Magnetization and Loss of Superconducting Cables with Helical (CORC) and Twisted Stacked Geometries -- FEM and Analytical Modelling for Accelerator Magnets

Mike Sumption, ***D. Kun***, M. Majoros, *C. Kovacs*, *C. Myers*, and E.W. Collings

Center for Superconducting and Magnetic Materials, MSE, The Ohio State University

Dong Kun
OSU and School of Electrical Engineering, Southeast University, Nanjing, China

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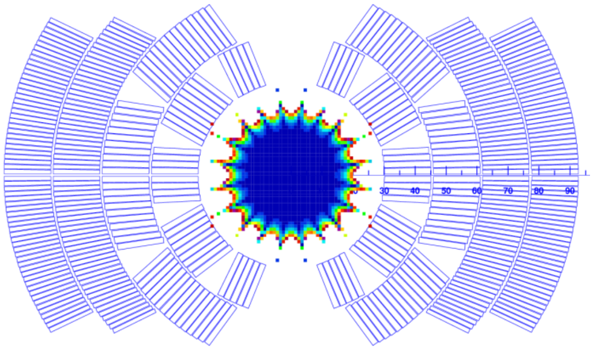
Department of Materials Science and Engineering

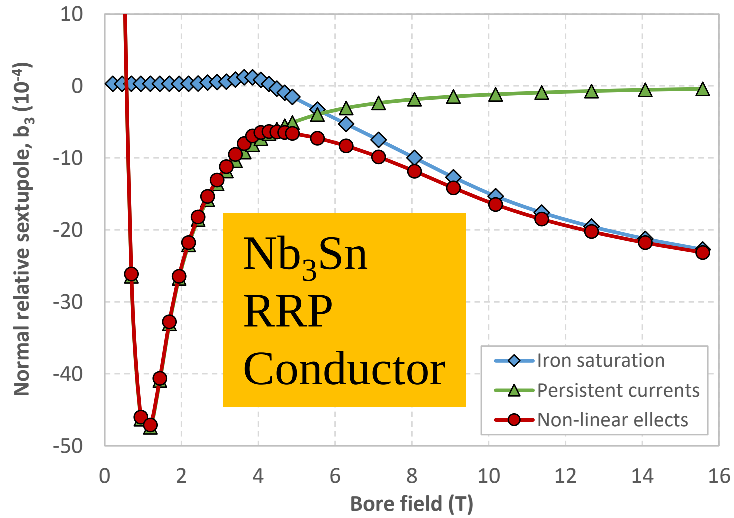
Outline of talk

- Motivation – accelerator quality
- Analysis of CORC and Twist Stack Cable Magnetization
- Comparison of Data and Theory
- Application to accelerators

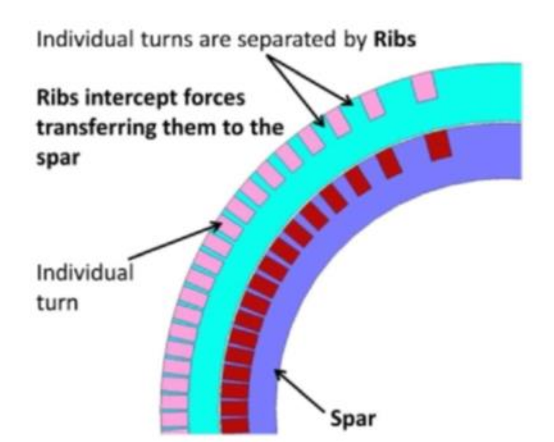
Motivation--Field Error in Accelerator Magnets



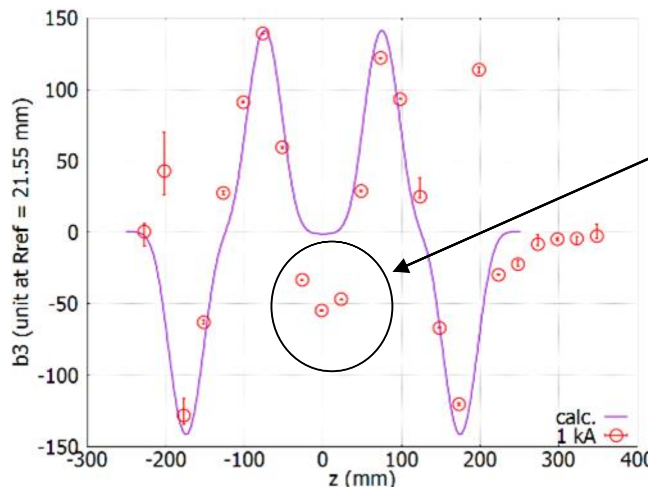
Cos theta



A Zlobin, “15 T dipole design concept, magnetic design and quench protection”, Presentation at the US MDP workshop Jan 2017



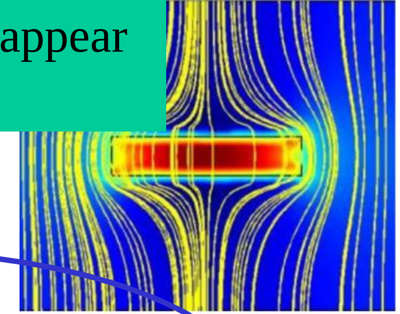
YBCO CORC Canted cos coil (Wang, LBNL 2018 MDP)



Magnetization related b3

What does the magnetization of HTS, esp YBCO, look like?

Summary of Loss expressions will appear in next edition handbook



$$Q{=}2N\mu_0 H_0 J_c a$$

$$N = \left(\frac{H_0}{H_d}\right) g\left\{\frac{H_0}{H_d}\right\}$$

$$g\left\{\frac{H_0}{H_d}\right\} = \frac{H_d}{H_0}\left[\frac{2H_d}{H_0}\ln\left(\cosh\left(\frac{H_0}{H_d}\right)\right) - \tanh\left(\frac{H_0}{H_d}\right)\right]$$

For flat strands with B ⊥ tape

1. For B perpendicular, $B >> B_p$

$$\Delta M = aJ_c$$

a is half width

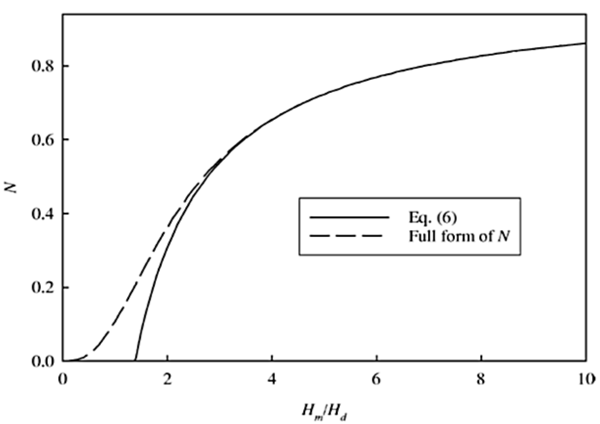
slabs

2. For B perpendicular, $B << B_p$

$$M = -\infty$$

As the width becomes infinite

3. For B perpendicular, $B \approx B_p$



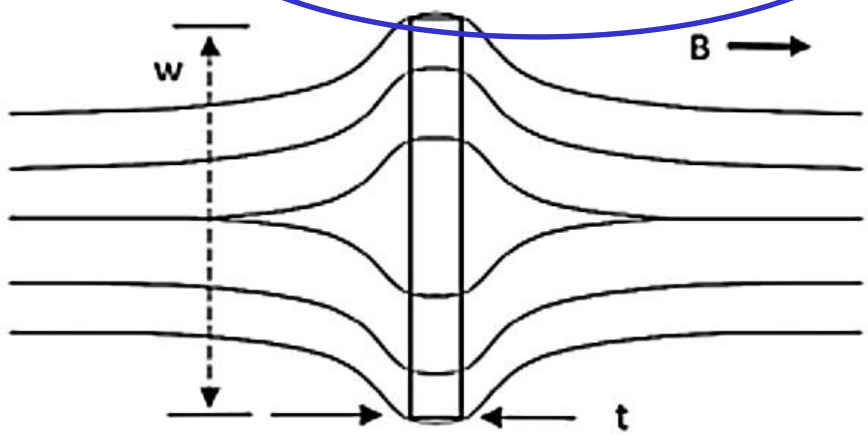


Figure 3. Field penetration into a thin slab (coated conductor).

The penetration field in this case is given by

$$H_p = \frac{J_c t}{\pi}\left[Ln\left(\frac{w}{t}+1\right)\right] = \frac{5}{2\pi}H_d\left[Ln\left(\frac{w}{t}+1\right)\right]$$

where $H_d = 0.4\,J_c t$ is a characteristic field. We note from Ref [16], that for $H_0/H_d > 3$

$$N \approx 1 - 2\left(\frac{H_d}{H_0}\right)Ln(2)$$

What does the magnetization of HTS, esp YBCO, look like - injection region?

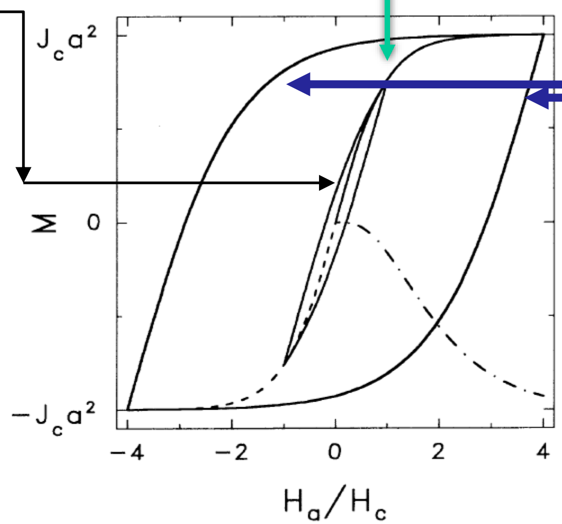
4. For B perpendicular, if we want M=f(H)

$$M_{\downarrow\uparrow} = \pm J_c a^2 \left[\tanh \frac{H_0}{H_c} + 2 \tanh \frac{H_a \mp H_0}{2H_c}\right]$$

$$M = \pi a^2 H_a (1 - H_a^2/3H_c^2)$$

$H_a << H_c$

$$M = J_c a^2 [1 - 2\exp(-2H_a/H_c)] \qquad H_a \gg H_c$$



$M_{\uparrow\downarrow} = M/L = J_c t a^2 = J_{cs} a^2$

a is half width of tape

H_a is applied field

$H_c = J_c/\pi$, where *J* is sheet current A/m

J_{cs} = usual $J_c{}^*t$

$H_0 = H_{max}$

$M_{\uparrow\downarrow}$ is moment per unit length

$M = m/Lta$

PHYSICAL REVIEW B VOLUME 48, NUMBER 17 1 NOVEMBER 1993-I

Type-II-superconductor strip with current in a perpendicular magnetic field

Ernst Helmut Brandt and Mikhail Indenbom*

CSMM Center fo

But What about Cables?

- A lot more Difficult for CORC and Twist stack!

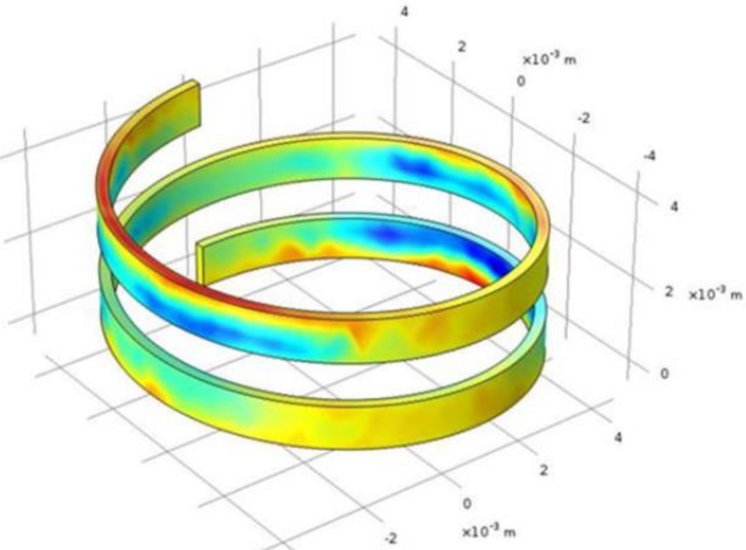
(helical, super high aspect ratio, node-hogging, multiple tape, tape-tape interaction, several loss components)

- Even Roebel has its complications!
- **<u>But, let us begin</u>**

Unravelling the CORC (and Twist Stack) Cable I

- Magnetization for coated conductor tapes is known
- A direct, analytic calculation for the loss of a CORC cable or a twist stack had not been performed, except at at $L_p \rightarrow \infty$, where
- $M_{hel} = \frac{2}{\pi} M_{tape}$ for an individual tape in a CORC or twist stack cable
- For all samples not in this limit (most samples), the magnetization is lower, but not known.
- The helical or twist geometry is a problem, as are the multiple layers of tape
- FEM approaches to the full problem are also not yet demonstrated because of large computational effort
- Desired is a simple expression to give the magnetization of CORC and twist stack cables
- Below we tackle this problem by first ignoring coupling and eddy currents, these can be added later
- We also simplify the problem to one tape in a helical or twisted geometry
- The individual tape response can be summed to give the cable response well above penetration field
- Magnetization at low fields (e.g. penetration fields) can be calculated later by computing the interactions among these layers
- Coupling current can be added back in later

Consider one tape of a CORC conductor – a helical wrap



This computation can be performed, but is quite demanding in terms of computation time

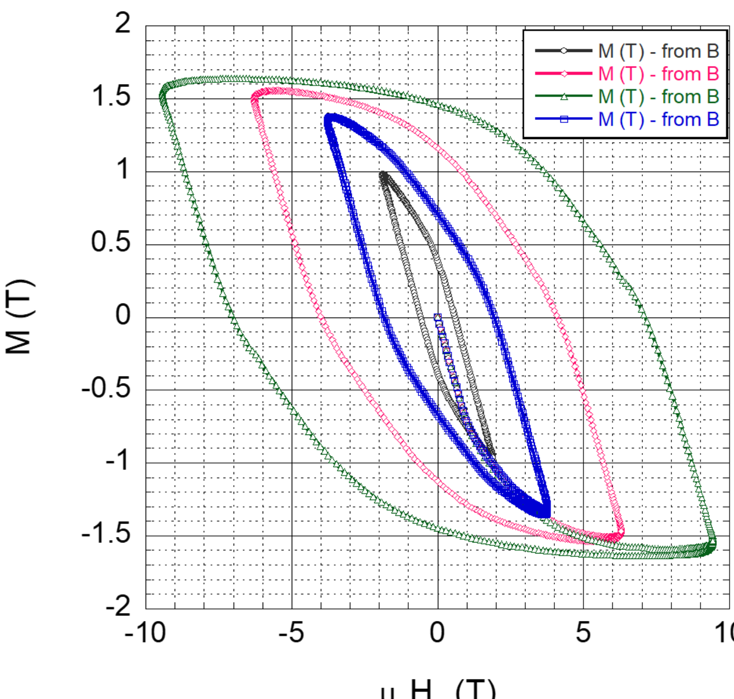
w_hel	1 mm	Tape width
th_hel	w_hel/4 = 0.25 mm	Tape thickness
r_hel	4.5 mm	Radius of helix axis
l_hel	(N_turns + 1)*w_hel + N_turns*gap = 5 mm	helix height
gap	w_hel = 1 mm	Gap between helix turns
pitch	w_hel + gap = 2 mm	Helix twist pitch
N_turns	2	Number of turns in helix
l_tape	N_turns*sqrt(pitch^2+(2*pi*r_hel)^2)=56.6899623 mm	Tape length in helix
V_tape	w_hel*th_hel*l_tape =14.17249058 mm^3	Tape volume in helix
J_c	10^{10} A/m^2	Critical current density

$$\Delta M_{tape} = J_c a = 10^{10}\left(\frac{0.001}{2}\right) = \frac{5x10^6 A}{m}$$

$$= 6.25\ T$$

5000 kA/m

$$M_{helix} = \frac{2}{\pi} M_0 \frac{1}{2} = 1.59\ x\ 10^6 \frac{A}{m} = 2\ T$$

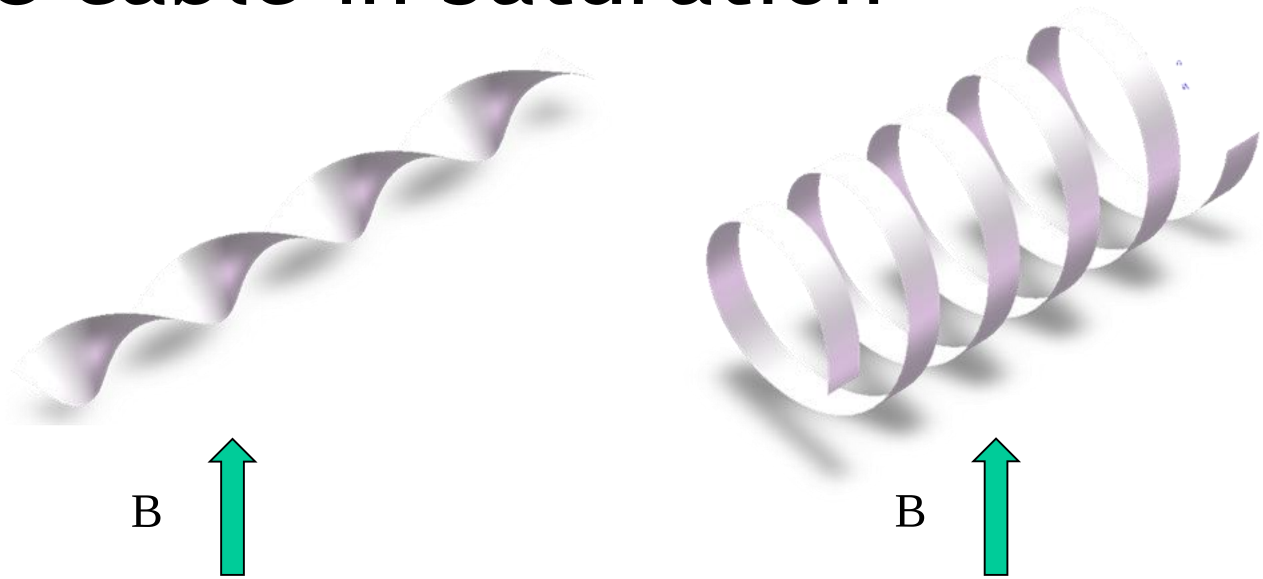


Magnetization of a helical Tape or CORC cable in Saturation

In general, in full penetration,

$$Q_0 = 2\mu_0 H_0 J_c w$$

(here w is the half width)



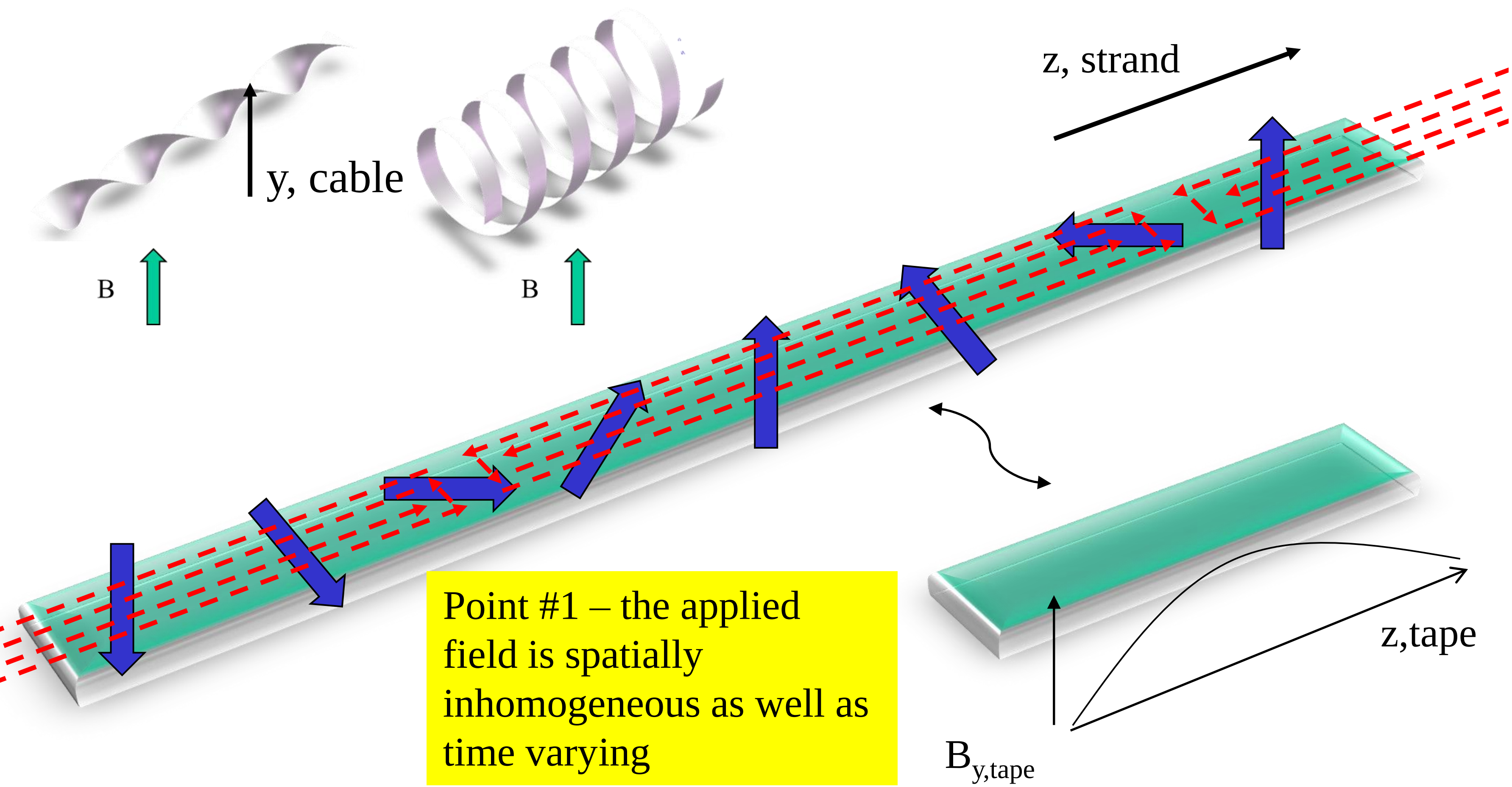
We might then imagine that that loss could be calculated by the simple expedient of integrating the average of Eq (5) over a spatial field cycle, such that

$$Q = \frac{2\mu_0 J_c w H_0}{L_p/2}\int_0^{\pi} sin\left(\frac{2\pi z}{L_p}\right) dz = \frac{2\mu_0 J_c w H_0}{L_p/2}\frac{L_p}{2\pi}(2) = \left(\frac{2}{\pi}\right) 2\mu_0 J_c w H_0 = \left(\frac{2}{\pi}\right) \boldsymbol{Q_0}$$

This leads to $M = (2/\pi)M_0$. **Is this true?** Yes if $L_p >> w$, but in general, not……

Carr, AC Loss and Macroscopic Theory of SC, p 189, 2nd Ed

Let us consider the general case -- Magnetization of a helical Tape or CORC cable in Saturation II

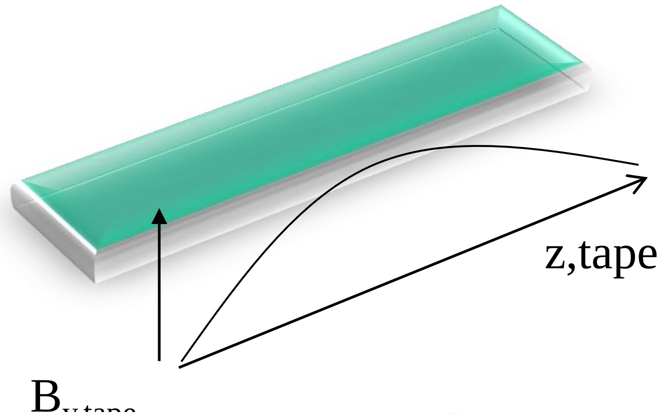


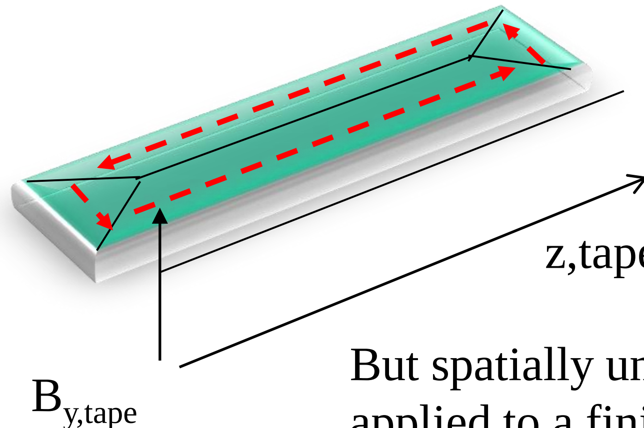
Point #1 – the applied field is spatially inhomogeneous as well as time varying

Magnetization of a helical Tape or CORC cable in Saturation III

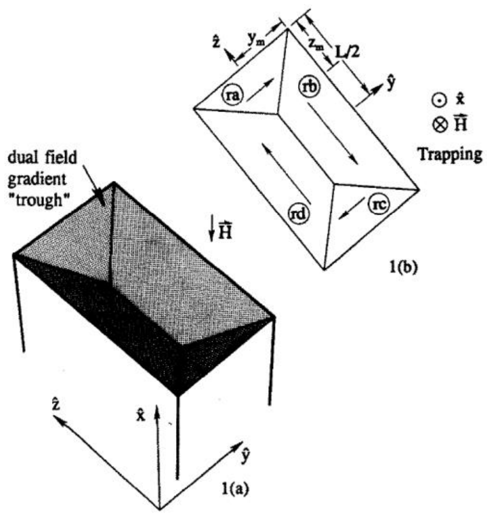
2. In general, currents in the presence of spatially inhomogeneous fields not a solved problem

3. The current flow is also spatially varying, leads to "end effects!"





But spatially uniform field applied to a finite length sample is a solved problem



2. E. M. Gyorgy, R. B. vanDover, K. A. Jackson *et al.*, *Appl. Phys. Lett* **55**, 283 (1989).
3. F. M. Sauerzopf, H. P. Wiesinger and H. W. Weber, *Cryogenics* **30**, 650 (1990).
4. S. Hu, H. Hojaji, A. Barkatt *et al.*, *Phys. Rev. B.* **43**, 2878 (1991).

$$\Delta M = J_c y_m \left(1 - \frac{2y_m}{3L}\right) \qquad L/2 > Z_m$$

$$\Delta M = J_c \frac{L}{2} \left(1 - \frac{2y_m}{3L}\right) \qquad L/2 < Z_m$$

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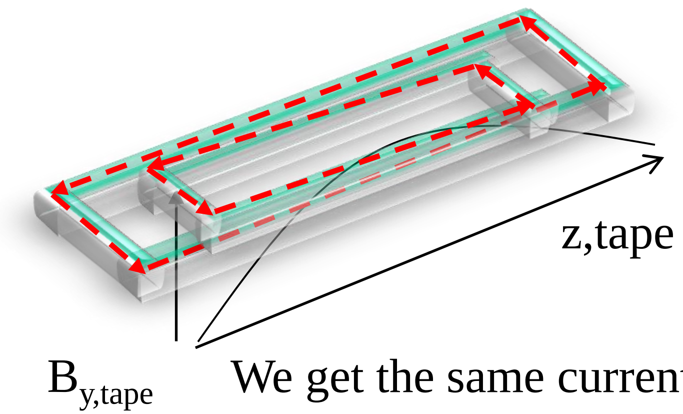
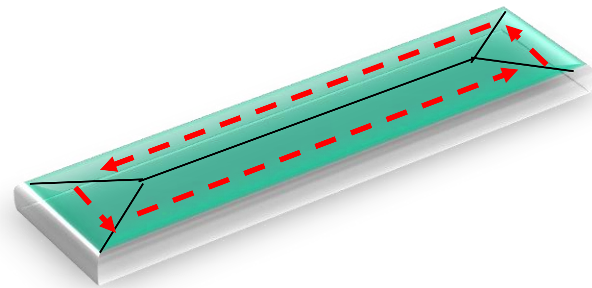
Magnetization of a helical Tape or CORC cable in Saturation IV



If we consider the field penetration layer by layer in a concentric shell configuration

We get the same current paths as the short sample in uniform field

If $B >> B_p$,
in this case, B (at L_p/2-w/2) > $J_c w/2$

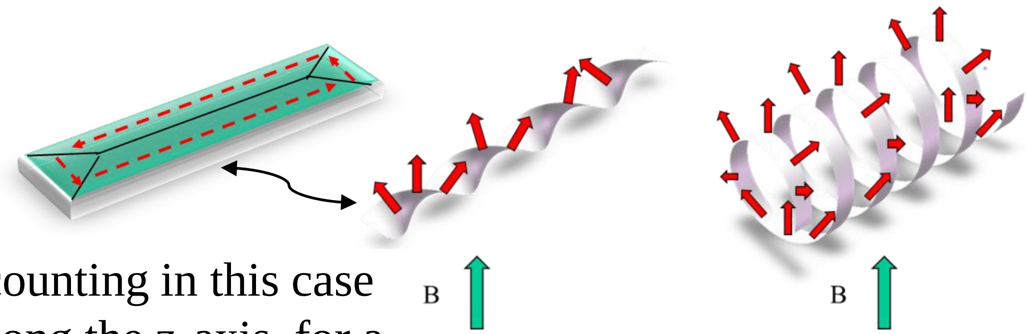


The local magnetization is changed, since M = <B>/μ0-<H> and <H> is lower
(*M* is reduced)

But, much more relevant for transforming back to the external field coordinates, the moment is the same as that of the finite sample in homogenous field (the demag leads to a lower local M)

Magnetization of a helical Tape or CORC cable in Saturation V

We can then use the moment of the short finite length calculation, breaking the twist or helix into a series of short samples



Integrating around the helix and accounting in this case for the component of the moment along the z-axis, for a twisted tape we get

$$\Delta M = \Delta M_0 \frac{2}{\pi}\left(1 - \frac{2y_m}{3L}\right) = \Delta M_0 \frac{2}{\pi}\left(1 - \frac{w}{3\frac{L_p}{2}}\right) = \Delta M_0 \frac{2}{\pi}\left(1 - \frac{2w}{3L_p}\right)$$

Analytic Result!

For the helix it will be the same, but with L_{eff} in place of L_p

$$L_{peff} = \sqrt{L_h^2 + \left(\pi D_h\right)^2}$$

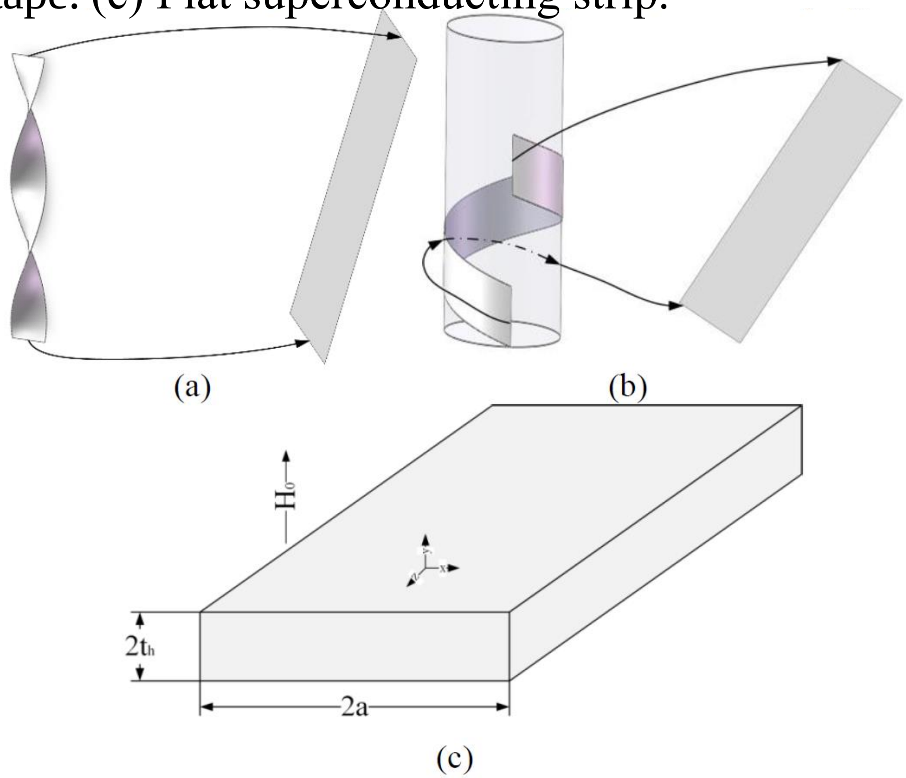
Twisted Tape: If $L_p > 20/3\ w$ (2.7 cm for 4 mm wide tape), $\Delta M_{twisted} \approx (2/\pi)\Delta M_{tape}$ with err < 10%

Helical/CORC Tape: **<u>Example 1:</u>** CORC Cable with L_h = 34 mm, OD = 4.76 mm, and L_{peff} =37 mm gives $\Delta M_{helical} \approx 0.85(2/\pi)\Delta M_{tape}$

<u>Example 2:</u> CORC wire with $L_h \approx 10$ mm, OD = 3 mm, L_{peff} = 13.7 mm, $\Delta M_{helical} \approx 0.80(2/\pi)\Delta M_{tape}$

Parallel FEM Approach – Again Unravelling the CORC (and Twist Stack) Cable

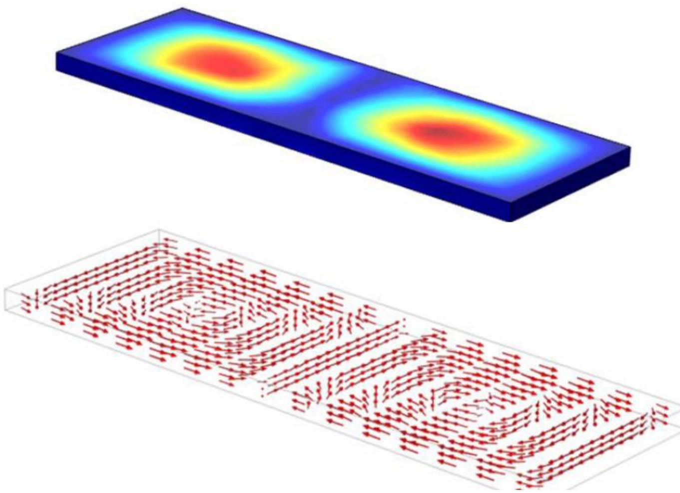
We consider first one tape from a CORC or a twist stack cable
Untwist the twisted superconducting cable into the mathematical model flat superconducting tape. (b) Unwind a single CORC tape into the flat superconducting tape. (c) Flat superconducting strip.



For a simple twisted Conductor, the twist pitch is straightforward, while for the helical wrap,

$$L_{peff} = \sqrt{L_h^2 + \left(\pi D_h\right)^2}$$

We then use Finite Element methods to calculate the Magnetization of a slab in a spatially inhomogeneous and time changing field



$$M = \left(\frac{1}{V} \int_V H_{local} \, dV - H_{applied} \right)$$

For a spatially uniform field

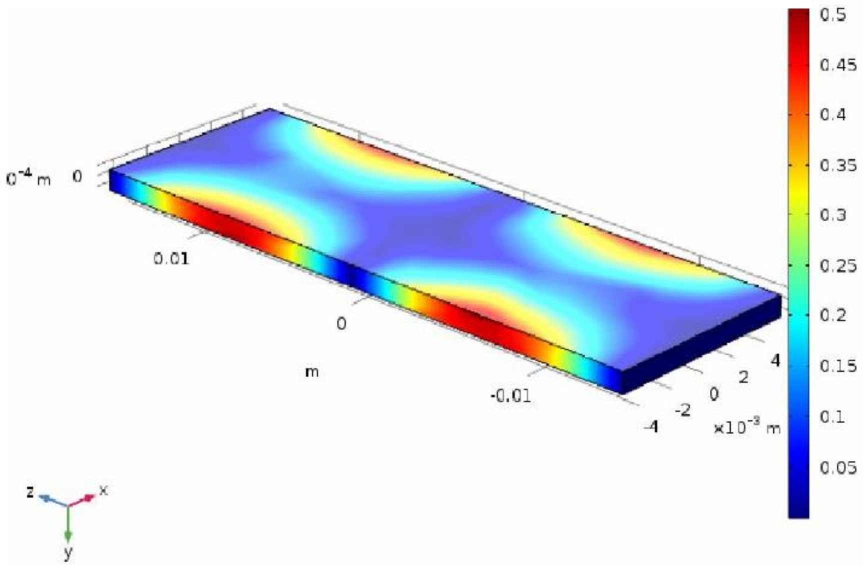
$$H_{applied} = H_{max} \sin(\omega t)$$

For a spatially varying field

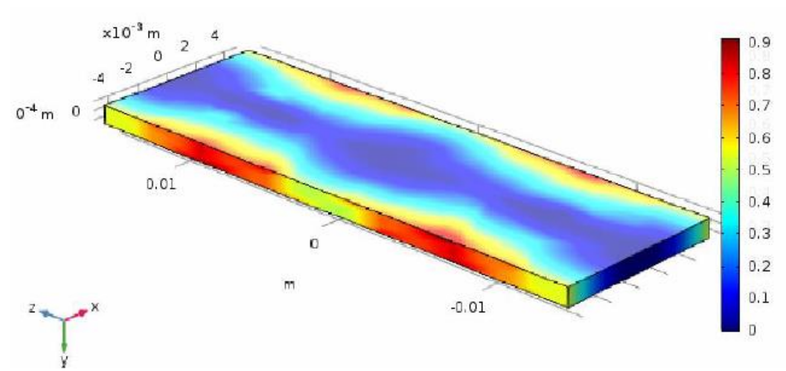
$$\boldsymbol{H_{applied} = H_{max} \sin(\omega t) \sin\left(\frac{2\pi z}{L_p}\right)}$$

- The expressions for M are the same,
- Only the applied field is different.
- Since $M=B/\mu-H$, the magnetization is the same except at very low fields

Simulations I

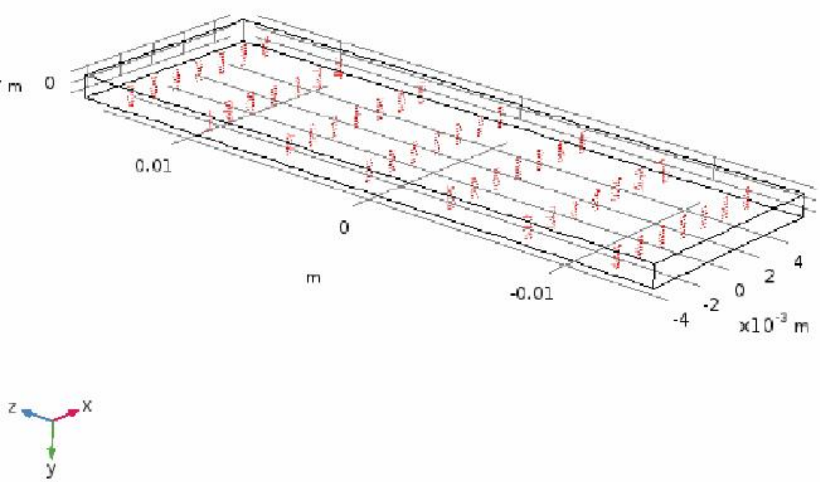


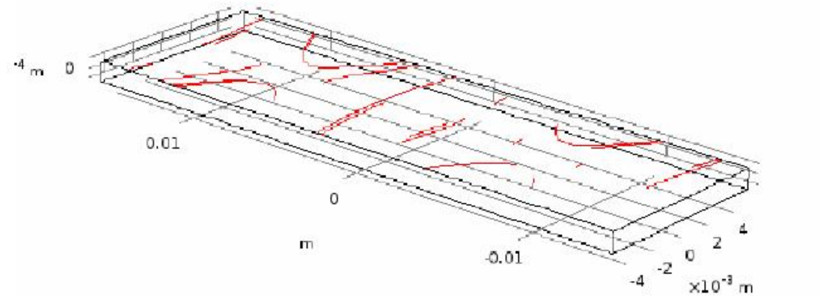
Normal Magnetic Field



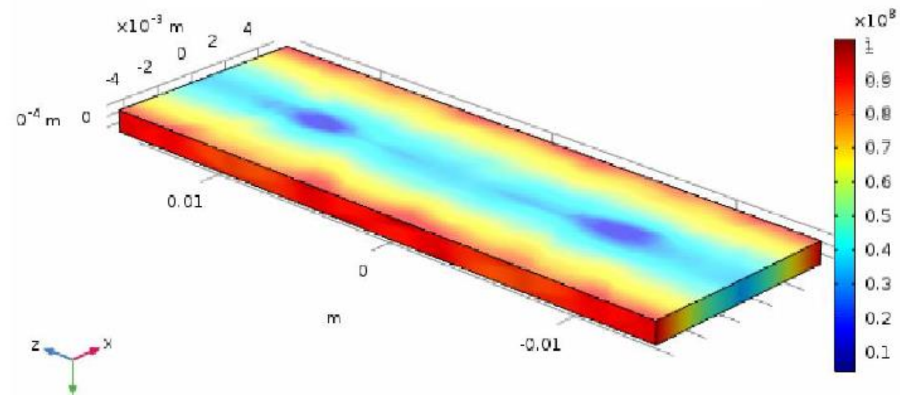
Electric Field

Simulations II –Electric field

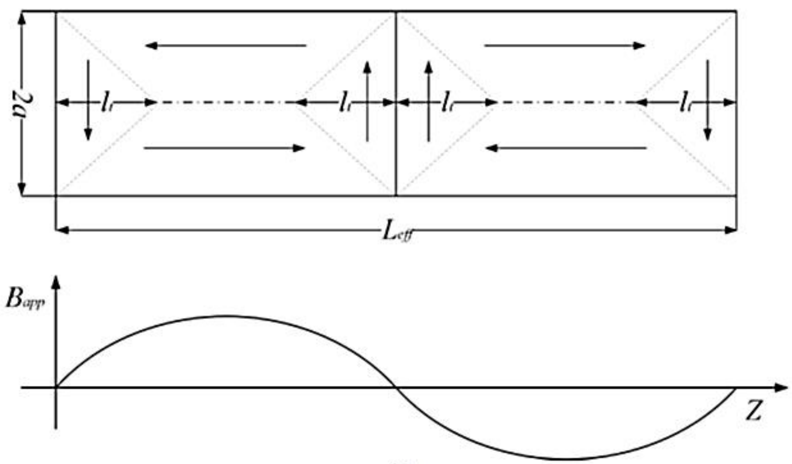




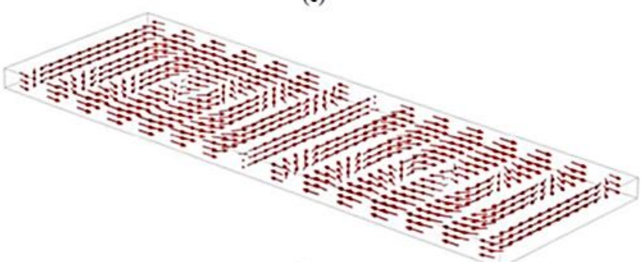
Supercurrent Density

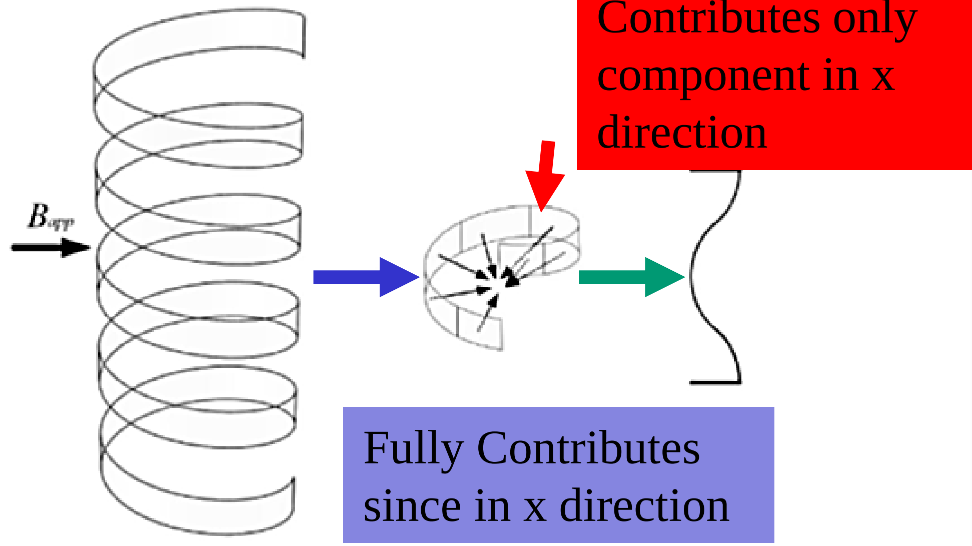


Then magnetic moments are re-assembled to generate the magnetization

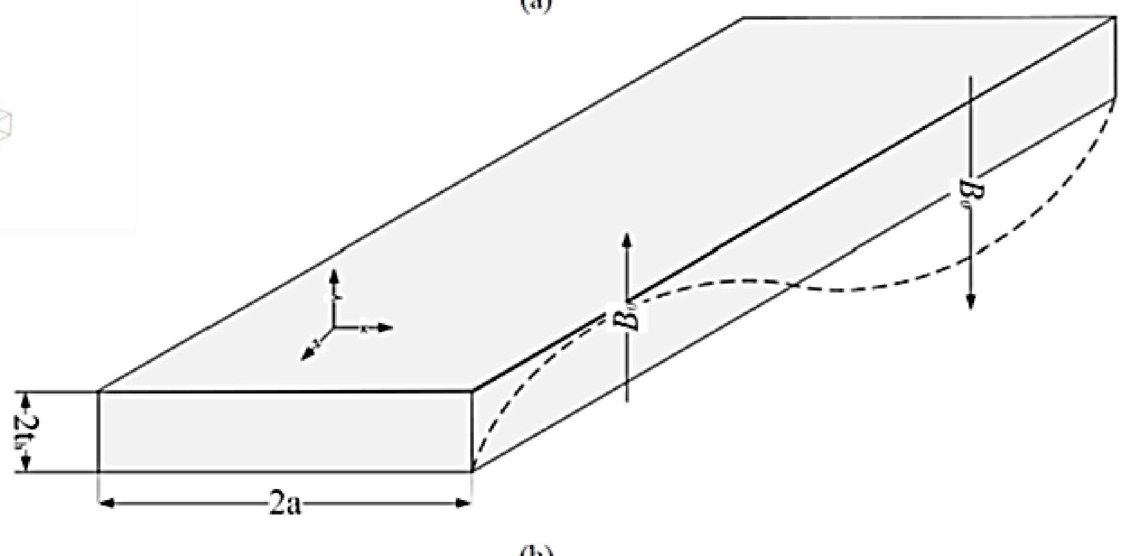


(c)


(d)



(a)



(b)

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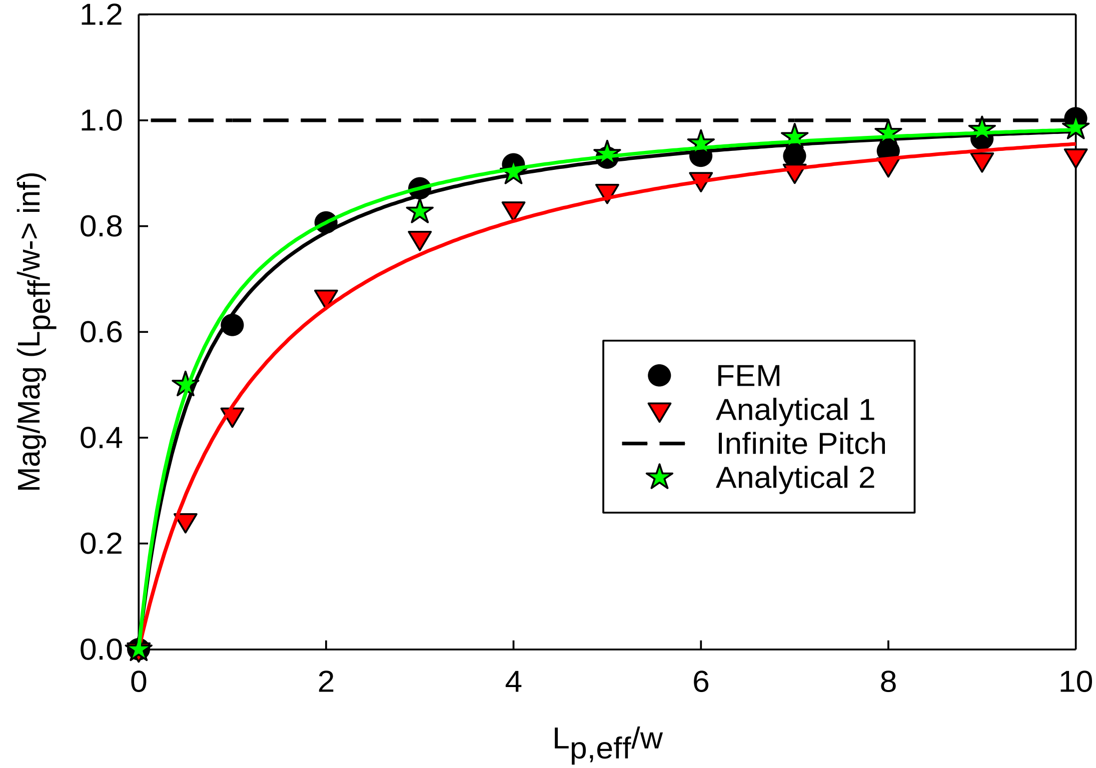
Comparison of FEM and analytic results

- Dashed line gives infinite pitch
- Shorter L_{peff}/w ratios give lower mag
- Agreement between FEM and analytic OK with Analytic 1
- Agreement even better when WF included – Analytic 2



Analytic 1

$$\Delta M = \Delta M_0 \frac{2}{\pi}\left(1 - \frac{2y_m}{3L}\right) = \Delta M_0 \frac{2}{\pi}\left(1 - \frac{w}{3\frac{L_p}{2}}\right) = \Delta M_0 \frac{2}{\pi}\left(1 - \frac{2w}{3L_p}\right)$$

Analytic 2

$$\Delta M = \Delta M_0 \frac{L_p}{\pi w} \cos\left[\frac{\pi L_p}{4}\left(1 - \frac{w}{L_p}\right)\right]$$

$$WF = \frac{L_p}{\pi w} \cos\left[\frac{\pi L_p}{4}\left(1 - \frac{w}{L_p}\right)\right]$$

Estimations for LBNL CORC samples

CORC A: 16-tape wire, wire OD 3.21 mm (including the heat shrink tubing), I_c = 4 kA at 4.2 K, self-field

For Tape A: I_c = 262 A per tape (0.04 mm thick, 2 mm wide, gives J_e = 262/.08 mm^2 = 3275 A/mm^2 = 3.27 x 10^9 A/m^2)

Magnetization Tape A: $M = J_c a/2$ = 3.27 x 10^9 A/m^2 * 10^{-3} m = 3270 kA/m

Magnetization CORC A: $M = (2/\pi) M_{tape}$*0.38*0.8= 633 kA/m [cable volume normalized]

Above B_p, but see below!

Magnetization CORC A: $M = (2/\pi) M_{tape}$*0.8= 1670 kA/m [strand volume normalized]

Penetration field CORC A: $B_p = \mu_0 J_{c,d} t_{wall}$ ***= 1.25x10^{-6}x3.27x10^9 A/m^{2}x3.79 x 10^{-4}m=1.55 T***

If this had been a single tape $B_p = (\mu_0 J_c t/\pi)[(\mathrm{Ln}(w/t)+1] \cong 120\ \mathrm{mT}$

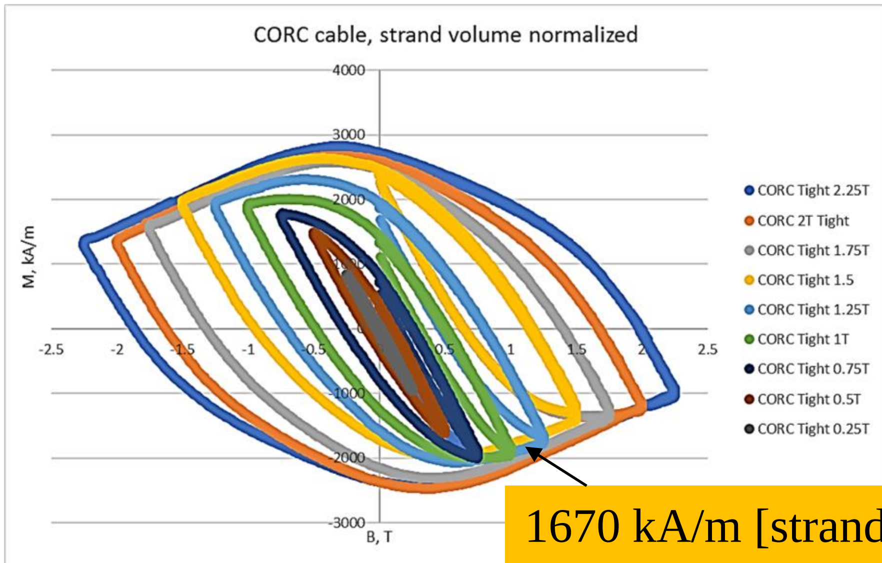
Note factor of 10 X difference in penetration fields of tapes and cables!
This difference is right in regime of injection field

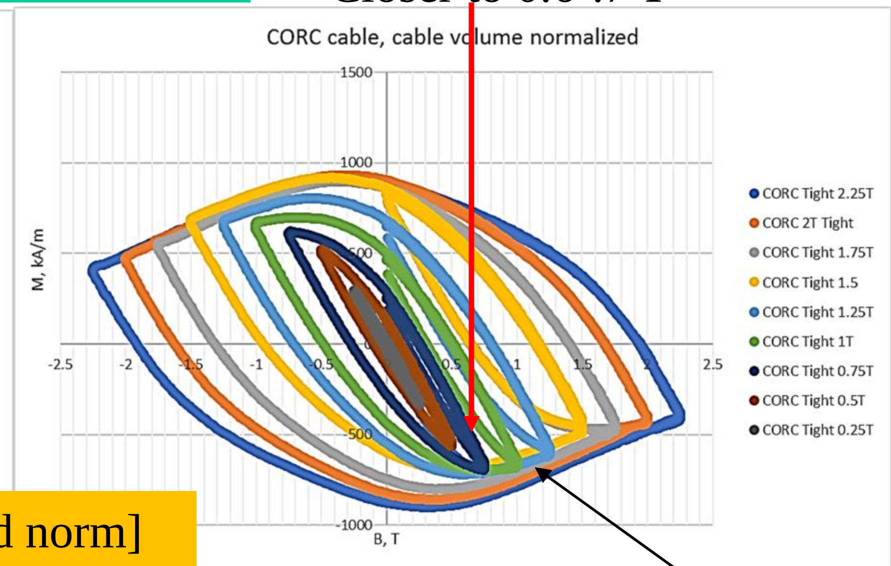
Compare to experiment: CORC Cable from LBNL – used canted cos dipole

Normalized to tape volume, 4 K result (C. Kovacs)
Measurements new OSU 3 T dipole, see 1 MPo=-2C-05

$$B_p = \mu_0 J_{c,d} t_{\text{wall}} = 1.55 \text{ T}$$

Closer to 0.6-.7 T





1670 kA/m [strand norm]
Value 2000 kA/m?

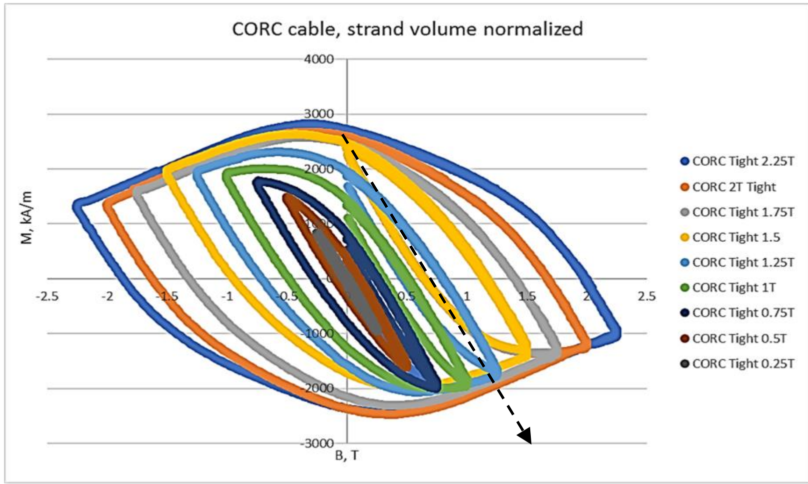
633 kA/m [cable norm]
Value 700-800 kA/m?

Cable	*# tapes*	*Cable Dim, mm*	*Cable I_c, A (4 K, SF)*	*Tape I_c, A (4K, SF)*	*Tape w, mm*	*Tape/cable*
CORC *Sample*	16 *# Segments*	3.21 (OD) *Pack dim, mm*	4000 *L, mm*	250 *V_{cable}, mm^3*	2 *V_{strand}, mm^3*	*0.34* *L_p, mm*
CORC	6	10 (OD)	94.2	4571	1591	6

Effects from strand-strand interaction?

	w	t	I	M (A/mm)	M (A/m)	M (kA/m)
tape	2	0.045	250	2777.778	2777778	2777.778

CORC Magnetization Near Injection



What happens to M is we don't start with a ZFC condition, or a full M-H sweep condition – e.g., a typical accelerator cycle?

Very generally, the CORC can be <u>treated as a simple tape of effective width</u>

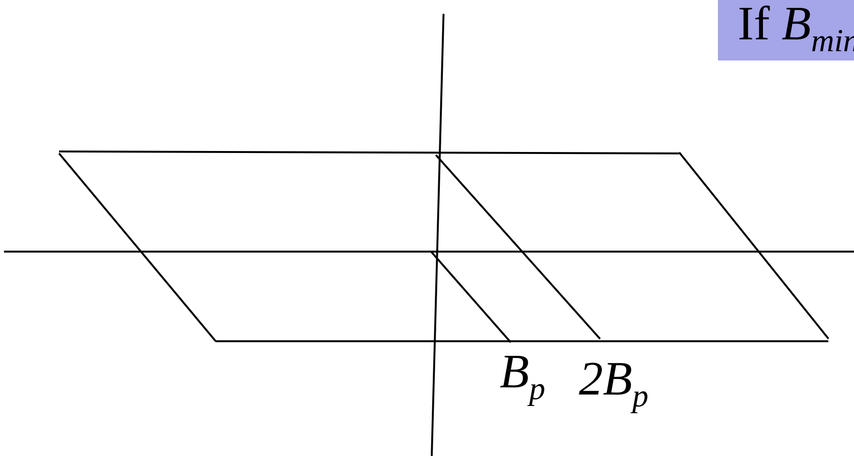
w_{eff} = (2/π)(fill factor)(1-$w/3L_p$)

If $B_{min} = 0$

$$M = M_{max}\left[1-\left(\frac{B}{B_p}\right)\right] = \frac{2}{\pi}\frac{J_c a}{2}FF\left[1-\frac{w}{3L_{p,eff}}\right]\left[1-\left(\frac{B}{B_p}\right)\right]$$

If $B_{min} \neq 0$

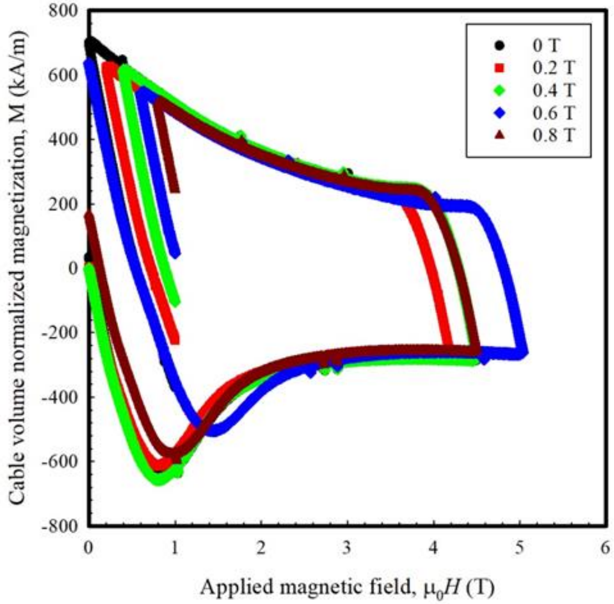
$$M = M_{max}\left[1-\left(\frac{B-B_{min}}{B_p}\right)\right] = \frac{2}{\pi}\frac{J_c a}{2}FF\left[1-\frac{w}{3L_{p,eff}}\right]\left[1-\left(\frac{B-B_{min}}{B_p}\right)\right]$$



In full penetration

$$M = M_{max} = \frac{2}{\pi}\frac{J_c a}{2}FF\left[1-\frac{w}{3L_{p,eff}}\right]$$

CORC Magnetization near Injection – influence of pre-cycle



See 1MPo-2C-03 Cory Myers Paper new OSU hall probe magnetometer system

$$M = \frac{2}{\pi}\frac{J_c a}{2} FF\left[1 - \frac{w}{3L_{p,eff}}\right]\left[1 - \left(\frac{B - B_{min}}{B_p}\right)\right]$$

TABLE I
MAGNETIZATION AND DRIFT VALUES

Sample type and hold field	M_0 (kA/m)	*Mpred (kA/m)*
CORC 0 T	-430	-420
CORC 0.2 T	-280	-210
CORC 0.6 T	19	210
CORC 0.8 T	180	420

Department of Materials Science and Engineering

Application to field error in accelerators

- Results not yet been put into field error calculations for magnets, will be magnet geometry dependent
- Nevertheless, some value for simply imagined “replacing” a NbTi or Nb_3Sn winding
- Taking the LHC as a reference, $b_3 \cong 3$, $M_{h,cable,1.9K,0.54T}$ = 10.3 kA/m
- Nb_3Sn deff 10 X that of NbTi, b_3 10 X higher ≅ 30-40 units.
- For HTS cable, $M_{inj} \cong$ 600-900 kA/m, suggesting b_3 values around **300 units** for a direct replacement (the current density at collision is roughly similar for these cables at their point of operation, so no correction is added for that).
- This is a very simple and rough estimate, and assumes no changes in the magnet to minimize these effects. As such, it is merely a starting point of data for inclusion in magnet design.

Summary

- *M-H* of CORC and twist stack in full penetration calculated by Analytic and FEM methods
- For long L_p, $M_{corc} = M_{twst} = (2/\pi)M_{tape}$
- *In general*

This CF ≈0.8

$$\Delta M = \Delta M_0 \frac{2}{\pi}\left(1 - \frac{2w}{3L_p}\right)$$

$$\Delta M = \Delta M_0 \frac{L_p}{\pi w} \cos\left[\frac{\pi L_p}{4}\left(1 - \frac{w}{L_p}\right)\right]$$

- Expressions for CORC Magnetization at arbitrary fields near injection have been developed as part of LBNL-OSU collaboration (X. Wang) YBCO data for error estimations

$$M = \frac{2}{\pi}\frac{J_c a}{2} FF\left[1 - \frac{w}{3L_{p,eff}}\right]\left[1 - \left(\frac{B - B_{min}}{B_p}\right)\right]$$

- Comparison to measurements made directly on CORC cable (from LBNL-ACT) give reasonable agreement, work remains
- "Direct replacement" leads to high b3, but just starting point