USE COMPUTERS WITH CARE

Maita Levine and Ray Rolwing
University of Cincinnati
Cincinnati, Ohio

In a widely used calculus textbook, the student is encouraged to use a computer in the numerical investigation of limits. As an example, students are instructed to calculate $(1+x)^{1/x}$ for x = 0.1, 0.01, 0.001, and so on, until the differences between two successive values of the function differ by less than 0.000001. The text then expresses the hope that the limit will have been found accurate to five decimal places and the following BASIC program is supplied to assist the student:

```
10    x = 1
20    E = 2
30    x = x/10
40    F = (1+x)↑(1/x)
50    PRINT x;F
60    IF ABS (F–E), 1.0E – 06 THEN GO TO 90
70    E = F
80    GO TO 30
90    PRINT "LIMIT REACHED"
100   END
```

Finally, the student is told that when the program is executed, the following output is generated:

x	$(1+x)^{1/x}$ (rounded to 5 decimal places)
0.1	2.59374
0.01	2.70481
0.001	2.71692
0.0001	2.71815
10^{-5}	2.71857
10^{-6}	2.71828
10^{-7}	2.71828
10^{-8}	2.71828

LIMIT REACHED

Unfortunately, on many computers the program doesn't work. The output from an IBM personal computer is given below:

x	$(1 + x)^{1/x}$ (rounded to 5 decimal places)
0.1	2.59374
0.01	2.70481
0.001	2.71678
10^{-4}	2.71667
10^{-5}	2.69542
10^{-6}	2.48166
10^{-7}	2.28484
10^{-8}	1
10^{-9}	1

LIMIT REACHED

And, the student concludes that the value of e is 1! A similar conclusion is reached using an NCR PC 4 (both single precision and double precision), IBM PC (double precision), and VAX (FORTRAN single precision).

Correct results were obtained from a FORTRAN double precision program on the VAX (for $x = 10^{-8}$, the computed value of the function to 5 decimal places was 2.71828) and from a Radio Shack EC–4004 calculator (for $x = 10^{-9}$, the computed value of the function to 9 decimal places was 2.718281827). It was impossible to get the wrong answer on the calculator since it isn't possible to key in $1 + 10^{-10}$. Hence, the student would correctly conclude that e is approximately 2.71828.

Attempts to deal with the problem on two additional calculators and on the Macintosh Computer produced similar results.

A program for the TI58C is given below:

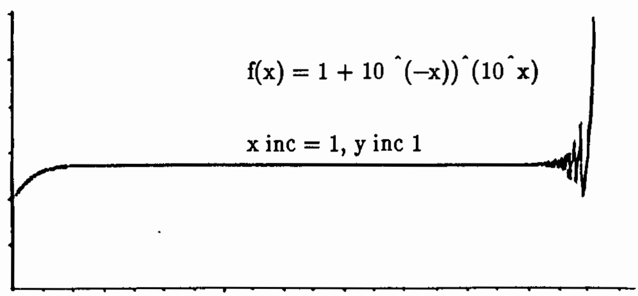
```
LRN
.1
STO 01
LBL
COS
RCL 01
+
1
=
STO 02
1
÷
RCL 01
=
STO 03
RCL 02
y^x
RCL 03
=
STO 04
```

RCL 04
PAUSE
RCL 01
÷
10
=
STO 01
GTO COS
R/S
LRN
RST
R/S

The output from the program is

2.59374246
2.70481383
2.71692393
2.71814593
2.71826824
2.71828407
2.71828183
2.71828183
2.71828183
2.71828183
2.71828183
2.71828183

Using the HP 28C manually gives an answer of 2.71828182845 for $x = 10^{-11}$. And, it is not possible to key in $1 + 10^{-12}$. Finally, the Math View Professional software program on the Macintosh SE computer produced the printout shown below. It is interesting to note that the graph shows the correct answer for $x < 10^{-17}$, it oscillates between $x = 10^{-17}$ and $x = 10^{-19}$, then continues to return the value 1.



All hardware and software tools must, of course, ultimately give a value of 1 for $(1 + x)^{1/x}$ because for small values of x, their degree of precision is not sufficient to carry out the computation. The advantages of using a non–programmable calculator, however, are the modest financial investment and the student's inability to key in a small enough value of x to produce the answer 1.

The moral of the story? Although computers are certainly a useful teaching tool, they must be used with caution. Rounding errors can produce surprising and incorrect results. And, in a first quarter calculus course, with students whose computer science background is minimal, it's helpful to discuss roundoff errors in connection with computer assignments.

References

Pollack, Seymour V., Ed., Studies in Computer Science, Studies in Mathematics, Vol. 22, M.A.A.; Washington, DC, 1982.

Sicks, Jon L., Investigating Secondary Mathematics With Computers, Prentice–Hall; Englewood Cliffs, NJ, 1985.

Smith, Jon M., Scientific Analysis on the Pocket Calculator, John Wiley & Sons, Inc.; New York, NY, 1975.

To bisect angle A draw any line m which cuts the sides in points B and C. Draw line n parallel to AB. Mark D so that AB = BD and E so that CE = AC. ED cuts m in F. AF bisects angle A.