Students' Misconceptions in Mathematics: Analysis of Remedies and What Research Says

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Abstract

It is common knowledge that students of all grade levels have misconceptions regarding various concepts in mathematics. This article is focused on analyzing this issue due to its importance in the teaching and learning of mathematics. Misconceptions from two areas of mathematics are presented; these include operations with fractions (arithmetic) and addition of exponents (algebra). In each area, the explanation of the misconception, steps that teachers can take to address the problem, and highlights of previous research relating to the misconception are presented.

What Are Misconceptions and How Do They Come About?

Misconceptions are misunderstandings and misinterpretations based on incorrect meanings. They are due to 'naive theories' that impede rational reasoning of learners. Misconceptions take various forms. For example, a correct understanding of money embodies the value of coin currency as non-related to its size. But, at the Pre-K level, children often hold a core misconception about money and the value of coins. Some students believe that nickels are more valuable than dimes because nickels are larger. Some elementary and even middle school students believe that 1/4 is larger than 1/2 because 4 is greater than 2. Additionally, a common misunderstanding is that the operation of multiplication will *always* increase a number. This impedes students' learning of the multiplication of a positive number by a fraction less than one.

As indicated by Ojose (2015), misconceptions "exist in part because of students' overriding need to make sense of the instruction that they receive" (p. xii). For example, the rules for adding fractions with like and unlike denominators are quite different. Moving from adding fractions with like denominators to adding fractions with unlike denominators requires learners to make sense of the different scenarios and make adjustments. According to Ojose (2015), the transition often creates cognitive conflicts and dissonance for learners because the process requires unlearning what has been previously learned.

It is important to understand how misconceptions manifest, based on the nature of school mathematics. From a student perspective, the rules may seem to change from one concept to another. For example, when decimals are introduced with addition, 0.4 + 0.7 equals 1.1 (one decimal place), but with multiplication of decimals, 0.4 × 0.7 equals 0.28 (two decimal places). The discrepancy from addition to multiplication with decimals could be a reason for learners to have misconceptions. Another dimension related to the nature of mathematics is that certain misconceived methods and errors in calculation could actually lead to correct solutions, possibly a significant reason as to why learners seem to hang on to them. For example, if 1/9 is divided by 1/3, the answer is 1/3. When given this problem, learners could also erroneously divide the numerators to get 1 and also divide the denominators to get 3, and thereby arriving at the correct answer of 1/3 (through a mathematically incorrect method). When this kind of situation happens, the onus is on the classroom teacher to identify and correct the misconception. In general, knowing the nature of a misconception and its source helps teachers to fathom ways of planning appropriate instruction that is beneficial to learners.

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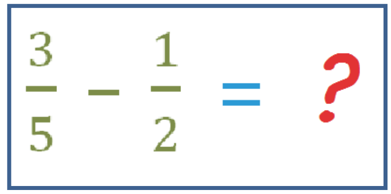
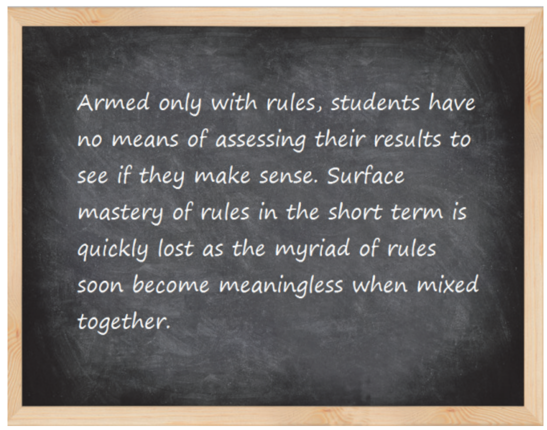
Misconception 1: Subtraction of Fractions



Question: Subtract: $\frac{3}{5} - \frac{1}{2}$

Likely Misconception: $\frac{3}{5} - \frac{1}{2} = \frac{2}{3}$

Explanation of Misconception: This misconception has to do with the misapplication of rules. The misconception could be associated with learners transitioning from operations with whole numbers to operations with fractions because, to them, the rules have changed. Because of previous "knowledge", the learner performed the subtraction operations distinctly with the numerators producing 2: $(3 - 1 = 2)$ and the denominators producing 3: $(5 - 2 = 3)$. Thus, the learner applied the wrong algorithm in solving the problem. There is no evidence of conceptual knowledge of fractions exhibited by the student. The learner could have manipulated the denominators to reflect same value before attempting to perform the subtraction operation.



***What Teachers Can Do*:** In problems involving fractions, it is important to impress upon learners that the numerator indicates the number of parts and the denominator indicates the *type* of part. Premature attention to rules for computation should be discouraged. Usually, the rules don't help learners think about the operations and what they mean. Armed only with rules, learners have no means of assessing their results to see if they make sense. Surface mastery of rules in the short term is quickly lost as the myriad of rules soon become meaningless when mixed together.

The following strategies are suggested:
1. Begin with simple contextual tasks,
2. Connect the meaning of fraction computation with whole number computation,
3. Let estimation and informal methods play a big role in the development of strategies, and
4. Explore using a variety of models and have learners defend their solutions using models.

Teachers will find that it is often possible to get answers with models that do not seem to help with pencil-and-pencil approaches.

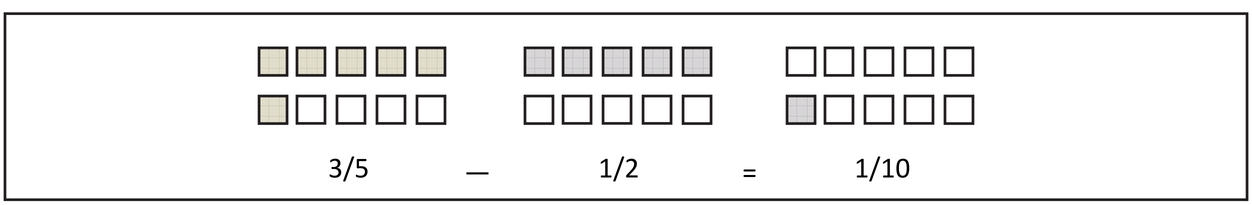


Figure 1. Illustration of $\frac{3}{5} - \frac{1}{2}$ using array technique

The ideas gleaned from models will help learners learn to think about the fraction and the operation, contribute to mental methods, and provide a useful background when they eventually learn the standard algorithms. One example of models which could be used to teach operations with fractions would be the use of an array of physical objects, like chips, to illustrate the concept of fractions with different denominators. An array form could be constructed, as shown in Figure 1 above. In the illustration, it shows that the fractions are first put in equivalent forms: $3/5 = 6/10$ and $1/2 = 5/10$. Then, the subtraction operation is performed. The illustration shows that $3/5 - 1/2 = 6/10 - 5/10 = 1/10$. Again, physically manipulating the objects (e.g., chips) to perform this task is beneficial to learners.

Research Note: Cramer et al. (2002) studied the effect of two different curricula on the initial learning of fractions by students in grades 4 and 5. One curricula was the commercial curriculum (CC) that could be described as traditional. The other was the Rational Number Project (RNP) curriculum that placed particular emphasis on the use of multiple physical models and translations within and between modes of representation – pictorial, manipulative, verbal, real-world, and symbolic. Students using RNP project materials earned statistically higher mean scores on the posttest and retention test on four (of six) subscales: concepts, order, transfer, and estimation. The result also showed differences in the quality of students' thinking as they solved order and estimation tasks involving fractions. RNP students approached such tasks conceptually by building on their constructed mental images of fractions. However, CC students relied more often on standard, often rote, procedures when solving identical fraction tasks. The program of study by the CC students did not include a wide variety of materials or regular use of manipulative experiences but focused instead on pictorial and symbolic modes of representation. Also, there were substantial differences in the amount of time devoted to various topics by teachers. For example, in the RNP group, a large amount of time was devoted to developing an understanding of the meaning of fraction symbol by making connections between the symbols and multiple physical models.

Misconception 2: Addition of Exponents

$$y^4 + y^4 = ?$$

Question. Simplify: $y^4 + y^4$

Likely Answer. $y^4 + y^4 = y^8$

Explanation of misconception. This misconception is connected with misapplication and overgeneralization of rules. The learner thinks that it is okay to add the powers because the base is the same for both terms. Instead of adding both powers, the correct thing to do would have been to add the coefficients of the two terms to attain $2y^4$.

What teachers can do. The teacher should attempt to analytically distinguish between $y^4 + y^4$ and $y^4 \cdot y^4$. Such distinction could reveal the learner misconception. The use of a graphic organizer to illustrate the concept is also suggested. Apprise learners that graphic organizers can be used as an learning aid when learning certain mathematical concepts, like properties of exponents.

The graphic illustration (Figure 2) demonstrates that y^4 factors to be $y \cdot y \cdot y \cdot y$ and represents $1y^4$. The sum of $1y^4$ and $1y^4$ results in $2y^4$. It would also be beneficial to represent the $y^4 \cdot y^4$ in graphic form. Learners would see how $y^4 \cdot y^4 = y^8$ and is therefore different from $y^4 + y^4$. Apart from exposing learners to the relationships between these concepts, this strategy can be useful in other situations, such as helping learners organize information.

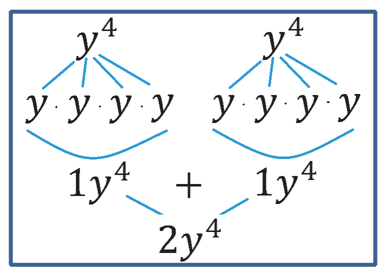
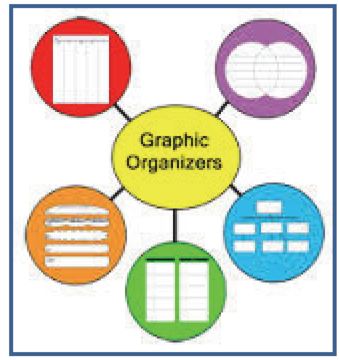


Figure 2. Graphic representation of Addition of exponents

The use of graphic organizers give structure to the concepts associated with exponents. For example, learners often confuse the role of exponents by thinking that 5^3 is the product of 5 and 3. However, if this concept is represented with the aid of a graphic organizer, learners would see how 5^3 is different from $5 \cdot 3$, diagrammatically. Teachers can use graphic organizers to reinforce learning, assess learning at multiple checkpoints, and identify misunderstanding of concepts. They can be used before, during, and after instruction. Teachers could use graphic organizers to brainstorm ideas, activate prior knowledge, and review concepts. They are also valuable tools in any activity which requires the use of critical thinking.

Graphic organizers appear to be beneficial as an instructional strategy that aid in the retention of learned information. Many learners benefit from a visual approach to brainstorming or organizing information. As learners become familiar with using graphic organizers, they will develop their own approaches and create their own organizers. Encourage learners to adapt them and create their own for more complex strategies and connections. Remember that there is no one right way to use graphic organizers; the best way is the way that works for each student.

Research note. Research supports the use of graphic organizers in facilitating and improving learning outcomes for a wide range of learners, not only in mathematics but in other subjects. Horton, Lovitt, & Bergerud (1990) reported on the value of graphic organizers to both middle school and high school students with or without disabilities as an organizational tool to promote the memory of content-related information. Other research (e.g., Jitendra,2002) indicates that organizers assists these same students in how to represent problem situations, such as searching for solutions to word problems in mathematics. Frequently, students with learning disabilities have difficulties recalling key information, making connections between broad concepts and detail, and solving mathematical word problems. According to Maccini & Ruhl (2000), students with learning disabilities might experience fluency difficulties with mathematical facts and with basic mathematical procedures. Teachers must be made aware that the use of graphic organizers is not only a valid instructional practice but a viable strategy that might lesson the difficulties that students with learning disabilities experience in mathematics (Gagnon & Maccini, 2000).



Ausubel (1963) believed that the manner in which knowledge is presented can influence learning. The appropriate organizer can help learners form relationships between previously acquired knowledge and new concepts. Research shows that graphic organizers are key to assisting learners improve academic performance. For example, Okebukola (1992) noted that data obtained in his study provide supportive evidence to indicate that the subjects in the study who were adjudged to be good at making concept maps exhibited superior performance in solving the three problems of the study. Also, Willerman & Mac Harc (1991) reported the relevance of graphic organizers in science. A control group of 40 eighth grader learners completed a unit on elements and compounds. An experimental group of 42 completed *concept maps* on same topic. Results of a one-tailed t-test demonstrated the usefulness of *concept maps* as graphic organizers. These results are consistent with the findings of Sneed & Snead (2004) who suggested that lower achieving learners appear to have success with the usage of concept mapping (advanced graphic organizers) in science.

Researchers (e.g. Ausubel and others mentioned here) have noted that graphic organizers aid comprehension for several reasons:

- They match the mind and, because it arranges information in a visual pattern that complements the framework of the mind, make it possible for information to be easily learned and understood;
- They demonstrate how concepts are linked to prior knowledge to aid comprehension;
- They aid the memory as opposed to recalling key points from an extended test;
- They help the learner retain information readily when higher thought processes are involved; and
- They engage the learner with a combination of the spoken word with printed text and diagrams.

Lenz et al. (2004) pointed out the significance of graphic organizers as related to students' ownership of the learning process. According to the researchers, creating a graphic organizer for an instructional lesson plan is an effective way to engage students in learning and it also provides a way to integrate an additional learning modality into instruction.

Conclusion

As indicated throughout this piece, misconceptions would always be experienced by learners due to the nature of mathematics. Be as it is, teachers need to be aware of their existence and ensure that misconceptions do not persist with learners for a longer period of time. For example, it will be detrimental to a grade four student who is still misplacing decimal points when multiplying decimals to move up to the fifth grade without adequate remedy. Research suggests that misconceptions that persist for years if undetected would negatively affect the future learning of mathematics. For example, Woodward, Baxter, & Howard (1994) pointed out that a continued, superficial understanding of mathematics allows learners to apply improper algorithms or repair strategies, eventually resulting in ingrained and deep-seated misconceptions.

Teachers should be sensitive and recognize that students come into their classrooms with misunderstandings and misconceptions. It is imperative that they work toward detecting the existing misconceptions that students may have and work purposefully to correct them. This is to avoid a situation whereby students move from one grade to another with these harmful misconceptions. Teachers should acknowledge that learners can overcome misconceptions by planning and consciously providing opportunities for learning through effective teaching strategies.

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