SO EVERYONE SHOULD BE TEACHING PROBLEM SOLVING?

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The math education community has been fairly successful in publicizing the message of the great need for problem solving skills to be taught in mathematics classrooms around the country. Since NCTM's Agenda for Action (1980) the consequent national, regional, state, and local efforts by the profession have communicated the concern about the problem solving abilities of students to educators and to lay people. If asked to list critical mathematics needs of students, most curriculum people, many teachers of mathematics, and even some building administrators would certainly list problem solving at or near the top of that list.

However, despite all the commotion that has been raised about problem solving, the fact is that little change has occurred in the mathematics classroom. The research on teacher training of Joyce and Showers (1980) at Stanford University provides insight into why we have failed to bring about change and what needs to be done in order to provide teachers with the necessary skills, confidence, and support to begin teaching problem solving in a meaningful way.

Joyce and Showers classified the outcomes of training into four levels of impact: 1) awareness; 2) the acquisition of concepts and organized knowledge; 3) the learning of principles and skills; and 4) the ability to apply those principles and skills in problem solving activities. At the awareness level, educators begin to realize the importance of problem solving and why it is so critical for their students. According to Joyce, the research says at this level about 5 percent of the teachers (possibly NCTM members?) will make a long-term change in their behavior in the classroom. At the conceptual and organized knowledge level, educators begin to understand what problem solving is, perhaps by becoming proficient at using various strategies themselves. In addition they see problem solving as a process, not a product. At this level of impact, the research says that only the same 5 percent of the teachers will make a long-term change in their behavior in the classroom. At the principles and skill level, educators learn to teach problem solving and to adapt their teaching of various problem solving strategies to students of varying ability. At this level there is potential for action by the teacher – and the teacher might

even try teaching problem solving – but again the research shows that only about 10 percent will adapt their new behavior over the long-term. Only at the fourth level, applications and problem solving, is the teacher apt to transfer the concepts, principles, and skills learned about problem solving to the classroom over the long-term. In order for the teacher to reach this fourth level, the teacher's experiences would include such things as modeling by a master teacher; practicing in simulated and/or classroom settings; receiving structured and/or open-ended feedback about performance in teaching problem solving; and, generally, some type of coaching. At this top level, approximately 80 percent or more of the teachers will continue to use this newly learned behavior over the long-term.

In view of the research, it becomes clear as to why only a few teachers have begun teaching problem solving. Almost all of the effort by NCTM, state, and local educators on problem solving has been at the awareness or concept level of impact. Training to help teachers reach the application and problem solving level has, for the most part, been nonexistent. Until this happens, the 95 percent will continue to teach in the same manner as they always have.

Attention and interest in problem solving will be rekindled again by NCTM's publication of Curriculum and Evaluation Standards for School Mathematics in the Spring of 1989. This is a critical time. A systematic approach and a massive effort in retraining teachers to teach problem solving is needed to achieve the goal of raising our students' problem solving skills. School districts need to provide vertical articulation in the mathematics curriculum to insure that problem solving skills are introduced, reinforced, maintained, and built upon as students progress through the grades.

One District's Approach to the Problem Solving Problem

The Stark County Local School System, an intermediate unit in northeastern Ohio, has created and published a comprehensive program for teaching problem solving in grades 1-8. Initially, problem solving strategies were identified along with the grade level(s) where they would be introduced and reinforced. (See Table 1.) These were incorporated as a part of the mathematics curriculum. All strategies from previous grade levels would be maintained in individual practice sessions. Teachers at each grade level who had received intensive training in problem solving instruction were selected to gather and organize problem solving resources around those strategies.

TABLE 1

Problem Solving Strategies Grade Level and Emphasis

Grade	Strategy
1st	Acting out Making a model (manipulatives) Finding a pattern (pictures, symbols) Drawing a picture Restating problem in own words
2nd	I.D. information Uses a chart, graph Guess and check
3rd	Write an open sentence Find a pattern (numeric) Make a model (visual)
4th	Make a table, diagram, etc. Make a simpler problem Acting out
5th	Working backwards Systematize – Find all possible solutions I.D. subgoal
6th	Finding patterns (operations) Guess and check Make a model
7th	Make tables, graphs, charts Generalize Write an open sentence (equations)
8th	Change your point of view Check for hidden assumptions Working backwards

The Resource Units have taken a strategies approach in that 3-5 strategies have been identified at each grade level for emphasis, and lessons have been designed to teach these strategies. Two large group lessons are provided for the introduction of the strategy. These are followed by a group of 12-20 problems that lend themselves to the use of that strategy. (See Table 2 for a sample.) This is called the Small Group Activities since teachers are encouraged to assign these problems to groups of 3-4 students to practice the strategy. After this section, an

individual set of problems is provided. This set also contains 12-20 problems and provides students the opportunity to use any strategies they have learned previously to solve the problems. (See Table 3 for a sample.)

TABLE 2

Small Group Problems with "Look for a Pattern"

B5 Sally is having a party. The first time the doorbell rings, one guest enters. If on successive rings a group enters that has 2 more persons than the group that entered before it, how many guests will have arrived after the 20th ring?

C6 Mrs. Ricketts gave her son, Tommy, 1 nickel on Monday. On Tuesday, she gave Tommy 5 nickels. On Wednesday, she gave him 14 nickels, and on Thursday she gave him 30 nickels. Mrs. Ricketts told Tommy that if he could figure out the pattern and tell her how many nickels he would get on the 10th day, Tommy could have them all on Friday. How many nickels should Tommy tell his mother he will get on the 10th day?

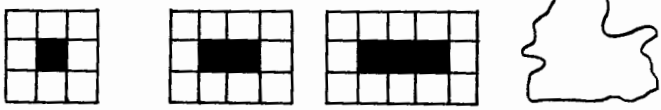
TABLE 3

Individual Practice

9. A square has two possible diagonals; a pentagon has five possible diagonals; and a hexagon has nine possible diagonals. How many diagonals would a 14-sided polygon have?

10. From A to B there are four possible air routes. From B to C there are five possible air routes. From C to D there are three possible air routes. How many different trips can be taken from A to D and back without taking the same route on any section of the return trip?

11.



1 shaded	2 shaded	3 shaded	? shaded
3 white	10 white	12 white	36 white

Teachers are encouraged to assign problems on the basis of the student ability and interest. Throughout the unit, teachers are provided potential questions and discussion points so that the strategies are overtly taught and one strategy may be distinguished from another. (See Table 4.)

TABLE 4
Discussion Suggestions

Discussion:

After students have had the opportunity to work in small groups with some or all of the above problems, call the class together to discuss the strategy of "looking for a pattern." Not only will this help solidify the strategy in long term memory, but it will also help to put the strategy in perspective. Questions such as the following might be helpful:

Q. What is involved in using the strategy "looking for a pattern?"
Q. What did making a table do for us in many problems?
Q. When would this strategy be appropriate and when not?
Q. When would you try this strategy?

At grades 4-8, a comprehensive end-of-the-unit assessment is provided to determine student progress. This assessment is based on an entirely different means of evaluation than that of typical mathematics tests. (See Table 5 for grade 6.) Students are asked under what circumstances a particular strategy might be employed. They are asked to identify conditions of a problem that might indicate a solution path could be found using a particular strategy, without having to solve the problem. Problems that they are asked to solve are awarded point values in descending order of 16-8-4-2 to negate the time pressure factor in completing the evaluation.

TABLE 5

Sixth Grade Assessment

Unit Check-Up

(30 pts.) 1. Give 2 reasons why you would try the following strategies.

a. Make a model

b. Looking for a pattern

c. Guess and check

(42 pts.) 2. What strategy would you <u>try</u> first to solve the following problems? Why did you pick that strategy? (Do not solve the problems.)

a. Fill in the following square so that all the sums (rows, columns, and main diagonals) are the same, using numbers 5-13 to fill in the square.

b. What are the next eight numbers after: 2, 3, 10, 12, 13, 20, 21, 22, 23?

c. Make a sum of exactly 100 by using only 16, 17, 23, 24, 29 and/or 39. (Any or all may be used more than once.)

d. An employee at the box factory ordered sheets that are 4 x 5 from which to make <u>topless</u> boxes. How can you cut one of these sheets to get four boxes? Can you find more that one way?

e. Can you find a way to predict whether any 3-digit number <u>is</u> or <u>is not</u> divisible by q without dividing out first? (For example, 275 is divisible by 11). (HINT: Try finding several first.)

f. If A = 1¢, B = 2¢, C = 3¢, ..., Z = 26¢, what words in the English language have the value of exactly $1.00?

(16-8-4-2 pts.) 3. Find a solution for the following. You do not have to work them in order.

a. Given three interlocked gears A, B and C which are side by side such that when one gear moves they all move. Gear A has 32 teeth, B has 16 teeth and C has 24 teeth. Gear B is in the middle of A and C. If A turns 3 times in a clockwise direction, how many times will C turn? In what direction?

b. When block T's are joined, the perimeter of the new figure is less than the sum of the perimeters of the separated T's. What is the perimeter of 20 block T's joined together?

c. The greatest common factor of two numbers is 30. Their least common multiple is 420. One number is 210. What is the other number?

TABLE 5, continued

d. During a football game, your team scored 18 points. In how many different ways could your team have made this score?

Touchdown (TD) = 6 pts.
Point after touchdown (PAT) = 1 pt.
Field goal (FG) = 3 pts.
Safety (S) = 2 pts.

The other significant part of the program is the inservice component. The teachers participate in 24-30 hours of inservice followed by observations of modeling of strategies by master teachers in their classrooms. (See Table 6.) Ideally, coaching and feedback are provided by other trained teachers at the same building site or at least a support network of trained teachers meet at the building site regularly to solve teaching difficulties and to reinforce successes.

TABLE 6
Inservice Schedule

Session 1:	A. Introduction to problem solving B. Identifying strategies
Session 2:	A. Focus on problem solving with manipulatives B. Teaching behaviors in problem solving sessions
Session 3:	A. Brainstorming B. Problem solving subskills C. Classroom management D. Peer practice teaching of a problem solving episode
Session 4:	A. Analysis of problem solving strategies B. Evaluation models C. Peer practice teaching of a problem solving episode

During the first sessions teachers are introduced to problem solving by solving problems from student materials. Problem solving strategies are brought

out as they are used. Modeling and identification of appropriate teacher behavior during problem solving sessions are key parts of the middle sessions as well as the discussion of classroom management issues. Cooperative groups of three or four are emphasized for use in the small group sessions where students investigate and learn to use the various strategies. Various techniques for using problem solving episodes on an almost daily basis are highlighted, including brainstorming, mathematics headlines, focus on subskills, and strategy practice.

After teachers have had opportunities to practice problem solving episodes in their own classrooms, a final session is focused on evaluation of students' progress in problem solving. Teachers are sensitized to the need to award grades or points on the basis of process goals such as proper use of a strategy rather than just on whether or not the solution is correct. Evaluation models such as Charles and Lester's (1982) are introduced for teacher practice. The end-of-the-unit assessment tests for grades 4-8 are also discussed.

Thus far we have had over 300 teachers go through the inservice program with additional teachers scheduled for the workshops each year. We are trying to build a support base in each local district and each building so that future workshop participants will have someone close to turn to as they try to incorporate problem solving into their classrooms. The program has not been easy to initiate, nor is it perfect. However, it is the kind of effort that needs to be made before change will take place in the mathematics classroom.

REFERENCES

Charles, Randell I., and Lester, Frank K., Jr. Problem Solving: What, Why and How. Palo Alto, CA: Dale Seymour Publications, 1982.

Joyce, Bruce R., and Showers, Beverly. "Improving Inservice Training: The Messages of Research". Educational Leadership, February 1980, 379–385.

"Mathematics Problem Solving Resource Units: Grades 1–8". Stark County Department of Education, 7800 Columbus Road, Louisville, OH, 1983.