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EXPONENTIATION IN PASCAL

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Most people will agree that Pascal is fast and elegant and is a good language to use when teaching programming. But while they're saying things like that they also want the language to do all sorts of things that may or may not be academic in nature. One good example of this confusion is the way that Pascal deals with some arithmetic functions. If I am strictly a programmer, then I want the language environment that I'm using to be a completely transparent vehicle for me to communicate my algorithms to the machine . . . and I want the thing to work with as little effort from me as possible. But if I'm a teacher, then I want a language environment that causes my students to actually think through what they want the machine to do. In fact, the reason that Pascal has so many adherents in the schools is precisely because Wirth designed it to make people learn rather than make the life of programmers easier.

Wirth chose to include a number of arithmetic functions into common Pascal, but he also chose not to include several. The philosophical decision to disallow certain functions can cause programmers to gnash their teeth a little bit while giving teachers a glorious opportunity to push their kids into doing some mathematical footwork.

The standard arithmetic functions of common Pascal include:

abs(x)	which returns the absolute value of the argument x
sqr(x)	which returns the square of x
sqrt(x)	which returns the square root of x
sin(x)	which returns the sine of x
cos(x)	which returns the cosine of x
arctan(x)	which returns the arctangent of x
ln(x)	which returns the natural logarithm of x
exp(x)	which returns base e raised to the power x
round(x)	which rounds x
trunc(x)	which truncates the real x to an integer

Now you may be wondering why the tangent, arcsine, and arccosine functions aren't on that list. The answer is that Wirth wanted the language to be as parsimonious as possible. Being an academician, Uncle Nick just assumes that everyone will have a calculus text around (you mean you don't?) with the necessary tables in the back to obtain all of the other trigonometric relationships from the sin, cos, and arctan functions. From Wirth's philosophical position predefining extra functions in Pascal would be redundant, and make the language less elegant and compact. Also, it would make it easier for students to breeze through trigonometric computations without actually learning how the functions are related.

Another arithmetic function that's not on that list is an exponentiation function, and this simple fact is what often separates the hackers from the real mathematical programmers. Unless you're specifically writing a program that calls for some trigonometric calculations, you can go a long time without having to make a call to arctan(x) . . . but it's a fairly common occurrence to come across exponentiated numbers. In fact, in school environments it seems as if our programs are dealing with exponents constantly: biology students are writing programs to assist them in calculating growth curves of bacteria cultures; physics students are writing programs to calculate the rate of absorption of radiant energy through different mediums; geometry students are writing programs to take the drudgery out of conics; business students are writing programs to calculate loan projections. We seem to be using the word mantissa a lot, but there it is . . . there isn't an exponentiation function in Pascal and I suspect that Dr. Wirth did it on purpose. Why? Because Pascal is first and last a teaching language. But where there's a will . . .

Let's go back and look at that list of functions again. We do have a ln(x) function which returns the natural logarithm of x, and we have the exp(x) function which represents e (the base of the natural log system), raised to the real or integer power of x (in other words, e^x). These functions exist primarily because logarithms are so common in formulas and Wirth wanted to simplify expressions like $(e^u-e^{-u})/2$ and ln(pi/2) so that they could be written out as (exp(u)-exp(-u))/2 and ln(3.141592654/2).

Now I know from high school algebra that I can use a property of exponents to derive a property of logarithms. For example, $125=5^3$; and $125^2=(5^3)^2=5^6=5^{2\cdot 3}$ so $\log_5 125^2=2\cdot 3=2\log_5 125$. Now in most of the textbooks we find that so long as m and b are positive real numbers, if b is not equal to 1 and if p is any real number, then the property of logarithms is expressed as . . .

$$\log_b m^p=p\log_b m$$

Since p is any real number, this property can be used to raise a number to a power or to extract a root. Suppose we want to find $(7.27)^5$. Well, $\log(7.27)^5 = 5 \log 7.27 = 5\ (0.8615) = 4.3075$. Therefore, $(7.27)^5$ has the approximate antilog of 4.3075 or approximately 20,300. It's these properties of logarithms which make them so popular in physics or biological calculations where large numbers need to be manipulated in small spaces.

Given all of this, we can come up with a simple formula which can be used to carry out exponentiation in Pascal:

$$a^n = \exp(n * \ln(a))$$

subject to the following restriction. The mantissa, a, must be a positive real or integer value. So . . . a mathematical expression like 7.27^5 would be written out in Pascal as exp(5*ln(7.27)). Get it? A mathematical expression like $9.87^{-3.51}$ would be written out in Pascal as exp(-3.51*ln(9.87)). The only thing that isn't going to fly is something like $(-5)^{0.15}$ because it has a negative mantissa.

So you see it is possible to do exponentiation in Pascal. In fact, you don't really need to understand the properties of logarithms so long as you are able to juggle the formula given and follow the rules. But it is nice to be in a position where you can teach logarithmic functions to your students and have a ready-made excuse to make a practical application of the instruction. Thanks, Dr. Wirth.

Here's a small program which calculates compound interest that demonstrates the use of this formula as a function. Notice that Function Power must be written before Function Interest because Interest calls Power, so when we call Interest from the main program it calls Power to itself.

```
Program Investment (input,output);

Var
     start,annualrate,earned   : real;
     years,days                : integer;

Function Power (x,y : real)  : real;
    {computes the value of x to the power of y}
Begin
     Power := exp (y * ln(x));
End;

Function Interest (start,annualrate : real; years,days : integer)
: real;
    {calculates interest earned using compound interest}
Begin
     Interest := start
                 * power (1+annualrate/100,years+days/365)
                 - start;
End;
```

```
Procedure Getdata;
Begin

    Write ('Input beginning amount - ');
    Readln (start);
    Writeln;
    Write ('Input annual rate of interest - ');
    Readln (annualrate);
    Writeln;
    Write ('Input years - ');
    Readln (years);
    Writeln;
    Write ('Input days - ');
    Readln (days);
End;


Begin    Main Program

    Getdata;
    Earned := Interest (start,annualrate,years,days);
    Writeln;
    Writeln ('Initial investment : $', start:10:2);
    Writeln ('Annual rate        :  ', annualrate:5:3);
    Writeln ('Duration of note   :  ',years,' years and ',days,'
days.');
    Writeln;
    Writeln (' Interest earned   " $',earned:10:2);

End. {of Investment}
```

OBJECTIVES IN THE MATHEMATICS CURRICULUM

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The mathematics curriculum is indeed experiencing rapid changes. The hand held calculator is no longer an innovative method in teaching-learning situations. Just a few years ago numerous educational journal articles contained content pertaining to the introduction and utilization of the calculator. As a whole, it is accepted that the hand held calculator is a must in the