an absolute spatial coordinate system. The attempt of Ledersteger [1951] to calculate the equatorial radius, and to connect America and Europe in this way, gave very promising results (±10 m in NS - direction and ±11 m in EW-direction). The Ellipsoid of Krassowskii (USSR) was determined in a similar way.

4. The Geopotential

4.1. Gravimetric Method

The geopotential function W is conveniently analyzed into a sphero potential U and a disturbing potential T so that

$$W = U + T.$$

The Spheropotential U is mathematically given; the disturbing potential T is connected with the undulation (geoid height) N by Bruns' theorem

$$T = \gamma N.$$

(γ = theoretical gravity), so that by finding N we obtain W at the same time.

The basic formula for N as a function of the gravity anomalies Δg was given by Stokes [1849]. This formula was extended and improved in several directions. Pizzetti [1911] extended it to arbitrary elevations. Sagrebin [1956] added terms of the order of the flattening.

By Stokes' formula we get the shape of the geoid, and we have to reduce gravity to the geoid. All the reduction methods are essentially equivalent (Jung [1956], p. 578, Moritz [1961b]), but for practical and geophysical reasons the isostatic reduction is prominent [Heiskanen and Vening Meinesz, 1958], p. 245.

Molodenskii (see [Molodenskii et al., 1962], p. 73) showed how to determine directly the potential on the earth's surface and its shape, without using the geoid. He gets an integral equation whose solution is, in first approximation, again given by Stokes' formula. Work in this direction has also been done by Hirvonen [1959, 1960] and others. A reconciliation of this "new theory" and the geoid has been given by Arnold [1961]. Another possibility is the "model earth" by de Graaff-Hunter [1957].

But also the classical reductions can be refined so that they are as accurate theoretically as the "new theory" [Moritz, 1962 a]. Therefore, all these solutions are equivalent theoretically and give, when properly applied, the same results. The gravimetric method, as such, is perfect; imperfect is only the material of gravity observations, since for a correct solution, gravity should be known all over the earth. But if this were the case, the figure of the earth and of the geopotential surfaces could be computed with very high accuracy, even over the oceans [Heiskanen 1951].

The accuracy of the geoid undulations depends not so much on a dense observation net but on a uniform coverage of the whole earth, since they are less affected by local irregularities of Δg than by larger gaps in the gravity material, even in very remote zones.

Since less accuracy in Δg is sufficient in these remote areas, the gaps might be filled out by a low order development in spherical harmonics. For instance, according to Molodenskii, et al., 1962, p. 164, if we approximate Δg outside a radius of 35° by a complete (not only zonal) development up to the 6th order, we get a standard error of N smaller than 1 meter. If the satellite observations could furnish all the necessary harmonics up to the 6th order, this development might be used for approximately taking the distant zones into account, as long as our gravity material is still insufficient. Another proposal along this line, but without detailed accuracy studies, is given in Jeffreys, [1962 , p. 144]. But even if we could use spherical harmonics for the distant zones, the geoid could, in this case, not be computed all over the earth.

The "Columbus Geoid" [Heiskanen, 1957] is computed on the basis of the free anomalies, corrected because of the elevation correction.

From these corrected anomalies the mean anomalies of squares of different size were evaluated.

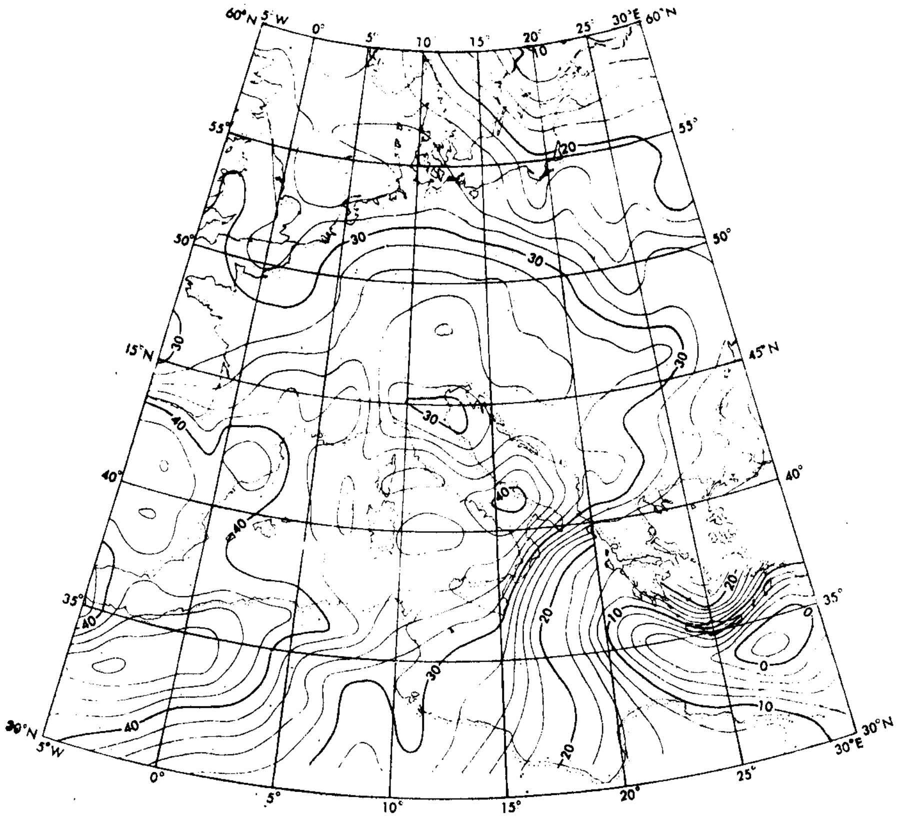
Where only a scarce gravity station net existed, we let the point value represent different sizes of squares and applied the weighted mean of point value and the isostatic anomalies zero. If we, for instance, used a point value 50 mgal as a representation of the squares; s = $0^{\circ}.5$, $1^{\circ}.0$, $2^{\circ}.0$, $5^{\circ}.0$, $10^{\circ}.0$ and $30^{\circ}.0$; the best mean anomaly values were 45, 40, 30, 16, 10, and 5 mgal. Outside of this boundary the isostatic anomaly 0 was used [Heiskanen and Vening Meinesz, 1958], p. 273.

This geoid has also been criticized; but in the long run, it seems to fit rather well in areas of reasonable gravity station net, with the geoid computed by other methods. No geoid was computed for the Southern hemisphere because of too scarce material. When we compare the geoids, we must remember that every gravity formula used corresponds to a different geoid.

As an example, a portion of the geoid in Europe of the Columbus geoid is repreduced. It refers to the International Ellipsoid, shown in Figure 2.

4.2. Astrogeodetic and Combined Methods

For the astrogeodetic geoid determination (astronomic leveling), observational material (astrogeodetic deflections of the vertical) over limited areas is sufficient, but one is able to determine the



The European geoid computed in Columbus, Ohio, 1954—57. The curves joining the same N-values are drafted at 2 m intervals. They indicate the shape of the geoid in Europe. In this area, the geoid is from 10 to 40 m too high, above the ellipsoid. The irregularities of the geoid are caused by the mass irregularities in Earth's interior.

Figure 2

geoid in these areas only. An example of the computation of the deflection of the vertical and the undulations of the geoid is the determination of these quantities in Finland by Ölander [1944]. A dense net of astronomic stations is necessary. If the astronomic stations are more distant (100 km, say), it is feasible to interpolate deflections gravimetrically, since the effect of the outer zones is almost constant (astrogravimetric leveling [Molodenskii et al., 1962]).

Astronomic leveling thus determines limited portions of the geoid, but only their shape and not their absolute orientation. However, with a good observational material, we can get a very high relative accuracy (±1 dm). The gravimetric method, on the other hand, gives the shape and absolute orientation of the geoid over the whole earth; but the practically obtainable accuracy is probably somewhat less (about ±1 m). For a worldwide geodetic system, the astrogeodetic method has to be supplemented by an absolute orientation either by physical geodesy, by satellite connection or by both methods. A tentative combination of the existing gravimetric, astrogeodetic, and satellite data to a world geodetic system was done by Kaula [1961]. [See also Fischer, 1959, 1960].

5. Constants of the Earth

5.1. The Equatorial Radius

For the scale problem of astronomy and geodesy, it is necessary to know the earth's equatorial radius as accurately as possible, to 10^{-6} relative accuracy or better. As already mentioned, this can be obtained by a combination of base line measurement, triangulation, and physical geodesy. Base lines furnish the scale, which is extended by triangulation to a larger net; and physical geodesy, combined with astronomic observations, gives the absolute position of the net with respect to the earth, thereby relating the size of the earth to the size of the net and obtaining the earth's radius in the scale of the net. (Actually, the earth's equatorial radius, since it depends on the reference ellipsoid used, is only a convenient visualization of the scale of the earth.)

5.2. The Equatorial Gravity

Absolute gravity must be measured on at least one point and then, by relative measurements, transferred to the equator. More rigorously, equatorial gravity, like the equatorial radius, is an idealized constant: it is the first term of the normal gravity formula, which is connected with an expansion of the gravity field of the earth in spherical harmonics; it furnishes "the scale of gravitation."

Equatorial gravity is needed for:

1. The gravitational attraction of the earth, which is indispensable for astronomy and satellite geodesy,

2. The product GM which is a fundamental constant.

5.3. The Harmonic Coefficients of the Earth's Gravitational Field

The expansion of the gravitational potential V of the earth (disregarding the centrifugal force) can be written

$$V = \frac{GM}{r} \left\{ 1 - \sum_{n=2}^{\infty} \left(\frac{a}{r}\right)^n \left[J_n P_n (\sin\varphi) + \sum_{m=1}^{n} J_{nm} P_{nm} (\sin\varphi) \cos m\lambda + \sum_{m=1}^{n} K_{nm} P_{nm} (\sin\varphi) \sin m\lambda\right]\right\}$$

G is Newton's gravitational constant; M the mass of the earth; r the radius vector; a the equatorial radius; φ, λ are geocentric coordinates; P_n and P_{nm} are conventional harmonics (zonal and tesseral); and J_n, J_{nm}, and K_{nm} are constant coefficients, characterizing the gravitational field of the earth.

This series can be well applied for higher elevations, say, for perturbing forces at satellite altitudes.

For terrestrial geodesy its significance is much less, for <u>the series is divergent on the earth's surface</u>. Therefore, even if we would know all the coefficients J_n, J_{nm}, and K_{nm}, we could not obtain the geopotential W(V + centrifugal potential) or the undulations N on the surface of the earth.

It converges only outside the smallest sphere S

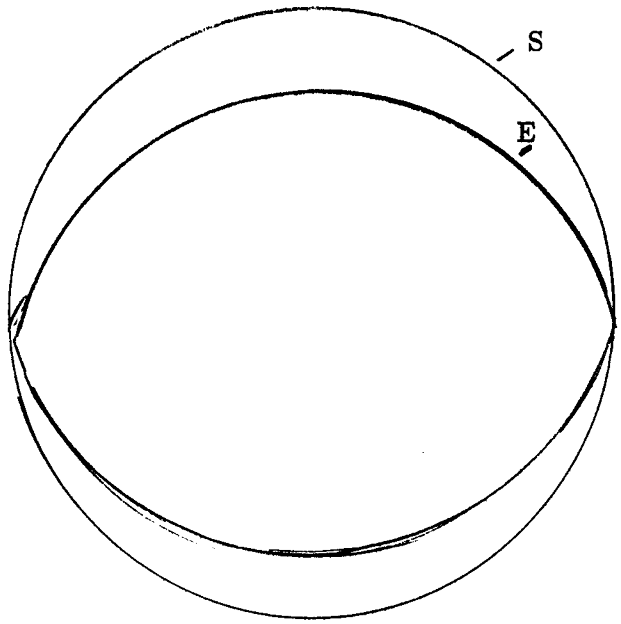


Figure 3

with the center in the earth's center, which completely encloses the earth [Moritz, 1961a]. The maximum distance between this sphere S and the earth E (at the poles) is about 20 km (Fig. 3).

On the earth, the only thing we can get by spherical harmonics is the potential field in large outlines, which suffices for determining the flattening of the earth, or for the influence of the distant zones, in Stokes' formula, but not for the other geodetic purposes. (Because of the divergence of the series, only low order terms can be used.)

Nevertheless, if gravity is known all over the earth, the harmonic coefficients can be computed to any order. Rigorously, the gravity anomalies should first be continued upward to the sphere S (Fig. 3), so that we have values on a sphere. Then, the harmonic coefficients are easily computed by an integration over the sphere. As long as we have no better material, such an upward continuation is, of course, purely academic.

The most recent computation of the lower spherical harmonics is that by Uotila [1962], where one also finds references to earlier work.

Accuracy studies on the computation of harmonic constants from gravity observations are given in another paper (Moritz) of this report. If, for instance, we assume an idealized gravity net with one station in each block 100 x 100 km (about $1^\circ \times 1^\circ$) all over the earth, the accuracy of the first zonal harmonic coefficients is given by the following table.

Table 4

Standard errors of the first zonal harmonic coefficients obtained from a gravity station net of one point per $1^\circ \times 1^\circ$ block all around the world

n	$m\{J_n\}$
2	0.2×10^{-6}
3	0.1×10^{-6}
4	0.07×10^{-6}
5	0.06×10^{-6}
6	0.05×10^{-6}

For the first few harmonics this is only slightly better than the values given by the authors, who determined the coefficients from satellite observations; but contrary to the satellite method, accuracy will improve for higher-order terms (approximately proportional to $\frac{1}{\sqrt{n}}$) and there is no correlation between the errors of the coefficients. The reason for this is that each coefficient is computed independently of the others, in contrast to the determination from orbital elements. Since correlation is as important an accuracy factor as the standard error, this is significant, too.

5.4. Flattening and Normal Figure of the Earth

From the low-order zonal harmonic coefficients, the constants of the "normal figure" of the earth (figure of complete hydrostatic equilibrium), above all the flattening, can be computed. Formulas are given, e.g., in [Heiskanen and Vening Meinesz, 1958], pp. 45-59. A remarkable new attack on this old problem is due to Ledersteger [1959, 1960, 1962]. A different method for determining the earth ellipsoid is described in [Molodenskii et al., 1962], p. 117.

6. Perturbing Forces at Satellite Altitudes

Formulas have been developed by Hirvonen ([1952], [1959], unpublished manuscript 1962). We may either use Δg directly or the "coating" with density

$$\mu = \Delta g + \frac{3\gamma}{2R} N,$$

provided we know the undulations N. The formulas using μ are simpler, and the remote zones have less influence. A detailed treatment will be given in a report "Computational Methods for the External Gravity Vector," which is in progress.

A preliminary investigation on the accuracy of this computation and on its influence on satellite orbits is found in a previous report on this Project (Moritz [1962d]). If, for instance, we again assume one observation in each 100 x 100 km block all over the earth, we find approximately the values of Table 5.

Table 5

Standard errors of the x-, y-, z- component of the gravity vector at 500 km elevation, of semi-major axis a, excentricity e, inclination i, and node Ω of a nearly circular orbit computed from a gravity station net of one point per $1^\circ \times 1^\circ$ block all around the world

$$m_x = m_y = 0.3 \text{ mgal},$$
$$m_z = 0.5 \text{ mgal},$$
$$m_a = 7 \text{ meter},$$
$$m_e = 1.0 \times 10^{-6}$$
$$m_i = m_\Omega = 0.11''$$

Naturally, this represents an ideal which is by no means already obtained. The large unobserved areas considerably impair the accuracy. The effect is similar as in the computation of the undulations N and of the harmonic coefficients.

As a matter of fact, once we have a reliable development in spherical harmonics, this might be profitably used for the computation of the perturbing forces, too.

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The following abbreviations are used:

IAG	International Association of Geodesy
MCRL	Mapping and Charting Research Laboratory, The Ohio State University
IGPC	Institute of Geodesy, Photogrammetry and Cartography, The Ohio State University
IIIAG	Isostatic Institute of the International Association of Geodesy

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