The Number Line Model for Conceptual Understanding of Fractions

Michele Heron, Kent State University at Stark <mheron@kent.edu>

Abstract

This paper outlines misconceptions related to the number line model for describing fractions. Examples of student work from a local research project are provided to illustrate these misconceptions. Instructional strategies for representations and academic language to be used when working with the number line model are described.

The Common Core Standards have been accepted as the updated mathematics standards in Ohio and beginning the school year of 2014-2015, the state tests will reflect the Common Core Standards rather than the Ohio Academic Standards. Common Core Standards have rearranged mathematics content; some content such as probability is introduced much later and other content such as fractions on a number line are introduced earlier. In addition to the content, eight Mathematical Practices are included. The Mathematical Practices emphasize concepts such as rigor, problem solving and modeling to encourage conceptual understanding of topics. Teachers have been making adjustments to their curriculum in preparation to implement the new standards. The purpose of this paper is to provide a better understanding of what research shows when teaching fractions on a number line. This concept was originally placed in the fourth grade in the Ohio Academic Standards; however, the National Council of Teachers of Mathematics *Principles and Standards* has stated that third graders should focus on representations of part to whole using models including a number line (NCTM, 2000), and now the Common Core Standards place this model in the third grade as well. Because there are several misconceptions surrounding this model, a goal of this article is to clarify these misconceptions and provide instructional strategies for teachers.

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Susan Lamon (2007) states " …fractions, ratios, and proportions arguably hold the distinction of being the…most difficult to teach, the most mathematically complex, the most cognitively challenging, the most essential to success in higher mathematics and science, and one of the most compelling research sites (p. 629)." Challenges associated with the number line model are extensive and should be understood prior to instruction. This article highlights what researchers have already suggested about the number line model as well as data from a local research project to suggest instructional strategies and organizational strategies when planning a unit that uses the number line model to teach fractions. Each of the challenges will require instruction that is flexible in types of questions and forms of assessment.

The Number Line Model and the Area Model Comparison

Typically, an area model such as circles, bars, or rectangles is used in the third grade to build understanding of the part to whole representation, the idea of fair share or equally proportioned pieces, and the idea of equivalence to show that 2/4 and 1/2 cover the same area of the model. The area model provides a strong visual model that is contextually similar to students' experiences with their own lives. Sharing a pizza fairly or sharing a candy bar fairly can be represented using the circle or rectangle model. The area model has weaknesses as well as strengths, including the difficulty of representing fractions such as 1/5 or 1/9 (Barnett-Clarke, Fisher, Marks, & Ross, 2010). The Common Core Standards do not require these fractions in third grade, however, subsequent grade levels will use these models and these fractions.

Challenge Number 1: The Whole

The first challenge is related to recognizing the whole or the unit or 1 with the number line model (Bright, Behr, Post & Wachsmuth, 1988). The transition to the number line model will require several conceptual shifts for students to make. The first concept shift to consider is what determines the whole. The area model makes the whole visible. Area models begin with the use of a whole circle, a length of a bar, or a rectangle to represent the whole. Eventually area models progress to include half a circle as the whole or multiple circles as the whole. The whole is then sliced into equal pieces to indicate the parts. The area model offers a simple visual example of the whole and fair share is contextually understandable because students share pizza or pie making the determination of equivalence visual (Barnett-Clarke, Fisher, Marks, & Ross, 2010). As a result, the whole is translated as the object. Number lines often contain more than 1 whole, however, students will use prior knowledge about the whole object and interpret the number line shown as the whole. Figure 1 illustrates how students will treat the object as the whole. In this case, the student interprets the distance from 0 to 5 as the whole rather than recognizing the number line has 5 wholes. This interpretation seems natural given that most models up to this point have been an image of something that counts as 1. A whole circle or a whole rectangle are shown and then partitioned. So when students begin using the number line model the transition is natural to count the entire number line as the unit. The following example in Figure 1 shows a sample of how students initially interpreted the whole.

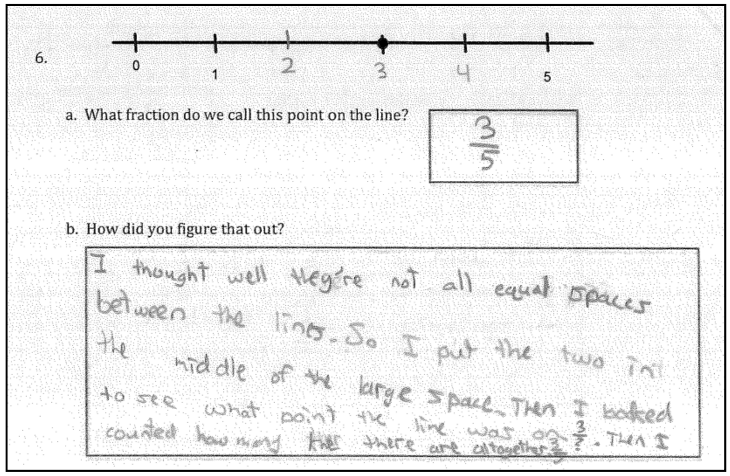

Figure 1

The student explanation shows that the entire number line is considered the whole where the number line is cut into 5 equal pieces and the point represents 3 of those pieces. During the study, several modifications to lessons were introduced including notation or how the whole was represented. One modification was the use of the language "from here to there" using hand motion to show where a whole would start and end. For example, a number line labeled from 0 to 3 has the potential of showing 3 different wholes. Students were asked to show a whole using this body language. So students could start at 1 and end at 2 or start at 0 and end at 1 or start at 2 and end at 3. Each of these motions made the idea of the whole visible while also highlighting that the entire number line was not necessarily the whole. Additionally, on paper students would illustrate the whole by circling that portion of the number line. These two illustrations supported the transition from the area model to the number line model through action and through visual support. Figure 2 shows an example of a student using a circle to indicate the whole. The initial problems posed to students on paper included the problem as well as space to describe how they determined the solution. These explanations were helpful in determining the type of misconception held by the student. Occasionally, the answer would be correct but the correct solution was based on incorrect reasoning. The types of misconceptions were tracked so lessons could be focused where the group had the greatest need.

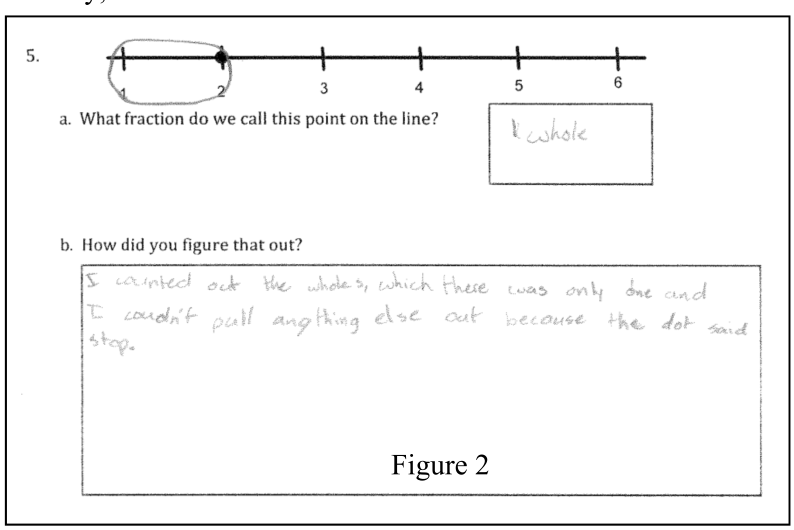

Figure 2

The explanation shows another level of language used with the class to illustrate the idea of walking from the left to the right and stopping at the point. The connection to walking and distance supported later language when improper fractions were introduced. This idea will be elaborated in a later section.

Challenge Number 2: Tick Marks

The next challenge for students relates to the tick marks used to show parts of the whole. Students tend to count the tick marks rather than the space between each tick mark (Bright, Behr, Post & Wachsmuth, 1988). The fraction bars used as an area model can support the transition between the two models. One of the activities used with the fourth graders early in lesson planning was creating a fraction bar by folding a long strip of paper. When the strip is folded in half, students are asked to draw the line where the fold is located then describe what the two parts would be named. This is visually similar to the fraction bar model but with the addition of drawing the 'tick mark' or the division for the two halves. Eventually, the students recognize that the lines or the folds are not what we focus on when naming the portions of the fraction bar. This is then translated to a line drawn on a board where students are asked to cut the line into two halves then another line into thirds. The focus is then placed on the sections of the line and not on the tick marks.

The language used while working with the number line model focus on reading the number line from left to right (in the same way that we read a book). The spaces between the tick marks are referenced as hops, so questions like 'how many hops to get across each whole' are asked to help students see the spaces as distances. This is helpful for games used later. Eventually, there will be a game where students will think of the whole as a large step and the parts of a whole as hops. So beginning the language with students using the step and hop will carry through the whole unit. The challenge is illustrated in several ways in the given examples. The example in Figure 3 shows how students will think of the tick mark as the object to count and not reference the spaces that should be counted.

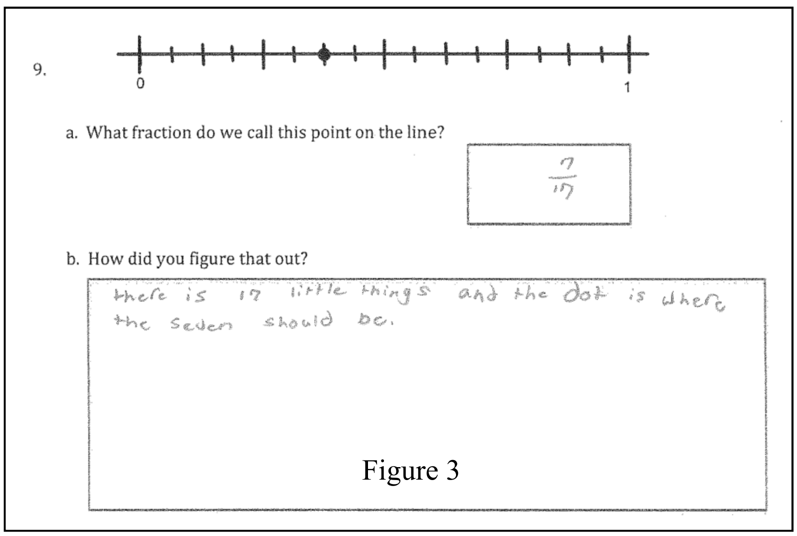

Figure 3

The explanation shows that the student counts the tick marks from left to right naming the point as the seventh tick mark out of seventeen tick marks. Another misconception is the combination of the tick mark distraction along with not reading the number line from left to right. The example in Figure 4 illustrates this misconception.

This student sees a ratio between the point and the number of tick marks instead of a point with a given distance from zero. The strategy to have students talk about moving to the point using the number of hops from zero illustrates a distance and direction rather than just the tick marks. A bulletin board with a number line with moveable tick marks was used daily. For example, one day the bulletin board will show a number line from 0 to 1 with tick marks to show fourths and a point on ¾. Students were asked to determine the number of hops from zero to get all the way across to 1. Using the idea of a hop helped students see the spaces they were jumping over to get to each of the tick marks. In this example, students would have to hop 4 times to get from 0

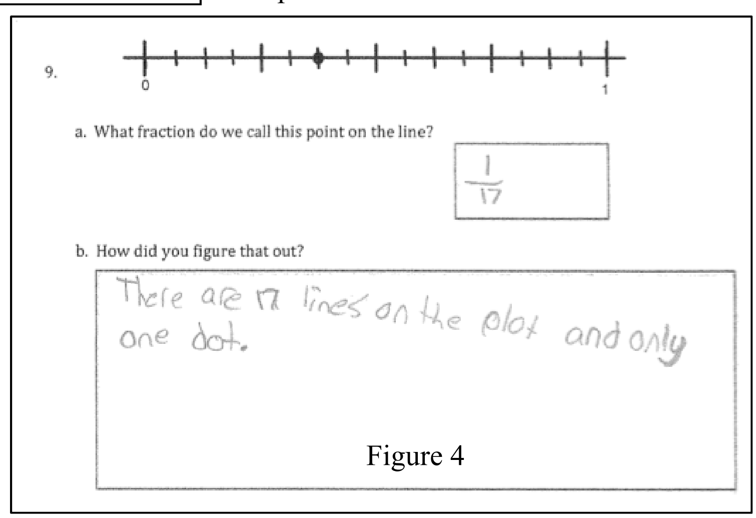

Figure 4

to 1. This provides the denominator. Then students determine the number of hops to get to the point at ¾ from 0. The 3 hops provide the numerator. Each student answered the question individually to provide formative assessment information and then one student would be asked to explain the solution using the language of hops. After students became comfortable with the language and the process, questions were also asked in reverse. Students were given the name of the fraction then asked to place the appropriate number of tick marks and the point in the correct position. These questions helped with the discussion about the distance between tick marks. Should these be the same distance? These questions supported student understanding of reading left to right, equal partitioning of the tick marks, and focusing on the spaces rather than the tick marks. This activity supports the third grade Common Core indicator that states students should understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts (CCSSI, 2010).

Challenge 3: Proportional Thinking

The next challenge is that students have difficulty with proportional thinking when using the number line model (Saxe et al., 2007). When using the area model, the idea of fair share or equal parts is less challenging because students have experience with objects like pizzas or pie where slicing the pizza into the same size pieces is expected and fair. The whole is typically the circle itself making the partitioning process seem natural. Because students already struggle with the idea of the whole, partitioning is also a challenge. Young students can understand the expectation of equal size pieces of a pizza using the circle model because they have a sense of fairness, however, fair share does not translate easily using the number line model. For this reason, the use of the term hop to show the spaces between tick marks requires that all participants use the same size hop every time. Figure 5 shows an example of how students can recognize that spaces are to be counted but do not understand that the distance for each space should be the same.

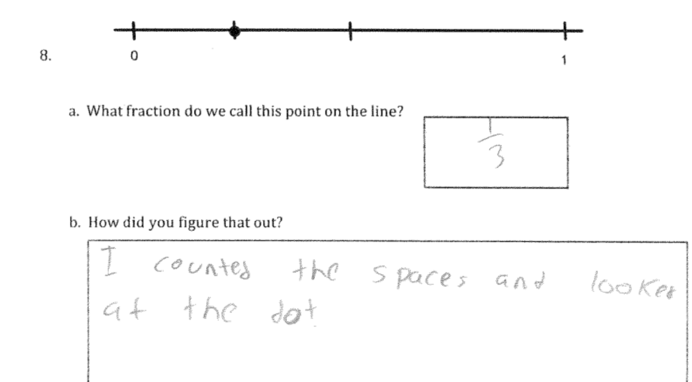

Figure 5

To support student understanding of this challenge, symbolic representations were introduced along with the language of hopping the same distance. The representation for the whole was the circle, the representation for the proportionally sized pieces of a whole were half loops. Students were expected to determine the number of hops all the way across the whole and also the number of hops to reach the point to determine the fraction. An example of the loops is shown in Figure 6. Some students count all the way across along the top of the number line and others along the bottom. The suggestion would be to encourage the loops that count all the way across to be on the bottom of the number line to match with the fraction notation.

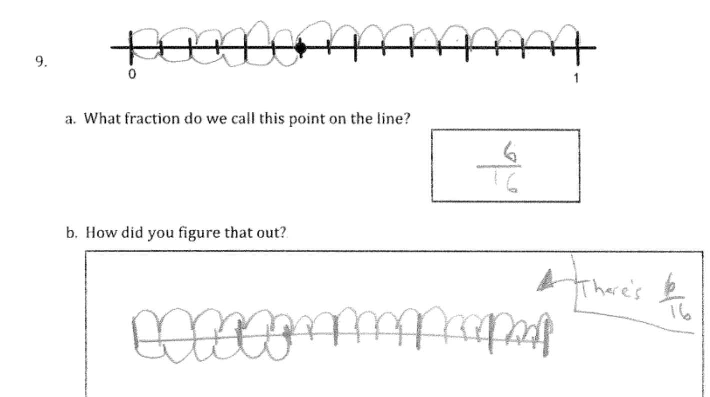

Figure 6

Conclusion

The three challenges were described to help support teachers plan for instruction and assessment when using the number line model for fraction understanding. The primary challenges relate to tick mark distractions, understanding the whole, and proportional understanding. Using consistent language supports and symbolic supports during the instruction will support student understanding and provide consistency for discussion and expectation. The concepts are not easily understood by students so continued support for a long period of time is suggested through elements such as problems of the day or the number line bulletin board. Encourage students to demonstrate their understanding with movement or words. Include a variety of question types to determine where misconceptions occur and what supports may be necessary for your students.

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Dr. Shelly Heron is an Assistant Professor at Kent State University at Stark. Her areas of interest include sociomathematical norms and self-regulation. She has presented several workshops at local, state, and national conferences.

Food for Thought

"The idea that knowledge or belief or learning *has* to be decomposable into sentence-sized gobbets is probably an illusion ..."

Dennett, D. C. (2013). *Intuition pumps and other tools for thinking*, 153. New York: W.W. Norton & Company.