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LABORATORY MEASUREMENTS OF ELASTIC CONSTANTS OF CARBONATE ROCKS FROM SOUTHWESTERN OHIO[1]

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Abstract. Improved laboratory techniques were developed for the measurement of seismic velocities, elastic constants, and loss factors in rock cores. The method used was the forced and free vibration of cylindrical cores in longitudinal, torsional, and flexural vibration modes. The results from Ohio sedimentary rock cores selected from the Brassfield limestone, the Cedarville dolomite, and the Columbus limestone were compared to limited field measurements using seismic techniques. This comparison demonstrates the advantages of the laboratory techniques, which can be used to study the effects of saturation, compostion, anisotropy, and grain size on elastic constants.

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The accurate measurement of dynamic elastic constants of rocks is important in many areas of geophysics, geology, and geological engineering. These dynamic measurements depend on setting the rocks into vibration and observing the characteristics of the motion; these vibrations can be studied *in situ* or in the laboratory. *In situ* responses of rock units to earthquakes or explosions can be recorded under natural field conditions of pressure, temperature, faults, and saturation. These *in situ* measurements have the disadvantages, however, of being difficult to interpret because of the usual complexity of the existing rock units and the lack of control of such variables as composition and saturation.

Laboratory measurement of dynamic elastic constants of rocks allows the experimenter to control the parameters of the rock specimens so that their effects on the dynamic responses can be found. This control over rock conditions makes laboratory measurements more versatile in developing and testing theories of rock behavior. For geophysical purposes, the pulse technique of Hughes (1960) has been used extensively. Small rock samples were covered with metal foil jackets, heated, and pressurized with water to simulate conditions deep in the earth. Mechanical impulses were then applied to the samples and their velocities through the samples timed. The apparatus is complex and expensive and because of the pressure of water on the jackets the specimens are not free to vibrate torsionally; therefore the shear wave velocity cannot be measured directly.

For engineering and surface geology studies, it is more important to obtain accurate inexpensive laboratory measurements of the shear modulus and Young's modulus of surface rocks under proposed power plant and dam sites (Richart *et al* 1970). For these purposes the resonant bar technique, first used by Birch (1942), appears more effective. This method used the longitudinal, torsional, or flexural vibration of cylindrical rock cores. The cores were excited into forced vibration and the resonant frequencies and internal friction was measured to give the elastic constants and seismic wave loss factors. The present study reports an improved resonant bar technique which has been used to measure the elastic constants of sedimentary rocks of southwestern Ohio.

MATERIALS AND METHODS

CORE SAMPLE PREPARATION

Core samples were obtained from 4 locations in southwestern Ohio. The Brassfield

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limestone was sampled at the quarry of the Southwestern Portland Cement Company on Ohio Route 235 just east of Interstate Highway 675. The Cedarville dolomite was sampled at two sites—the American Aggregates quarry, off Ohio Route 4, opposite the George Rogers Clark Park southwest of Springfield and at the Trekker quarry in Cedarville. The Columbus limestone was sampled at the Marble Cliff quarry on the Scioto River in Columbus. All of these formations have been described in the literature and stratigraphic descriptions of the Brassfield limestone and the Cedarville dolomite are found in Norris *et al* (1950). A reference by Stout (1941) includes descriptions of the Brassfield, Cedarville formations, and the Columbus limestone.

At each field site large rock samples about 20 cm across were carefully chosen to be free of obvious planes of weakness, cracks, or extensive weathering. These samples were cut with a rock saw to form flat sides parallel and perpendicular to the bedding planes. The samples were inspected and those which appeared to have no defects were clamped onto a drill press and cored with a diamond-set, hollow-core drill. The rock cores produced were 1.335 cm in diameter and 8 to 15 cm long. The cutting was performed slowly with a large supply of cooling water to minimize core damage from heat and vibration. Nevertheless, only a small proportion of holes drilled produced unbroken cores.

Cores were obtained both perpendicular and parallel to the bedding planes from each of the 4 sets of samples. The longest cores were selected and their ends were squared off with a carborundum disk saw used for cutting glass tubing. The cores were dried thoroughly and weighed to ± 1 mg and their diameters and lengths were measured to ± 0.003 cm. An average of 3 or 4 diameter measurements was used. The individual diameter measurements differed at most by ± 0.005 cm. Variation was apparently due to drill wander during cutting or due to the varying hardness of the sample cores over their lengths.

LABORATORY PROCEDURES

Each specimen was supported by 3 pointed bolts about their midpoints for torsional and longitudinal vibration and by two knife edges 0.224 of length from the ends for the flexural vibration. These were the node points for the fundamental modes of vibration in the 3 respective arrangements.

The eddy-current technique of Armstrong and Dickinson (1965) was used for torsional excitation, as shown in figure 1A. Two coils were placed at right angles to the specimen near one end and 2 powerful magnets were placed orthogonal to both the coils and the specimen. When an oscillating voltage with a frequency in the kilohertz range is applied to the coils, a metallic specimen experiences induced eddy currents which cause an oscillating force due to the magnetic field. With non-metallic samples, such as rock cores, thin metal sleeves must be cemented over the sample ends to generate the eddy currents.

The same principle was used for detecting the motion of the specimen by an identical set of coils and magnets at the opposite end. Here the motion of the conductor in the magnetic field produced eddy currents which induced a voltage in the detection coils. In order to minimize spurious signals from the input voltage and from extraneous electrical fields, the pickup coils were mounted at right angles to the driver coils and the pickup end of the sample was enclosed in a metal box. The output of these coils was typically 1–10 millivolts rms at resonance for rock cores. For an aluminum test rod with low internal friction, the output was as high as several hundred millivolts.

The excitation system used for the longitudinal vibration, figure 1B, and for the flexural vibration, figure 1C, was the acoustic force from a small audio loudspeaker placed very near the end or side of the core, respectively. We used air to couple the motion of the speaker to the core. The force was increased by cementing a thin mica disk to the speaker dome and placing the speaker so that the mica disk was parallel to the core and about 1 mm from it. The rest of the speaker cone was covered with a damping layer to prevent unwanted noise. The motions of the cores in both longitudinal and flexural vibration were detected by a small piezoelectric crystal taken from a small microphone. The crystal was placed in light contact with the cores to detect the motion.

The vibration system electronics (shown in figure 2) employed an audio oscillator to generate a drive signal in the frequency range of the specimen resonance and a digital frequency counter to measure the resonant frequency. The input voltage was developed through a potentiometer, audio amplifier, and a switch (to interrupt the input for damping measurements). A step-up transformer was used to attain 75 volts across the torsional drive coils. The output signal from the pickup coils or the crystal was routed through a band-pass filter to remove noise and interference, monitored for waveform on an oscilloscope, and measured by an AC voltmeter. For internal friction measurements, the signal was amplified and fed to a specially designed Q meter (Simpson and Sosin 1977).

The internal friction of rock cores can be measured by the damping observed when the specimens are vibrating. If a specimen is set into vibration by an oscillating force, its response reaches a peak at a certain frequency of vibration called the *resonant frequency* (Thomson 1972). The sharpness of this peak and its amplitude are determined by the internal friction of the specimen.

The ratio of this peak response to the zero-frequency response with the same force is called the quality factor Q. The quality factor is usually measured by setting the specimen into vibration and then observing the rate at which the successive peak amplitudes decrease after the force is removed. A second measure of internal friction is the damping ratio ζ. This is the ratio between the actual damping in the specimen and the critical damping, which is the amount of damping that would just prevent

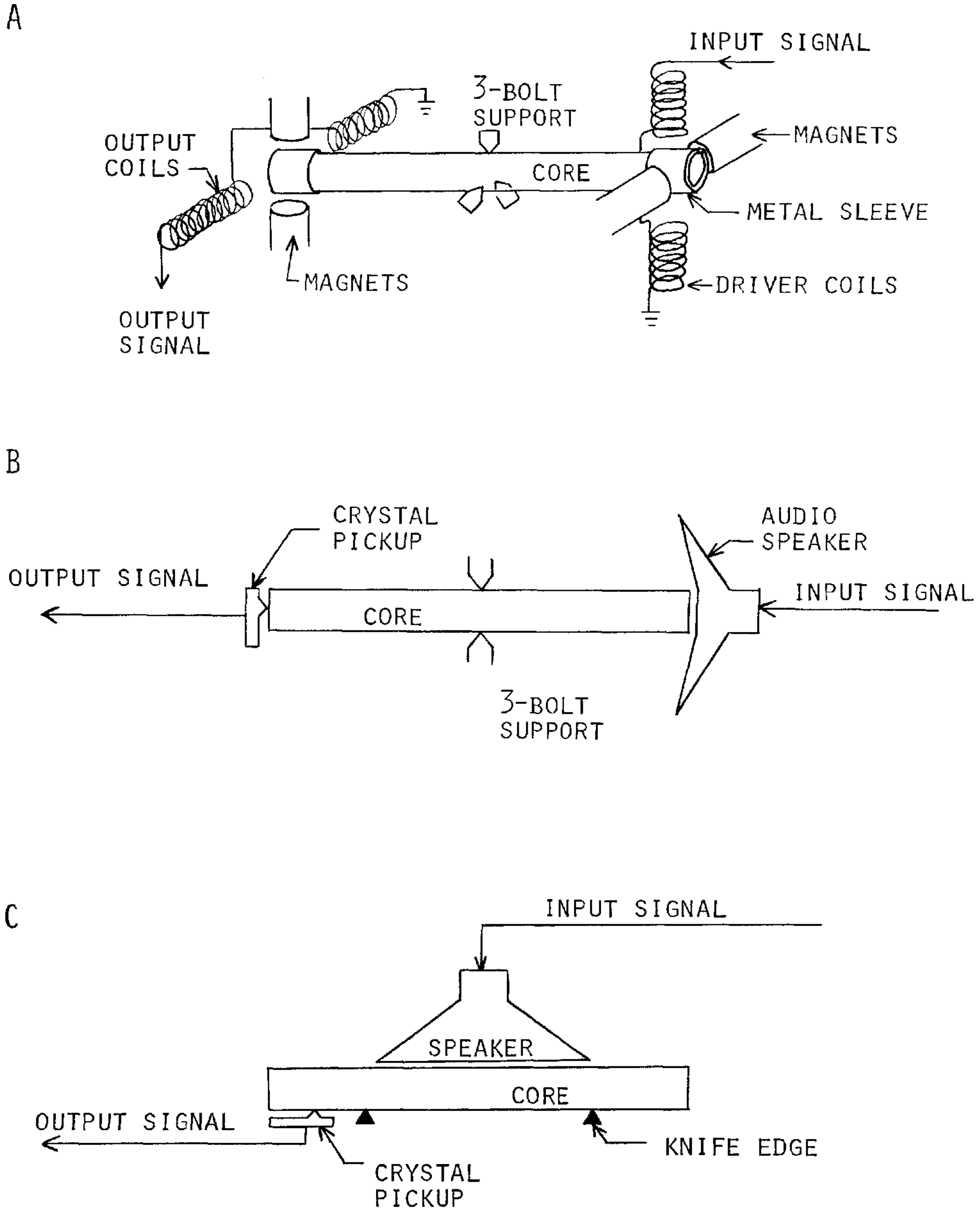

FIGURE 1. Laboratory vibration measurement apparatus. A) torsional vibration set-up; B) longitudinal vibration; C) flexural vibration.

any oscillatory motion. The damping ratio and the quality factor are related by the expression $2\zeta = 1/Q$. A complete discussion of damping in vibrating systems is given in Thomson (1972).

The apparatus was first used to measure the elastic constants of metal samples for which handbook data were available. An aluminum test rod, which has very low internal friction, was used to determine the lower limits of the apparatus for measuring damping at atmospheric pressure. The highest value of Q for the aluminum rod was 53,400 in torsion. The damping ratio ζ from air damping and support system damping in the torsional apparatus is thus no more than 9.36×10^{-6} for a sample mass of 43.4 g. The damping measured in the longitudinal and flexural modes was much higher.

Since the material damping of the aluminum in these modes is theoretically expected to be commensurate with the damping in the torsional mode, it is concluded that these values represent purely air and apparatus damping. These two sources gave total damping ratios of $\zeta = 3.25 \times 10^{-4}$ for a Q of 1539 and $\zeta = 4.33 \times 10^{-4}$ for a Q fof 1156 for the 43.4-gram aluminum sample in longitudinal and flexural vibration, respectively. For core damping measurements, this apparatus damping was subtracted from the observed damping to give the actual rock material damping.

The damping contributed by the crystal pickup alone was evaluated by using it as the sensor for torsional vibration of the aluminum rod in place of the eddy-current coils. With the crystal on the end of the sample near one edge, the damping ratio was 5.37×10^{-4} for the 43.4-g sample. Within the accuracy of these measurements one may conclude that the damping measured in the longitudinal and flexural modes for the aluminum rod represent chiefly the crystal damping. Additional details of the apparatus are described in Pearson (1977). The torsional system represents the state of the art for measuring shear modulus; the longitudinal and flexural vibration systems represent more conventional methods for measuring Young's modulus.

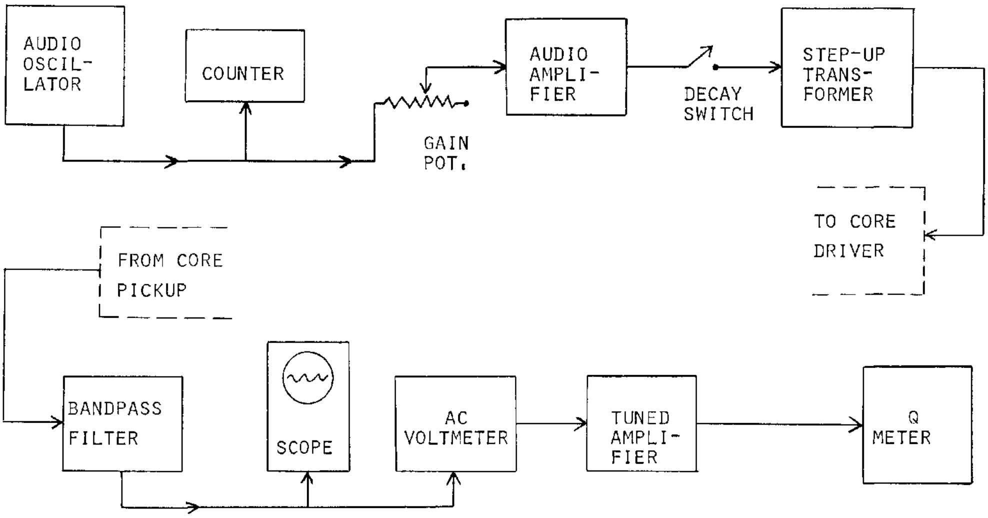

FIGURE 2. Block diagram of vibration system electronics.

RESULTS

Twenty rock cores were measured, with at least 2 cores each in the horizontal and vertical directions from every field site. Each sample was measured in torsional vibration with 4 different sets of thin aluminum sleeves 1.25 cm long cemented over each end to provide the eddy currents. Following Armstrong (1971), the frequencies measured with each set of sleeves were plotted versus sleeve mass and extrapolated to zero mass. This procedure eliminated the systematic error in frequency caused by the added mass of the sleeves and also by any added stiffness proportional to the mass.

The results are summarized in table 1, where each of the 4 sites is represented by horizontal (H) cores, drilled parallel to the bedding planes, and by vertical (V) cores, drilled perpendicular to the bedding planes. The densities (ρ) of these carbonate rocks were all between 2400 and 2700 kg/m^3. The value of shear modulus (G) was measured from torsional vibration:

$$(1) \qquad G = 4L^2\rho f_T^2$$

where L is the core length, ρ is the density, and f_T is the lowest torsional resonant frequency. Note that this equation is not a function of the core diameter.

The average value of Young's modulus (E) measured in the longitudinal and flexural vibration is:

$$(2) \qquad E = \frac{4L^2\rho}{U^2} f_L^2$$

for longitudinal motion, where f_L is the lowest longitudinal natural frequency of the core, and U is a correction factor,

TABLE 1

Laboratory Values of Elastic Constants for Some Carbonate Rocks of Southwestern Ohio.

Measurement	Brassfield limestone		Columbus limestone		Cedarville dolomite (Springfield)		Cedarville dolomite (Cedarville)	
	H(3)*	V(3)+	H(3)	V(2)	H(2)	V(2)	H(2)	V(3)
ρ, kg/m³	2687±3	2651±46	2500±66	2428±166	2529±4	2595±22	2554±59	2436±36
G, 10^{10} N/m²	2.71±.07	2.19±.35	2.12±.03	1.72±.12	2.42±.32	2.08±?	2.69±.19	1.78±.11
E, 10^{10} N/m²	6.78±.29	5.02±.93	5.03±.33	4.18±.12	5.25±.13	5.07±.75	7.12±.26	4.39±1.08
σ	0.26	0.15	0.22	0.19	0.09	0.22	0.34	0.23
Q_T	596±161	259±41	332±64	249±41	272±1	199±?	179±?	149±27
Q_L	464±42	187±45	218±62	203±90	178±28	185±91	48.0±?	58.5±?
Q_F	427±102	210±53	128±?	168±50	311±?	—	302±?	137±?

*H(3) indicates measurment of three horizontal cores, parallel to the bedding plane.

+V(3) indicates measurement of three vertical cores, perpendicular to the bedding plane.

? indicates only one sample was measured; standard deviation could not be calculated.

which takes into account the change in volume of the rod as it deforms:

$$U = 1 - \left(\frac{\pi\sigma d}{4L}\right)^2 \qquad (3)$$

where σ is Poisson's ratio and d is the rod diameter. The Rayleigh correction (U) is small for cores that are as long as the ones used in this study. Equations (1), (2) and (3) are all discussed in Schreiber *et al* (1973). The value of E can also be determined from the frequency of flexual vibrations (Spinner *et al* 1960):

$$E = 1.261886\left(\frac{L^2 f_F}{d}\right)^2 \rho T \qquad (4)$$

where f_F is the lowest flexural frequency and T is a tabulated correction factor which is a function of the thickness of the rod, d/L, and Poisson's ratio. The factor T was evaluated by Tefft and his values are reproduced in Schreiber *et al* (1973).

An iterative technique was necessary because the calculation is dependent on Poisson's ratio. An assumed value of $\sigma = 0.25$ was used to calculate E; from this value Poisson's ratio was calculated by the relation:

$$\sigma = \frac{E}{2G} - 1 \qquad (5)$$

The value of E was then re-calculated, using the new value of σ, until it converged; usually 1 or 2 iterations sufficed. Because equation (5) is strictly true only for a homogeneous, isotropic material, the values of Poisson's ratio tabulated in table 1 are subject to some uncertainty. All of the rocks tested were distinctly aniosotropic. Fortunately, the value of E is not strongly affected by the value of σ over the normal observed range.

Table 1 shows the observed values of Q, or quality factor, for the cores in torsional, longitudinal, and flexural vibration. These values have been corrected for the apparatus damping. The quality factor is related to the damping ratio by the relation $Q = 1/2\zeta$.

These samples show a large variability in measured elastic constants, which is due to variations in the rock material itself. The rocks are not isotropic; the horizontal cores are more homogeneous and competent than the vertical cores. The vertical cores show a greater variation in both G and E, which is apparently due to variations across thin beds. The horizontal cores also show generally higher values of Q, indicating less damping across grain boundaries. These differences between the horizontal and vertical cores show that horizontally travelling seismic waves should show greater velocity and less attenuation then vertically travelling waves. There are distinct differences in the elastic constants and the degree of anisotropy of the Cedarville dolomite between 2 locations 20 km apart.

It should be noted that the cores represent only the most competent portions of the parent strata. Any layers with cracks, planes of weakness, incomplete fossils, or large voids would not produce unbroken cores and are thus underrepresented in these samples. The seismic velocities computed from these measured elastic constants are seen to be higher than the measured field values, indicating little or no damage to the cores during drilling. The differences between laboratory and field velocities are therefore a measure of the imperfections in the field strata.

FIELD MEASUREMENTS

A limited field investigation was performed to validate the laboratory results. In normal field seismic work, only the pressure-wave velocity is measured; the slower shear wave is more difficult to detect. In the application of seismic design criteria to dams, power plants, and buildings in earthquake zones, however, the shear wave velocity is perhaps the most important characteristic of the bedrock (Stokoe and Woods 1972). For foundations, the most important vibration is the Rayleigh wave, whose velocity is strongly dependent on the shear wave velocity (Richart *et al* 1970). The addition of pore fluid between grain boundaries in rocks can raise the pressure-wave velocity while the bearing capacity of the rocks is actually decreased. The shear wave velocity is not affected by pore pressure. Because of the importance of the shear wave velocity, it was decided to attempt the measurement of shear waves in the field.

The field measurements were taken with a portable 12-channel Geospace seismic unit and 4 groups of geophones oriented vertically, longitudinally, and transversely. A truck-mounted auger was used to drill holes 0.5 to 2.5 meters deep; 0.25 to 1.0 kilogram of 40% dynamite was placed at the bottom and tamped with soil and water. The 4 groups of phones were spaced at nominal 16-meter intervals, with the closest varying from 13 to 47 meters from the shot.

The field investigation demonstrated the difficulty of making shear wave measurements. In the one shot at the

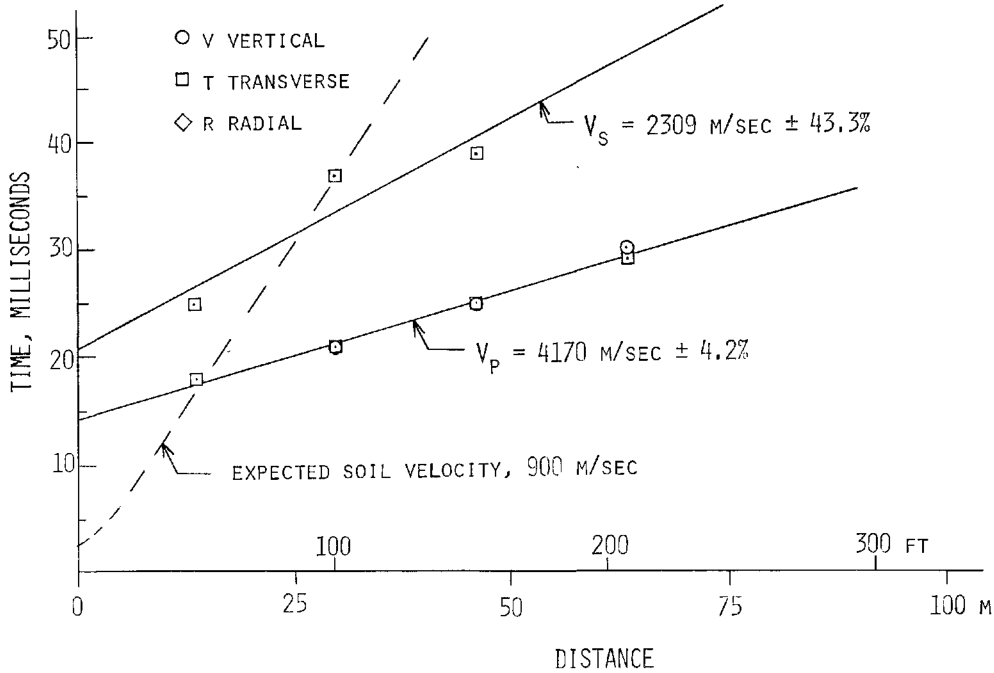

FIGURE 3. Cedarville dolomite (Springfield) first and second arrivals from two seismic shots. The arrival time of the shot energy is plotted against the distance from the geophone. The slope of the least squares straight line through these points is the seismic velocity.

Cedarville dolomite (at Cedarville) no second arrivals could be definitely identified. The Columbus limestone was not measured. The remaining sites, the Brassfield limestone and the Cedarville dolomite (at Springfield), showed some second arrivals which could be identified as shear waves.

An example of the results from the Cedarville dolomite at Springfield is shown in figure 3. The arrival time of the shot energy is plotted against the distance of the geophone from the shot; the slope of the least-squares-fit straight line through these points is the seismic velocity. Because of the difficulty in determining the response of the geophones to a shear wave shortly after they have responded to a pressure wave, the measured shear wave velocities are subject to a large uncertainty.

A summary of the laboratory measurements compared to the field measurements is shown in table 2. No field measurements were made for the Columbus limestone, and the shear wave velocity for the Cedarville dolomite at Cedarville could not be obtained. In most cases the laboratory values are higher than the field values. This was expected, due to the great difficulty in obtaining intact cores and the fact that only the most competent strata are represented by the cores. The overall agreement, however, is remarkably good. Typically, laboratory and field measurements may differ by a factor of 2 (Goldsmith and Austin 1964).

TABLE 2

Comparison of Laboratory and Field Values of Seismic Velocities.

Location	Measurement	Velocity (m/sec) Lab	Velocity (m/sec) Field	Difference
Brassfield limestone	V_P	4999	4808	+ 3.8%
	V_S	3022	3048	− 0.9%
Columbus limestone	V_P	5488	*	*
	V_S	2784	*	*
Cedarville dolomite	V_P	4967	4170	+16.0%
(Springfield)	V_S	2962	2309	+22.0%
Cedarville dolomite	V_P	5557	4900	+11.8%
(Cedarville)	V_S	2974	—	—

*Not attempted.
—Not obtained.

CONCLUSIONS AND RECOMMENDATIONS

Improved laboratory methods have been developed to measure directly and accurately the important elastic constants of rock cores. These methods provide simple and direct means for controlling the conditions of the measurements and are particularly effective for obtaining the shear modulus. By contrast, the field measurement of shear modulus has been found by many investigators to be a very difficult task.

The techniques described in this report can be applied to any competent rock which can be cored successfully. They are recommended for studying the effects of saturation, composition, anisotropy, and grain structure on elastic constants. For typical rock materials with Q less than 1000, there is no need to perform the measurements in a vacuum.

The results of this study also indicate the great variability of elastic constants in many formations over short distances. This variability should lend caution to the application of measured data from cores or from seismic work even a short distance away from the point of interest.

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