Problem Solving is About Seeing Relationships

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Introduction

As you read this installment of *Contest Corner,* the 2011 State Tournament of Mathematics will be history and your attention will be on other competitions or perhaps our 2012 tournament. The *Math to Success* web site (www.mathtosuccess.com) outlines five important steps to success in mathematics and mathematics contests that are based on the simple concept – keep practicing. The steps include:

1. Presenting students with specially tailored questions;
2. Providing students with opportunities to compete with others in the same grade;
3. Providing instant feedback after each practice with correct answers and hints/ explanations;
4. Giving students a chance to correct questions previously missed by providing similar questions in the next practice.

Past installments of *Contest Corner* have provided short examples of in-class competitions. Mathematics is about seeing relationships and discovering patterns. To illustrate, let's consider a problem (Flick, 1981, p. 546) that is based in high school Geometry and AP Calculus.

A. A right circular cylinder of height y is rolled from a rectangle of width x, length y, and perimeter $2x + 2y = p_1$ (where p_1 is any positive real number) as seen in Figure 1. Find the values x and y (in terms of p_1) that will maximize the volume of the cylinder.

B. Another right circular cylinder is formed by revolving a rectangle about one of its sides. The rectangle has perimeter p_2 (see figure 2). Find the dimensions of the rectangle that maximizes the volume of this cylinder.

C. State a conjecture regarding the dimensions of the cylinder formed in A and B.

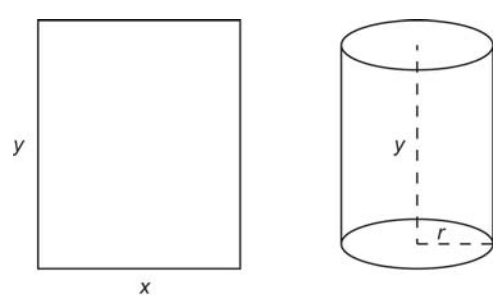

Fig 1 Right circular cylinder formed from rolled rectangle

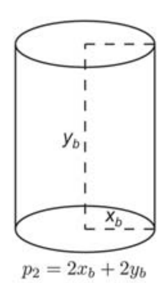

Fig 2 Right circular cylinder formed by revolving a rectangle

Solution to Part A

Let x = circumference (C) of the circle forming base of the cylinder with radius r and height y (because the rectangle is bent to form the cylinder). Therefore, $C = x = 2\pi r$. Solving for r gives $r = \frac{x}{2\pi}$. The perimeter of the rectangle is $p_1 = 2x + 2y$ with its height y. Therefore, $h = y = \frac{p_1}{2} - x$. Since the volume of a cylinder is $V = r^2 h$, the volume of the cylinder is given by

$$V = \pi \left(\frac{x}{2\pi} \right)^2 \left(\frac{p_1}{2} - x \right) = \frac{p_1 x^2 - 2x^3}{8\pi}$$

The maximum volume is found by finding $\frac{dV}{dx} = \frac{p_1 x - 3x^2}{4\pi} = 0$.

Thus $x = \frac{p_1}{3}$ and $y =_1 2 - \frac{p_1}{3} = \frac{p_1}{6}$.

> *Answer*: The dimensions of the rectangle in Part (A) are $x = \frac{p_1}{3}$ and $y = \frac{p_1}{6}$.

Solution to Part B

In Figure 2, the cylinder is formed by rotating the given rectangle about the side y_b and x_b is the radius. Therefore, its volume is given by V_2 = \pi \left(x_b \right)^2 \left(y_b \ right). As in Part A, $p_2 = 2x_b + 2y_b$, so $y_b = \frac{p_2}{2} - x_b$.

Hence, $V_2 = \pi \left(x_b \right)^2 \left(\frac{p_2}{2} - x_b \right) = \frac{p_2 \pi (x_b)^2 - 2\pi (x_b)^3}{2}$.

The maximum is found by finding
$\frac{dV_2}{dx_b} = p_2 \pi x_b - 3\pi \left(x_b \right)^2 = 0$. Thus $x_b = \frac{p_2}{3}$ and $y_b = \frac{p_2}{6}$.

> *Answer*: The dimensions of the rectangle in Part (B) are $x_b = \frac{p_2}{3}$ and $y_b = \frac{p_2}{6}$.

Solution to Part C

This problem is an example that allows students to see and discover relationships. Questions (A) and (B) are general problems. We discovered this relationship when a student was solving a specific case of question (A). The student misinterpreted the question and worked it as in question (B). The similarity seemed curious so we then solved the general problem given here to discover the relationship concluded in (C).

> *Answer*: The dimensions of the rectangle that maximizes the volume of the cylinder of revolution are the same dimensions needed to maximize the volume of the cylinder rolled from the rectangle.

Mathematics is about problem solving and discovery. Keep your students practicing!

References

Flick, T. M. (1981). Encounter with introductory calculus. *The Mathematics Teacher, 74*(7), 546-547. National Council of Teacher of Mathematics.

Math to Success Systems. (2011). *Math to Success: Where Practice Brings Success*. Retrieved January 18, 2011 from http://www.mathtosuccess.com/.

Think About It!

ENGAGE THE BRAIN!

"One further aspect of attention became evident after IP [information processing] theorist conducted studies of skill acquisition. Studies showed that when people are first learning a skill, they have to think about what they are doing."

Byrnes, J. P., (2001). *Minds, brains and learning: Understanding the psychological and educational relevance of neuroscientific research, 76.* The Guilford Press. New York.

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