

## Contest Corner: Developing Problem Solving Skills and Attitudes in our K-12 Students

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**The** Common Core State Standards for Mathematical Practice require students to explain their mathematical thinking, to represent their ideas and to make sense of other students' ideas, (CCSSMP, 2010). These Principles encourage a shift in how math is taught that will ultimately change what students know about math and how they perceive mathematics. Problem solving is no longer a topic in mathematics, but rather is the methodology every student should engage in daily. How do we change our classrooms into problem solving environments that empower all students with the tools necessary to think critically, reason abstractly, and communicate mathematically?

George Polya, often referred to as the father of problem solving in mathematics, said

“Mathematics, you see, is not a spectator sport. To understand mathematics means to be able to do mathematics. And what does it mean doing mathematics? In the first place it means to be able to solve mathematical problems. To achieve the higher aims I am talking about, there are some general tactics of problem solving—the right attitude for problem solving and ability to attack all kinds of problems, not only simple problems that can be solved with simple arithmetic, but more complicated problems of engineering, physics and so on, which will be further developed in the high school. But the foundations of problem solving should be started in the elementary school. And so I think an essential point in the elementary school is to introduce children to the tactics of problem solving—not to solve this or that kind of problem, not to make just long divisions or some such thing, but to develop a general attitude and ability for the solution of problems”. (California Mathematics Council).

Polya, goes on to relate his ideas to the Ancient Chinese Proverb, “there is allegedly a Chinese proverb that says, “I hear and I forget. I see and I remember. I do and I understand.” So “I hear and I forget.” What you just hear you forget quickly. Good advice is very quickly forgotten. What you see with your own eyes is remembered better, but you really understand it when you do it with your own hands. So there must be more than just priority of action and perception in our teaching.” (California Mathematics Council).

Therefore the schools, especially elementary schools, are today in an evolution; an evolution of developing problem solvers. In his book, *How to Solve It*, Polya created a four-step process considered the basics for any problem solving. Those four steps consist of:

1. Understand the Problem
2. Devise a Plan
3. Carryout the Plan
4. Look Back

What do these steps mean? What must one do to “Understand the Problem.” Typically this is where the student reads the problem, and attempts to understand what is known and unknown. What is the question? What facts are given? Next, the students will “Devise a Plan,” which requires the student to translate the problem from words to mathematical symbols, such as an equation, or a mathematical drawing, or a table or chart. Then a student must “Carry out the Plan” by solving for the unknown, recognizing a pattern, or whatever is needed to answer the question asked in the given problem. Lastly, the student will “Look Back” at his/her answer to interpret the solution to the problem. Relate the solution back to the context of the problem. Does this solution offer a reasonable answer for the question asked in the problem?

Dr. Dendane suggest in his paper *Skills Needed for Mathematical Problem Solving* (2009), that looking back be part of each of the first three steps of the process as well. Students should start over if what he/she is doing does not make sense, rather than continuing with a plan that does not make sense towards answering the question. By checking each phase along the way, the student can confirm that the plan makes sense and is leading towards a solution that that makes sense for the given problem.

Typically teachers expect students to complete all four steps, rather than practicing each of the steps, so that students are given the time and opportunities to become confident and proficient with each of the four steps in the process of solving problems.

The National Council of Teachers of Mathematics' *Principles and Standards for School Mathematics* recommended the following problem-solving standards for all students in 2000.

Instructional [programs](#) from pre-kindergarten through grade 12 should enable all students to:

- Build new mathematical knowledge through problem solving,
- Solve problems that arise in mathematics and in other contexts,
- [Apply](#) and adapt a variety of appropriate strategies to solve problems, and
- Monitor and reflect on the process of mathematical problem solving.

The Common Core reiterates the expectation that all students should be competent problem solvers. According to Heidi Janzen, a former classroom teacher and mathematics specialist “Problem-solving skills can be fostered in pre-kindergarten and even earlier by instilling in children a sense of exploration and a spirit of perseverance.”

Ms. Janzen states that problem solving requires cognitive skills such as logical thinking, reasoning, and creativity that require students to move beyond simply using algorithms or other routine processes. Students need to be able to recall previously learned mathematical skills and learn how to apply them to problem solving situations.

To be successful in solving problems, research has shown that more than mathematical skills and the ability to apply these skills are needed. Many students have adverse attitudes towards problem solving. These attitudes are often brought about by continued difficulty with problem solving and the eventual loss of confidence in one’s ability to problem solve.

The first standard for mathematical practice is *make sense of problems and persevere in solving them*, as such, teachers must address the affective issues. Heidi Janzen identifies the issues as motivation, confidence, perseverance, flexibility, and risk taking behavior. If elementary teachers engage their students in problem solving activities that allow the students to become confident in their own ability to solve problems, by exposing students to different problems on a regular basis, focusing on the different phases of problem solving, and allow the students to develop a repertoire of problem solving strategies, these affective issues can be avoided, so that they do not spiral out of control in middle and secondary mathematics.

In answer to our opening question: How do we change our classrooms into problem solving environments that empower all students with the tools necessary to think critically, reason abstractly, and communicate mathematically, Polya offered the following advice in a lecture to mathematics teachers,

“Be interested in mathematics. Know mathematics. Know about the ways of learning mathematics. The best way to learn anything is to discover it by yourself. Try to read the faces of your students, try to see their expectations and difficulties, put yourself in their

place. Give them not only information, but “ know-how” attitudes of mind, the habit of methodical work. Let your students learn guessing. Let them learn proving. Look out for such features of the problem at hand as may be useful in solving the problems to come – try to disclose the general pattern that lies behind the present concrete situation. Do not give away your whole secret at once – let the students guess before you tell it – let them find out by themselves as much as is feasible. Suggest it; do not force it down their throats.”

Developing college and career ready students who are skilled problem solvers is a goal set by the *Common Core State standards for Mathematics*. If such goal is to be reached, educators from Pre-K through 12<sup>th</sup> grade must continually engage their students in problem solving tasks. They must educate the students on how to problem solve, and allow students the time and opportunity to develop the strategies needed, to be competent problem solvers who are confident in his or her own ability to solve problems on their own or in group situations.

Perhaps this vision will become reality and Mathematical Contest will involve all students not just the few that are part of a schools math team. Perhaps Mathematical Contest can be introduced in Early Childhood and continue throughout secondary years. What a wonderful vision. Consider the following sites, when looking for engaging problems.

#### **The Mathematics Assessment Project: MAP**

Assessment tasks and problem solving lesson plans for middle and high school students can be found on this website. <http://map.mathshell.org/materials/tasks.php>

#### **The Ohio Resource Center: ORC**

Includes Language Arts, Science, Social Studies and Mathematics Resources for all grade levels. Great Math Problems can be found in Stella’s Stunners for grades 6 -12; the Problem Corner has awesome math problems for grades 3 – 12. The Mathematics Mini Collections and Standards offer math lessons that cover various topics many of which include a problem-solving context.

<http://www.ohiorc.org/for/math/>

#### **National Council of Teachers of Mathematics**

Lessons and resources are available for grades pre-K through 12<sup>th</sup> grade. See

<http://www.nctm.org/resources/default.aspx?id=230>

#### **K-5 Math Teaching Resources**

This site provides an extensive collection of free resources, math games, and hands-on math activities aligned with the Common Core State Standards for Mathematics. Our math printables are suitable for use in math centers, small group, or whole class settings. Instructions for each activity are presented in large print on a task card in child-friendly language to enable students to work on tasks independently after a brief introduction to the task. All files are in PDF format and can be accessed using Adobe Reader.

<http://www.k-5mathteachingresources.com/> These are just a few to get you started.

#### **Bibliography**

Chaudhary, H. P. (2013, May 8). Five steps to improve mathematics problem solving. myHSEBnotes.com. Retrieved December 27, 2013.

California Mathematics Council, Infinity Wall. George Polya, 1887-1985. Retrieved December 27, 2013. <http://www.cmc-math.org/members/infinity/polya.html>

Dendane, A. (2009, April 13-16). Skills needed for mathematical problem solving. Paper presented at the tenth annual research conference, United Arab Emirates University, Al-Ain, United Arab Emirates. Abstract retrieved from [http://www.analyzemath.com/math\\_problems/paper\\_1.html](http://www.analyzemath.com/math_problems/paper_1.html).

Janzen, H. Developing problem solvers in mathematics. Glencoe Online. Retrieved December, 27, 2013.

Mongeau, L. (2014, January 20). Common core standards bring dramatic changes to elementary school math. EdSource.org. Retrieved January 20, 2014.

National Council of Teachers of Mathematics, (2000). *Principles and standards for school mathematics*. Nctm.org. Retrieved December 20, 2013.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics*. Washington, DC: Authors. Retrieved December 20, 2013.



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## Food for Thought

Researchers have known for decades that none of the sweeping assertions about left brain/right brain differences are supported by solid science. Although they were not shouting from the mountaintops, these scientists had unimpeachable evidence that the popular culture versions of the left brain/right brain theory do not capture how the brain really works.

In a 1984 essay published in the journal *Neuropsychologia*, [Roger] Sperry warned that experimentally observed polarity in right-left cognitive style is an idea in general with which it is very easy to run wild . . . it is important to remember that the two hemispheres in the normal intact brain tend regularly to function closely together as a unit.

And, of course, there was the issue that the patients on whom all of the conclusions were based were just that-patients. They had abnormal brains (otherwise they would never have been operated on). So, there was a lingering question of just how strongly the results from these patients could be assumed to apply to normal people, whose brains are not split into two.

Kosslyn, S.M. & Miller, G. W. (2013). *Top brain, bottom brain: Surprising insights into how you think*, 63-64. New York:Simon & Schuster.

Note: Nobel Laureate Roger Sperry's research on split brained monkeys and humans was the basis for the popular, but incorrect "Left-brain, Right-brain" movement. A split brain has the corpus callosum severed, thus removing communication between the left and right sides of the brain. *edl*