Classroom Activity

Circular Table Seating: An Application of the Fundamental Principle of Counting

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Introduction

The Fundamental Principle of Counting (FPC) can be applied in a variety of intriguing settings. We will employ this principle in counting seating arrangements on a circular table. Let's consider specifically counting four seating arrangements around a circular table, that is, the number of ways four people can be seated in four chairs around a circular table. We will specify three similar but distinct settings.

Setting 1

The 4 chairs are all distinguishable from each other. This leads to a straightforward application of the FPC. The first person can be seated in 4 ways. When this has been done, there remain (4-1) ways to seat the second person. For the third person, (4-2) seats are available. Finally, for the remaining person 4 there remains one seating possibility. Altogether, these 4 persons can be seated in 4(4-1)(4-2)(4-3) or 4! ways. Figure 1 illustrates this solution graphically.

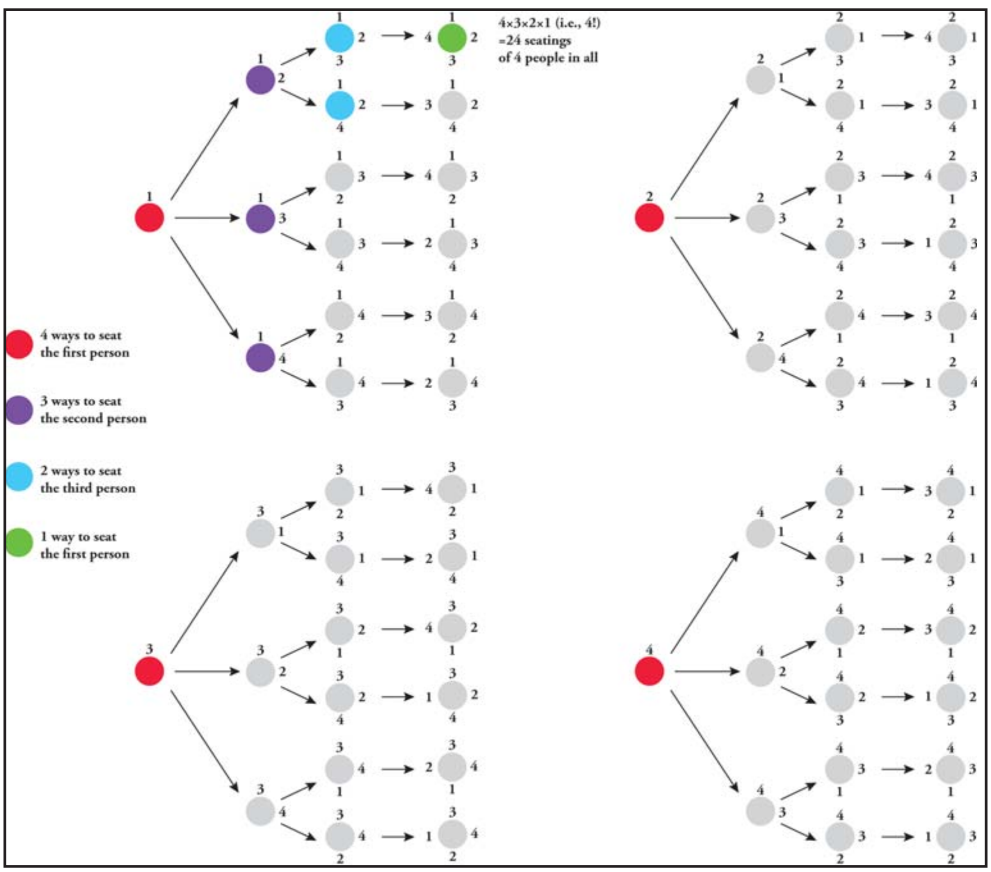

Fig 1 Graphical representation of solution for Setting 1

Setting 2

The 4 chair positions are indistinguishable. Consequently, it does not matter which chair the initial person uses. However, it is significant who sits on each person's left and right. The FPC selection sequence then proceeds as follows:

- Person 1 can sit anywhere; his or her position is irrelevant.
- We must next choose the person to sit to the left of Person 1. There are (4-1) ways of choosing this Person 2.
- We next choose Person 3, who will sit to the left of Person 2. This selection can be done in (4-2) ways.
- There is then one position available for Person 4.

The total number of seating possibilities is then (4-1) (4-2) (4-3) or (4-1)! The interface between Settings 1 and 2 can be explained in a related way. After the Setting 1 seating arrangement has been determined, the 4 persons can rotate around the table, occupying 4 arrangements that are indistinguishable in Setting 2. The number of Setting 2 arrangements is thus $\frac{4!}{4} = (4-1)!$ Figure 2 illustrates this solution graphically.

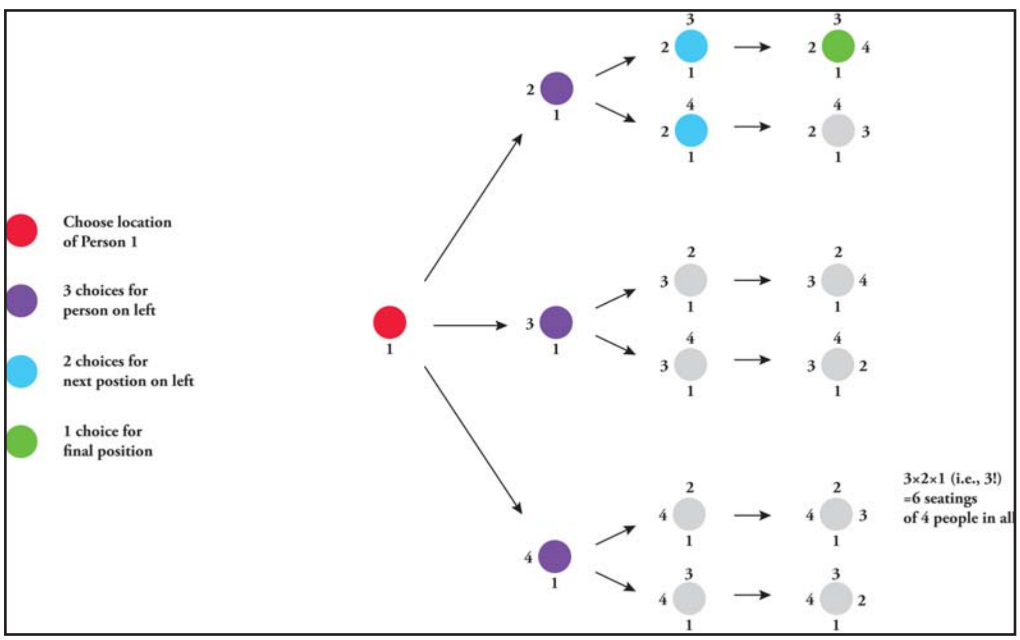

Fig 2 Graphical representation of solution for Setting 2

Note that each of six rows in the listing of Setting 1 contains four arrangements that are indistinguishable in Setting 2. Consequently, for Setting 2 we will list only one arrangement from each of those six rows.

Setting 3

The 4 chair positions are indistinguishable, as in Setting 2. But in this setting it makes no difference whether Joe is sitting to Liz's left, for example, or to her right. The initial phase of the analysis of Setting 3 follows the Setting 2 process, yielding (4-1)! possibilities. But one more step is then needed. Each possible Setting 2 arrangement can be paired with another arrangement in which each person *x* seats to the right of "Person 1" moves to a seat x seats to Person 1's left. By the standard of Setting 3, these two arrangements are indistinguishable. Consequently, there are exactly half as many arrangements in Setting 3 as there were in Setting 2. Setting 3 thus allows $\frac{(4-1)!}{2}$ distinct arrangements. Figure 3 illustrates this solution graphically.

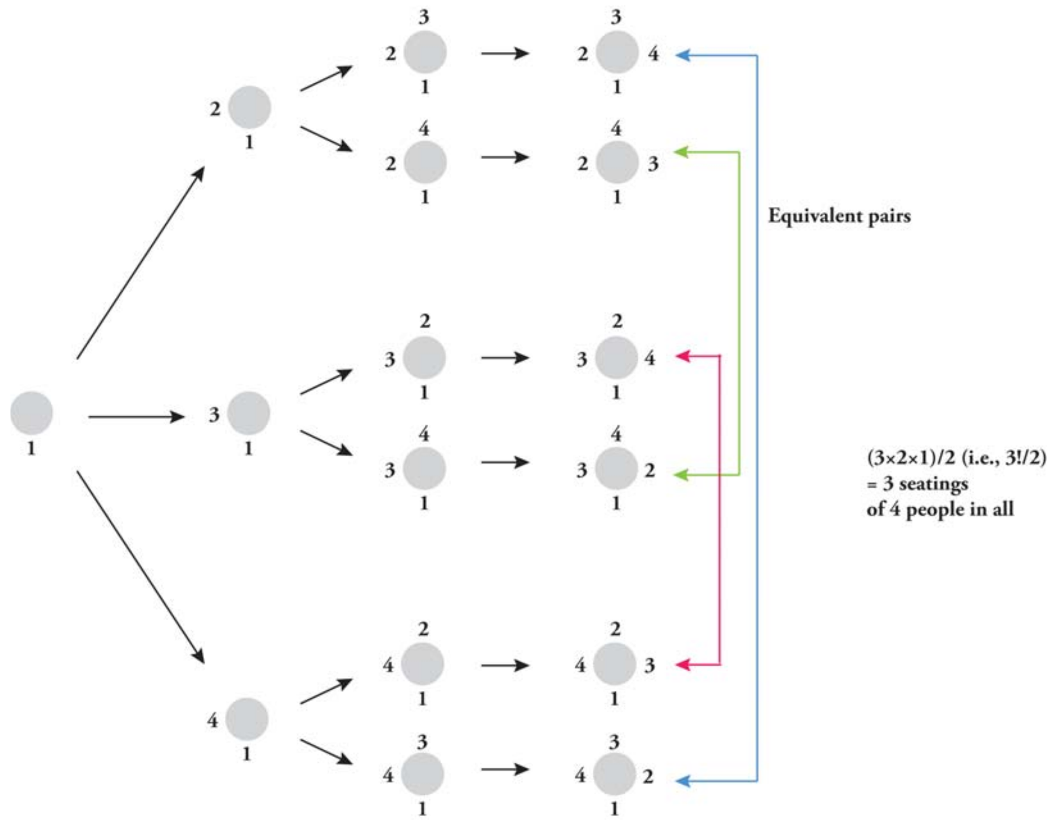

Fig 3 Graphical representation of solution for Setting 3

Again, each of the three rows of Setting 2 contained two arrangements which are indistinguishable in Setting 3. Consequently, we list for Setting 3 only one arrangement for each of those rows.

Extending Tasks

Extending this process for 10 persons, the number of arrangements for each setting is as follows.

Setting	Arrangements for 10 people
1	10! = 3,628,800
2	9! = 362,880
3	$\frac{9!}{2}$ =181,440

These numbers are too large to readily facilitate a listing of possible arrangements. But for 4 persons, this listing is manageable. Teachers and their students are encouraged to find other settings in which the FPC could be exemplified.