The Physics of Massive Star Death

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Introduction

Massive stars end their lives by producing core-collapse supernovae (ccSNe), spectacularly bright explosions that can outshine the rest of their host galaxy's 100 billion stars for more than a month (e.g., Rest et al., 2011). Supernovae affect the evolution and formation of entire galaxies, injecting matter and energy in the interstellar medium (e.g., Efstathiou, 2000) and fostering birth of the next generation of stars (e.g., Springel & Hernquist, 2003). Supernovae are also an important source of chemical enrichment of the Universe, producing carbon, oxygen and iron among other elements (e.g., Matteucci & Greggio, 1986). Despite decades of observations of hundreds of supernovae, detailed numerical calculations and theoretical efforts, the mechanism of explosion remains unknown (e.g., Colgate & White, 1966; Bethe & Wilson, 1985; Burrows & Goshy, 1993; Burrows et al., 1995; Buras et al., 2006a; Marek & Janka, 2009).

Stars produce energy by converting light elements to heavier nuclei by thermonuclear fusion. In the current paradigm, stars more massive than about 10 masses of our Sun ($M_{\odot}$) eventually develop an iron core surrounded by onion-like shells made of consecutively lighter elements (e.g., Woosley et al., 2002). Nuclear fusion in these shells increases the mass of the iron core. When the iron core mass reaches the Chandrasekhar limit of 1.4 $M_{\odot}$, it suddenly collapses and removes pressure support for the top layers, which start to fall to the center too. The density in the center rises, and when it reaches a value comparable to the density of an atomic nucleus, the hard core repulsion of the strong nuclear force halts the collapse. The dense compact object in the center

[1]This research was done in collaboration with my advisor Todd A. Thompson.

is called a proto-neutron star (PNS). At this instant in time, the matter falling in to the center bounces off of the PNS, forming an outgoing sound wave that steepens into a shock wave while travelling outwards. However, the shock wave does not propagate all the way through the overlying progenitor star, but stops its progress due to energy losses from neutrino emission, and due to ram pressure of the infalling matter of the progenitor star – an accretion shock forms, which converts cold free-falling matter of the progenitor star to a hot slowly-moving gas that gradually settles on the PNS. The accretion shock phase lasts at least several hundreds of milliseconds after collapse, a very long time for an object this dense and hot.

The explosion is initiated when the shock wave starts moving out again and travels through the progenitor star. However, even the most sophisticated numerical simulations of ccSNe ultimately fail in reviving the outward movement of the shock (Rampp & Janka, 2000; Bruenn et al., 2001; Liebendörfer et al., 2001; Mezzacappa et al., 2001; Thompson et al., 2003; Buras et al., 2006a,b). This "supernova problem" has important consequences for the astrophysics: there are no grounds for theoretical predictions for which stars will explode as ccSNe and what the explosion energies will be, and whether the compact remnant in the center will be a neutron star or a black hole, and what will be the distribution of their masses (see Fryer, 1999; O'Connor & Ott, 2011, for attempts to derive these distributions). More generally, ccSNe are a crucial component of the Universe and the lack of knowledge of the explosion mechanism has great ramifications.

It has been known that the matter below the shock wave heats up by absorption of neutrinos coming from the PNS and from cooling of the accreting gas, and this likely plays an important role in driving the explosion. Specifically, Burrows & Goshy (1993) showed that a steady-state accretion shock turns into explosion when the neutrino flux from the core $L_{\nu,\,\mathrm{core}}$ exceeds a critical value ($L_{\nu,\,\mathrm{core}}^{\mathrm{crit}}$) – the "neutrino mechanism". However, it is not known why $L_{\nu,\,\mathrm{core}}^{\mathrm{crit}}$ exists, and its dependence on the properties of the PNS (mass M, radius r_ν) has not been systematically explored. Furthermore, a reliable and physically motivated explosion condition equivalent to $L_{\nu,\,\mathrm{core}}^{\mathrm{crit}}$ is needed to diagnose the approach of sophisticated multi-dimensional simulations to an explosion (Scheck et al., 2006; Murphy & Burrows, 2008).

In order to understand the physics of $L_{\nu,\,\rm core}^{\rm crit}$, I solve the one-dimensional steady-state accretion flow between the surface of the PNS at radius r_ν and the accretion shock at radius $r_{\rm S}$ at three levels of sophistication: (1) an isothermal flow, (2) an analytic model including simple heating and cooling of the matter by neutrino processes, and (3) a full numerical calculation, that allows for tracking of variations in composition of the matter, and that also includes, for the first time in this context, a simple scheme for radiation transport of neutrinos and antineutrinos that couples to the energy deposition in the accretion flow. Detailed results are given in Pejcha & Thompson (2011).

Isothermal supernova

Here, I present the simplest version of the problem — an isothermal accretion flow bounded by a shock — and I show how this problem is crucial in understanding the existence of $L_{\nu,\,\rm core}^{\rm crit}$ in the more complete problem. Isothermal flows have constant sound speed c_T everywhere, and hence constant temperature. Changing c_T means changing the temperature, and this is conceptually similar to changes in the heating rate in the full problem, parameterized by the core neutrino luminosity $L_{\nu,\,\rm core}$.

The velocity structure of spherical steady-state isothermal flows (Bondi, 1952; Velli, 2001) is described by the equation

$$\left(\mathscr{M} - \frac{1}{\mathscr{M}}\right)\frac{{\rm d}\mathscr{M}}{{\rm d}x} = \frac{2}{x} - \frac{1}{2x^2}, \qquad (1)$$

where $\mathscr{M} = v/c_T$ is the Mach number, v is the fluid velocity, c_T is the isothermal sound speed, $x = rc_T^2/(2GM)$ is the rescaled radial coordinate r, G is the Newton's constant, and M is the mass of the central object. We show solutions to equation (1) for a fixed mass accretion rate $\dot{M} = 4\pi r^2\rho v$ in Figure 1. It is possible for a standing shock wave in the flow to exist at a point that satisfies the two Rankine-Hugoniot shock jump conditions

$$\rho^-\mathscr{M}^- \quad = \quad \rho^+\mathscr{M}^+, \qquad (2)$$

$$\rho^-(\mathscr{M}^-)^2 + \rho^- \quad = \quad \rho^+ x^{-1}, \qquad (3)$$

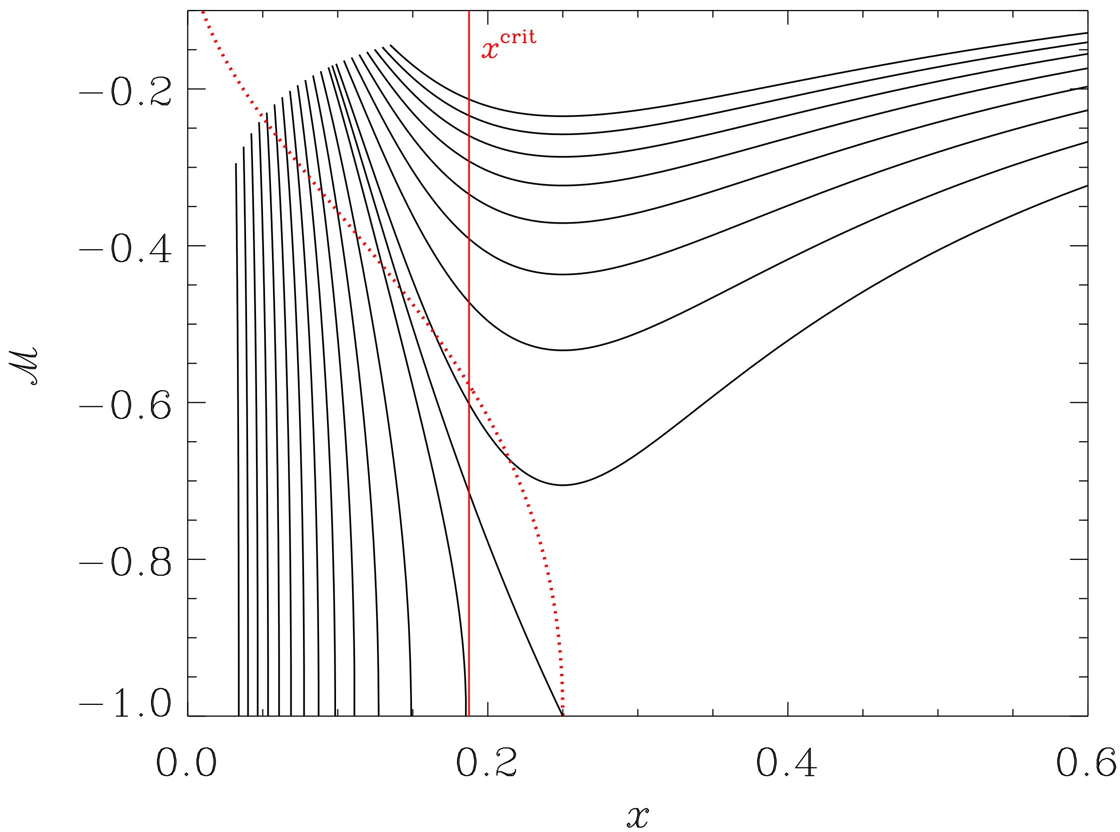

Figure 1: Isothermal accretion plotted in the space of Mach number $\mathscr{M}$ and rescaled radial coordinate $x = rc_T^2/(2GM)$. Solid black lines show solutions to eq. (1) with $\dot{M} = -1\,M_\odot$ s^{-1} and $M = 1.4\,M_\odot$ starting from $r_\nu = 30\,$km, and with fixed velocity $v(r_\nu) = \dot{M}/(4\pi r_\nu^2 \rho_\nu)$, where $\rho_\nu = 3\times10^{10}\,$g cm^3. The value of c_T^2 increases from 4×10^{18} (black solid line starting at lowest x) to 1.68×10^{19} cm^2 s^{-2} (highest line) in the steps of 6.4×10^{17} cm^2 s^{-2}. Red dotted line is velocity just downstream of a shock positioned at any x, assuming that the upstream flow is in pressure-less free fall. The critical value $x^{\rm crit}$ is shown with a vertical red solid line.

which express conservation of mass and momentum, respectively, and assume that matter upstream of the shock is in pressure-less free fall. Here, ρ is the mass density and the $+$ and $-$ superscripts correspond to the quantities evaluated just upstream and downstream of the shock, respectively. The physically relevant solution to equations (2–3) is $\mathscr{M}^- = (\sqrt{x^{-1}-4} - x^{-1/2})/2$. For the sake of clarity, we do not show the upstream profile in Figure 1.

We can see from Figure 1 that a steady-state solution with a shock is not possible for every value of c_T. For low values of c_T, the surface r_ν of the PNS lies above a viable position of a shock (denoted by the red dotted line), and hence no solution for shock is possible. If we step up in c_T,

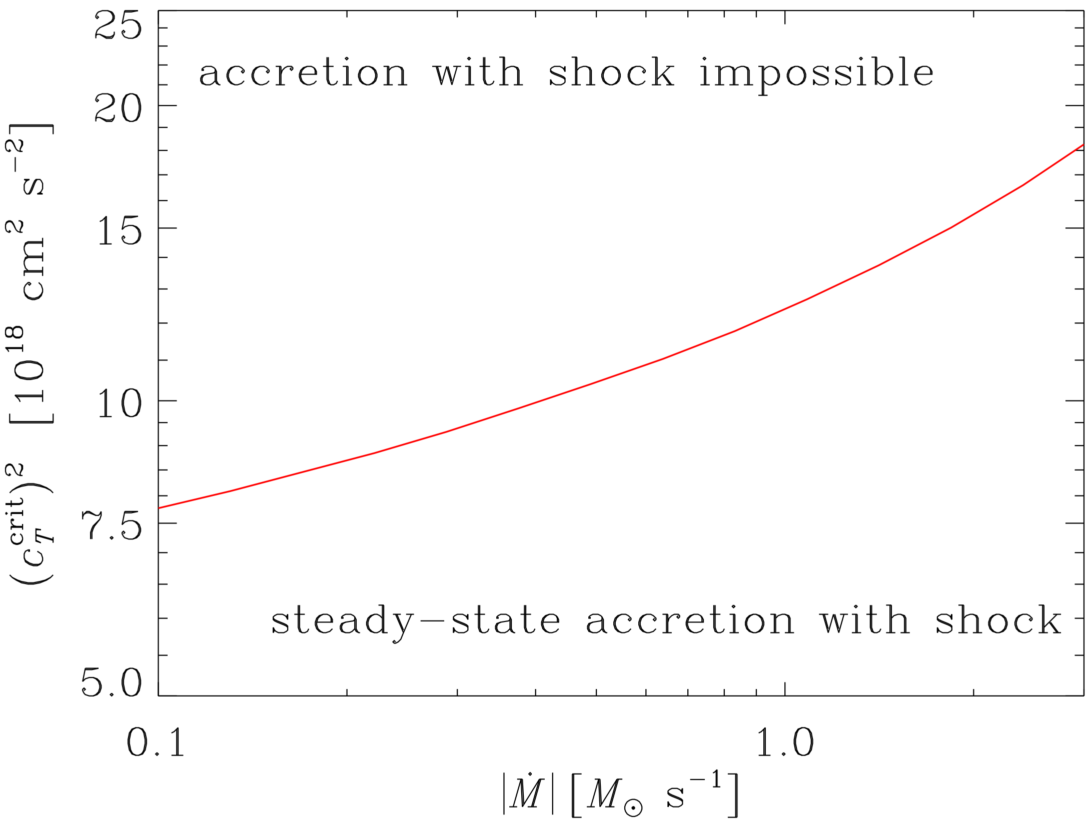

Figure 2: The maximum isothermal sound speed $c_T^{\rm crit}$ that allows for a shock in the flow at a given $\dot{M}$. The PNS parameters — M, r_ν, ρ_ν — are the same as in Figure 1.

the shock appears at r_ν and increases its radius. At even higher values of c_T (solid curves starting at higher x), a second solution appears where there are two intersections between the solid curve of interest and the red dotted line (the outer solution is at $x = 0.25$ and $\mathscr{M} = -1$). Increasing c_T further, the two solutions come closer together and finally merge at $x^{\rm crit}$ for a critical value $c_T^{\rm crit}$. For $c_T > c_T^{\rm crit}$ it is not possible to have a shock in the flow, because it is not possible to satisfy the shock jump conditions at any radius. That is, the uppermost solid curves do not intersect the dotted curve.

In Figure 2, I plot values of $c_T^{\rm crit}$ as a function of mass accretion rate $\dot{M}$, the "critical curve". The upward curvature of the critical curve is caused by exponential nearly-hydrostatic density profile below the shock, which is seen also in more elaborate calculations (Pejcha & Thompson, 2011).

A simplified supernova evolution from the stalled shock to explosion can be pictured with the help of Figure 2, similarly to Burrows & Goshy (1993). The system starts on the right at high $|\dot{M}|$ and with $c_T < c_T^{\rm crit}$. It evolves in a sequence of steady-states to the left decreasing $|\dot{M}|$ and c_T. If the model trajectory crosses the red line, that is if at some point $c_T > c_T^{\rm crit}$, no steady state is possible (Yamasaki & Yamada, 2005, 2006) and the flow reorganizes itself most likely to a supersonic wind – a supernova explosion (Burrows, 1987; Burrows & Goshy, 1993; Burrows et al., 1995; Yamasaki & Yamada, 2005).

The position of the critical point $x^{\rm crit}$ can be calculated exactly by substituting equations (2) and (3) into equation (1). We obtain exactly $x^{\rm crit} = 3/16$ implying that the critical condition for existence of a standoff shock is

$$\frac{(c_T^{\rm crit})^2}{v_{\rm esc}^2} = \frac{3}{16} = 0.1875, \tag{4}$$

where $v_{\rm esc}^2 = 2GM/r$ is the square of the escape velocity. I call the condition of equation (4) the "antesonic" condition, because the critical point lies below the sonic point[2], and it occurs at a time before a supersonic wind with a sonic point is established in supernova explosion.

Results from more realistic calculations

Although the isothermal model presented above is very rudimentary, it clearly states the underlying physical principle: there is a maximum value of the controlling parameter that allows for steady-state solution with a shock, and this is because the shock jump conditions in mass, momentum, and energy cannot be satisfied anywhere for flows with parameter value higher than critical. The isothermal model with $c_T^{\rm crit}$ is general and does not depend on any specific heating mechanism. In Pejcha & Thompson (2011) we explicitly show using calculations with more realistic microphysics and simple approximation to radiation transport, that exactly the same mechanism as in isothermal flows is responsible for the presence of $L_{\nu,\,\rm core}^{\rm crit}$ in supernovae.

Is the critical condition for explosion in the isothermal case (eq. [4]) present also in more complete calculations? In Pejcha & Thompson (2011) we found that if we generalize the left-

[2]Sonic point is a well-known point of interest of equation (1), which occurs at $\mathscr{M} = -1$ and $x = 0.25$ to preserve the continuity of the solution.

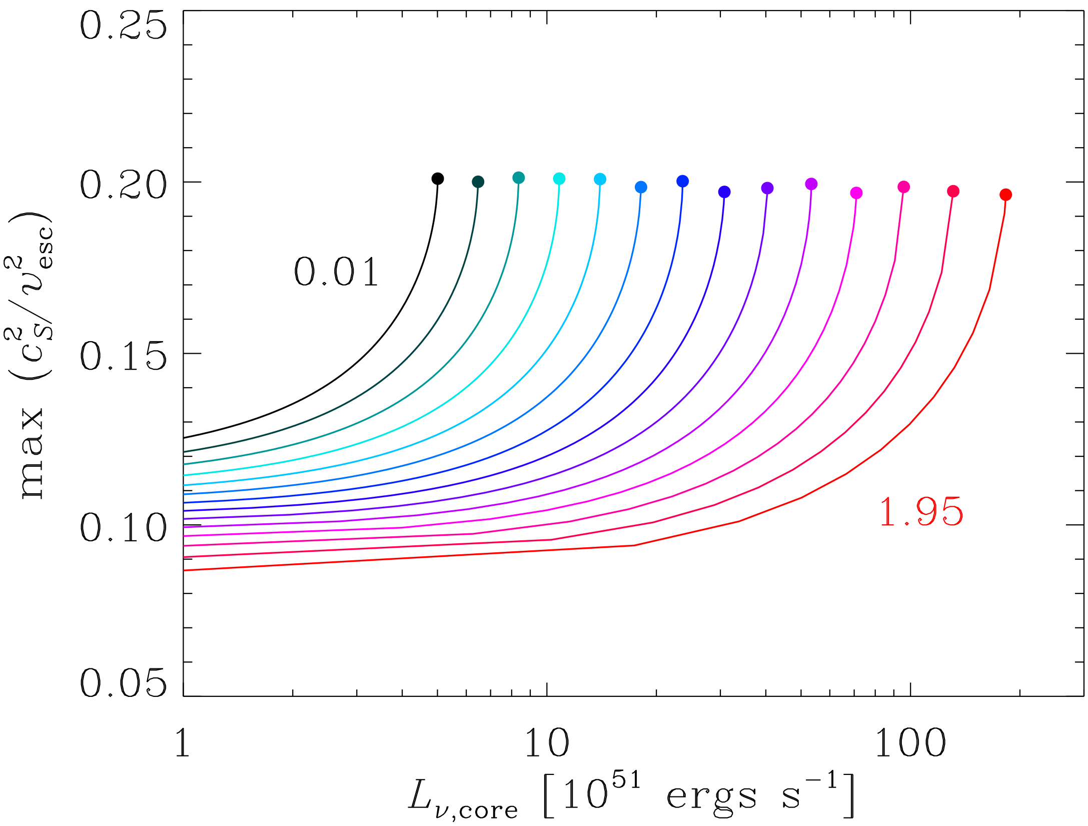

Figure 3: The antesonic condition. Maximum value of the ratio of the adiabatic sound speed c_S to the escape velocity $v_{\rm esc}$ as a function of $L_{\nu,\,\rm core}$ (individual lines). Colors distinguish between different mass accretion rates $\dot{M}$ with lowest and highest values denoted (in $M_\odot$ s^{-1}). Filled circles mark $L_{\nu,\,\rm core}^{\rm crit}$. We see that at $L_{\nu,\,\rm core}^{\rm crit}$ the value of the antesonic ratio is remarkably constant. Although the critical condition plotted here appears to be a local condition on the sound speed in the accretion flow, our analysis of isothermal flows shows that the quantity $\max\,(c_S^2/v_{\rm esc}^2)$ is a merely a scalar metric for solution space of Euler equations and thus it is a global condition.

hand side of equation (4) to $\max\,(c_S^2/v_{\rm esc}^2)$, where $c_S = (\partial P/\partial\rho)_S$ is the adiabatic sound speed at constant entropy S, then the factor on the right is 0.193 to within 5% for a very wide range of $\dot{M}$, M, and r_ν, as is illustrated in Figure 3. Furthermore, the antesonic condition holds even after quite drastic changes to the microphysics; the value of the numerical factor is close to 0.2 in most cases. This is much better consistency and robustness than for other conditions proposed in the literature (Bethe & Wilson, 1985; Thompson, 2000; Janka, 2001; Thompson & Murray, 2001; Thompson et al., 2005; Buras et al., 2006b; Scheck et al., 2008; Murphy & Burrows, 2008), which vary by 50% or more at $L_{\nu,\,\rm core}^{\rm crit}$, as shown in Pejcha & Thompson (2011).

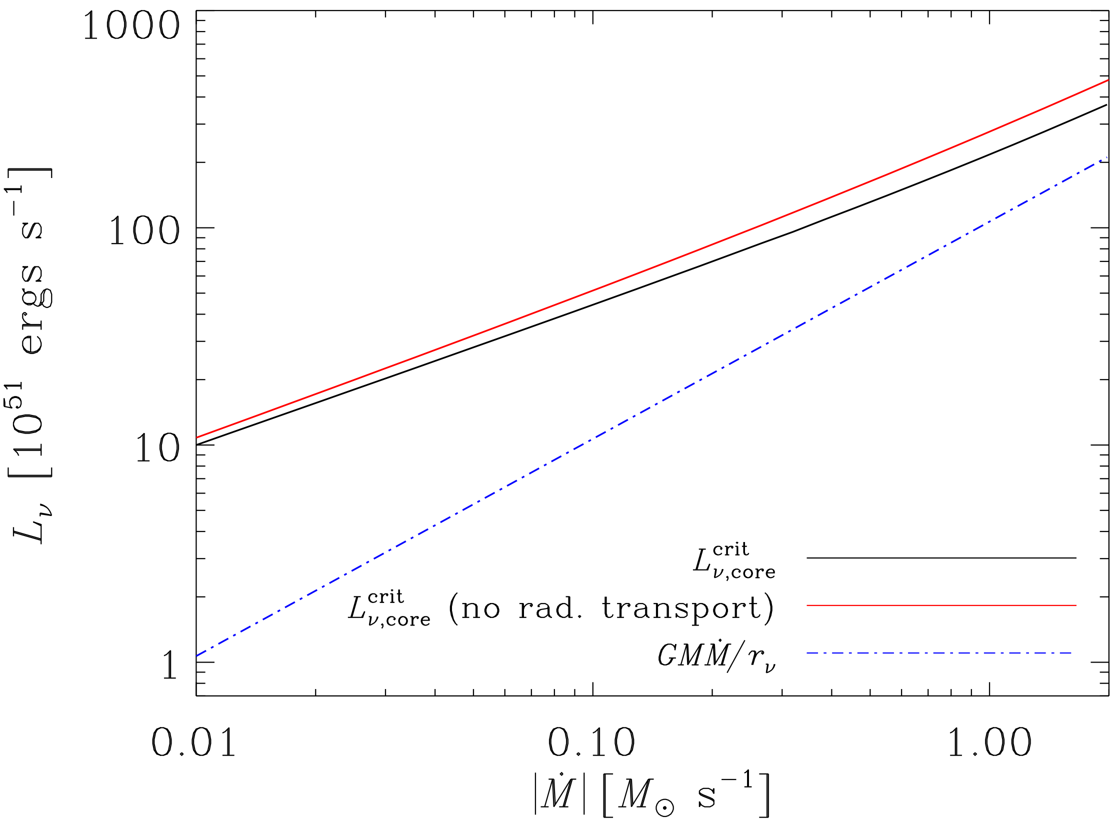

Figure 4: Effect of radiation transport and accretion luminosity on the critical curve. The black solid line shows critical core neutrino luminosity $L^{\rm crit}_{\nu,\,\rm core}$ for $M = 1.6\,M_\odot$ and $r_\nu = 40\,$km. The blue dash-and-dotted line is the maximum possible accretion luminosity, $GM\dot{M}/r_\nu$. The red solid line is a critical curve for the same parameters except that the neutrino radiation transport was shut off ($\mathrm{d}L_\nu/\mathrm{d}r = 0$).

In more realistic calculation described in Pejcha & Thompson (2011), I included simple neutrino radiation transport. It is then possible to assess the importance of the accretion luminosity (the release of gravitational potential energy of infalling matter through cooling by neutrinos). Figure 4 shows comparison of critical curve from our fiducial calculation (black line) with the maximum possible accretion luminosity $GM\dot{M}/r_\nu$ (blue dash-dotted line). We see that $L^{\rm crit}_{\nu,\,\rm core}$ is always much higher than the maximum possible accretion luminosity, maybe except very high $|\dot{M}|$. However, these high values of $|\dot{M}|$ occur early on in the supernova evolution, when the steady-state approximation is not justified. Figure 4 shows also that a mere fact of including radiation transport lowers $L^{\rm crit}_{\nu,\,\rm core}$ considerably: the red solid line shows critical curve calculated using the same pa-

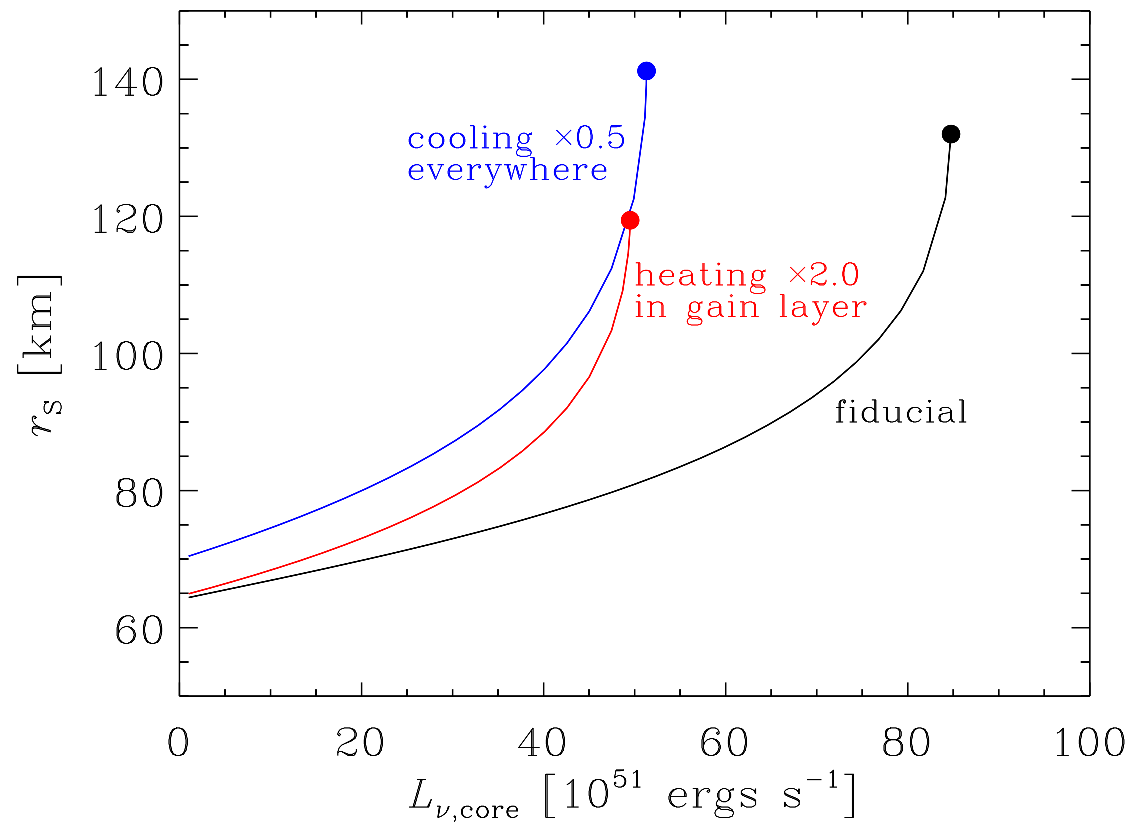

Figure 5: Illustration of effects of modified heating and cooling on $r_{\rm S}$ (solid lines) and $L^{\rm crit}_{\nu,\,\rm core}$ (filled circles). Decreasing cooling (blue) and increasing heating (red) both act to decrease $L^{\rm crit}_{\nu,\,\rm core}$, and to increase $r_{\rm S}$ at fixed $L_{\nu,\,\rm core}$ with respect to the fiducial calculation (black). However, lower cooling efficiency increases $r_{\rm S}$ at $L^{\rm crit}_{\nu,\,\rm core}$, whereas higher heating decreases $r_{\rm S}$ at $L^{\rm crit}_{\nu,\,\rm core}$.

rameters, but with radiation transport turned off. Accounting for radiation transport lowers $L^{\rm crit}_{\nu,\,\rm core}$ by 8% to 23%, depending on $\dot{M}$. These results show that the neutrino luminosity emanating from the PNS has to be the major source of the neutrino luminosity that revives the explosion.

An interesting result from recent time-dependent simulations of supernovae (Ohnishi et al., 2006; Iwakami et al., 2008; Murphy & Burrows, 2008; Nordhaus et al., 2010) is that $L^{\rm crit}_{\nu,\,\rm core}$ is a function of dimension of the simulation. Specifically, simulations in 3D have $L^{\rm crit}_{\nu,\,\rm core}$ lower than those in 2D, which in turn have $L^{\rm crit}_{\nu,\,\rm core}$ lower than 1D. While going from 1D to 2D allows for convection, it is not clear what is gained by going from 2D to 3D. The analytic model of $L^{\rm crit}_{\nu,\,\rm core}$ I developed (details in Pejcha & Thompson, 2011) suggests, that the accretion flows in higher dimensions are less efficient in cooling themselves rather than more efficient in heating. This is illustrated in Figure 5, where I plot shock radius $r_{\rm S}$ as a function of $L_{\nu,\,\rm core}$; values of $r_{\rm S}$ at $L_{\nu,\,\rm core}^{\rm crit}$ are denoted with filled circles.. When compared to fiducial calculation (black), an artificial decrease of cooling or an increase of heating have the same effect of reducing $L_{\nu,\,\rm core}^{\rm crit}$ and increasing $r_{\rm S}$ for a fixed $L_{\nu,\,\rm core}$. However, reduced cooling gives higher values of $r_{\rm S}$ at $L_{\nu,\,\rm core}^{\rm crit}$ than both the fiducial calculation and increased heating. While multidimensional simulations show larger shock radii than 1D simulations (Burrows et al., 1995; Nordhaus et al., 2010), even at explosion (Ohnishi et al., 2006; Iwakami et al., 2008), we conclude in contrast to expectations that it is the decreased cooling rather than increased heating that lowers $L_{\nu,\,\rm core}^{\rm crit}$ in higher-dimensional simulations.

Conclusions

For the first time, I explicitly show that there is a direct connection between the topology of solutions of a simple isothermal accretion flow with shock and the neutrino mechanism. I find that there is a maximum isothermal sound speed $c_T^{\rm crit}$ — essentially the temperature of the gas — above which it is impossible to maintain steady-state accretion shock – at higher sound speeds the flow cannot conserve energy and momentum throughout the shock. This mechanism for $c_T^{\rm crit}$ is quite general and it is explicitly shown in Pejcha & Thompson (2011) that it is equivalent to $L_{\nu,\,\rm core}^{\rm crit}$ in the neutrino mechanism of supernovae. Furthermore, the isothermal model provides an explosion condition: when the ratio of c_T^2 to the square of the local escape velocity exceeds 0.188, explosion occurs. I show that in the detailed calculation appropriate for the supernova context the value of this ratio at $L_{\nu,\,\rm core}^{\rm crit}$ is 0.193 with a scatter of only 5% over a very wide range of mass accretion rates, and masses and radii of the PNS. This "antesonic" explosion condition is a much better diagnosis of $L_{\nu,\,\rm core}^{\rm crit}$ than other conditions proposed in the literature.

My results on the physics of the neutrino mechanism of ccSNe reveal necessary ingredients for a successful supernova explosion, what is the nature of $L_{\nu,\,\rm core}^{\rm crit}$ and how does it depend on parameters of the PNS and microphysics. While more work is clearly required to determine viability of the neutrino mechanism, my results can already be used to diagnose multi-dimensional simulations and to constrain some of the observable properties of ccSNe, which is my current work in progress.

References

Bethe, H. A., & Wilson, J. R. 1985, The Astrophysical Journal, 295, 14

Bondi, H. 1952, Monthly Notices of the Royal Astronomical Society, 112, 195

Bruenn, S. W., De Nisco, K. R., & Mezzacappa, A. 2001, The Astrophysical Journal, 560, 326

Buras, R., Rampp, M., Janka, H.-T., & Kifonidis, K. 2006a, Astronomy & Astrophysics, 447, 1049

Buras, R., Janka, H.-T., Rampp, M., & Kifonidis, K. 2006b, Astronomy & Astrophysics, 457, 281

Burrows, A. 1987, The Astrophysical Journal Letters, 318, L57

Burrows, A., & Goshy, J. 1993, The Astrophysical Journal Letters, 416, L75

Burrows, A., Hayes, J., & Fryxell, B. A. 1995, The Astrophysical Journal, 450, 830

Colgate, S. A., & White, R. H. 1966, The Astrophysical Journal, 143, 626

Efstathiou, G. 2000, Monthly Notices of the Royal Astronomical Society, 317, 697

Fryer, C. L. 1999, The Astrophysical Journal, 522, 413

Iwakami, W., Kotake, K., Ohnishi, N., Yamada, S., & Sawada, K. 2008, The Astrophysical Journal, 678, 1207

Janka, H.-T. 2001, Astronomy & Astrophysics, 368, 527

Liebendörfer, M., Mezzacappa, A., Thielemann, F.-K., Messer, O. E., Hix, W. R., & Bruenn, S. W. 2001, Physical Review D, 63, 103004

Marek, A., & Janka, H.-T. 2009, The Astrophysical Journal, 694, 664

Matteucci, F., & Greggio, L. 1986, Astronomy & Astrophysics, 154, 279

Mezzacappa, A., Liebendörfer, M., Messer, O. E., Hix, W. R., Thielemann, F.-K., & Bruenn, S. W. 2001, Physical Review Letters, 86, 1935

Murphy, J. W., & Burrows, A. 2008, The Astrophysical Journal, 688, 1159

Nordhaus, J., Burrows, A., Almgren, A., & Bell, J. 2010, The Astrophysical Journal, 720, 694

O'Connor, E., & Ott, C. D. 2011, The Astrophysical Journal, 730, 70

Ohnishi, N., Kotake, K., & Yamada, S. 2006, The Astrophysical Journal, 641, 1018

Pejcha, O., & Thompson, T. A. 2011, The Astrophysical Journal, submitted

Rampp, M., & Janka, H.-T. 2000, The Astrophysical Journal Letters, 539, L33

Rest, A., et al. 2011, The Astrophysical Journal, 729, 88

Scheck, L., Kifonidis, K., Janka, H.-T., Müller, E. 2006, Astronomy & Astrophysics, 457, 963

Scheck, L., Janka, H.-T., Foglizzo, T., & Kifonidis, K. 2008, Astronomy & Astrophysics, 477, 931

Springel, V., & Hernquist, L. 2003, Monthly Notices of the Royal Astronomical Society, 339, 289

Thompson, C. 2000, The Astrophysical Journal, 534, 915

Thompson, C., & Murray, N. 2001, The Astrophysical Journal, 560, 339

Thompson, T. A., Burrows, A., & Pinto, P. A. 2003, The Astrophysical Journal, 592, 434

Thompson, T. A., Quataert, E., & Burrows, A. 2005, The Astrophysical Journal, 620, 861

Velli, M. 2001, Astrophysics & Space Science, 277, 157

Woosley, S. E., Heger, A., & Weaver, T. A. 2002, Reviews of Modern Physics, 74, 1015

Yamasaki, T., & Yamada, S. 2005, The Astrophysical Journal, 623, 1000

Yamasaki, T., & Yamada, S. 2006, The Astrophysical Journal, 650, 291