Reports of the Department of Geodetic Science

Report No. 103

ANALYTICAL BLOCK TRIANGULATION WITH FICTITIOUS DATA

by

Sanjib K. Ghosh
John F. Kenefick
Arthur S. Brown

Final Report on Research
Conducted by

The International Society of Photogrammetry
Commission III
Group Study for Experimental Researches
on
Analytical Block Triangulation

The Ohio State University
Department of Geodetic Science
Columbus, Ohio 43210

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FOREWORD

This report was prepared by Dr. Sanjib K. Ghosh, Associate Professor and Research Supervisor and two of his Associates: Mr. John F. Kenefick and Captain Arthur S. Brown, U.S. Army.

The research was performed under the sponsorship of the International Society of Photogrammetry, Commission III (Mr. Charles M. Theurer, Chairman of the Working Group).

The study is presented in two sections as follows:

Report No. 1: A study on the use of 60% sidelap as compared to 20% with or without additional control in the center of the block.

Report No. 2: A study on the use of horizon data and the problem of weighting in the adjustment phase of the block triangulation.

The system of coordinates used in this study was a right-handed rectangular one, where Y axis coincides with the direction of the strips and Z (elevations) are positive upwards from the reference datum plane. The rotations are: ω around X, φ around Y and κ around Z.

The principal investigator for Report No. 1 was Mr. John F. Kenefick, Graduate Research Assistant; and for Report No. 2 was Captain Arthur S. Brown, U. S. Army, Graduate Research Assistant. The primary and preliminary reporting are credited to these two researchers who worked under the advice and guidance of Dr. Sanjib K. Ghosh, Associate Professor.

For a more thorough description on the research beyond what is presented in this report, the reader is referred to the two Master of Science Theses submitted by Kenefick and Brown in March 1968 at The Ohio State University, Department of Geodetic Science.

The investigations were supported by a grant-in-aid from The Ohio State University, College of Arts and Sciences. Data for the investigations were supplied by Commission III of the International Society of Photogrammetry as part of the International Study. Computer services were rendered free of charge by The Ohio State University Numerical Computation Laboratory. The Department of Geodetic Science, The Ohio State University (Dr. U. A. Uotila, Chairman) provided the printing costs.

Thanks are extended to all the above organizations and individuals for their support and assistance.

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Center of Work

Department of Geodetic Science
The Ohio State University
Columbus, Ohio 43210 U.S.A.

Report No. 1

on

Analytical Block Triangulation

1. Material and Equipment

IBM readouts containing all information on the fictitious data generated for this experimental research were obtained from Mr. Charles M Theurer, Chairman of this working group. Computations and data analyses were performed at the facilities of the Ohio State University, Department of Geodetic Science, and the OSU Computer Center (IBM 7094).

2. Personnel

The adjustment and programming were primarily performed by Mr. John F. Kenefick, Research Assistant under the advice of Dr. S. K. Ghosh. Mr. Fletcher W. Twitty assisted in the cartographic representations. Mrs. Vera N. Hoff assisted in typing. Dr. Sanjib K. Ghosh supervised the entire project.

3. Characteristics of the Block Adjustment

The use of 60% sidelap as compared to 20%, or nearabout, between strips has been suggested expecting that this would contribute to a better solution in Block Triangulation. This is expected not only from the geometrical point of view (to provide better tie), but also from the point of view of obtaining more independent observations for the control (pass) points (thus increasing the number of redundant observation equations that will contribute to the final set of normal equations).

The aim of this research was to determine whether 60% sidelap photography provides a better plan and elevation block adjustment than 20% sidelap photography and, if so, to what extent. With this objective, various combinations of strips to form various sub-blocks were studied with the data obtained from the principal block, consisting of nine strips of twenty photographs each. This block of fictitious data was obtained from Commission III of the International Society of Photogrammetry. The data used in the tests will be described for the specific cases.

The photographs are near vertical and have a focal length of 152.00 mm with a format of 230 x 230 mm. An average flying height of 11,000 m above the terrain results in a photo scale of 1:66,000 (approximately).

The pictures (images) are arranged in a 3 x 3 block pattern. Image coordinates are perturbed from their theoretically correct positions with random errors of $\pm$ 6 microns in x and y and have systematic errors with a maximum value of 20 microns.

Errorless local geocentric coordinates are given for exposure stations and ground points. The exposure stations deviate from straight lines of flight, randomly, by $\pm$ 100 m in X and Y and $\pm$ 50 m in Z. Systematic deviations are as high as 200 m in X and Y and 100 m in Z. Elevations in the terrain vary, randomly, by $\pm$ 300 m.

4. Mathematical Basis

The well known equations giving the condition of collinearity were used in these investigations as follows:

$$x = -f\frac{(X-X_o)a_{11} + (Y-Y_o)a_{12} + (Z-Z_o)a_{13}}{(X-X_o)a_{31} + (Y-Y_o)a_{32} + (Z-Z_o)a_{33}}$$

$$y = -f\frac{(X-X_o)a_{21} + (Y-Y_o)a_{22} + (Z-Z_o)a_{23}}{(X-X_o)a_{31} + (Y-Y_o)a_{32} + (Z-Z_o)a_{33}} \qquad (1)$$

Where,

- x, y are image coordinates of a point with respect to the principal point as origin in each picture.
- f is the calibrated focal length of the camera.
- X,Y,Z are the ground coordinates of the imaged point.
- X_o,Y_o,Z_o are the ground coordinates of the projection center at the instant of exposure.
- $a_{11}\ldots a_{33}$ are the elements of a sequential rotation matrix, involving rotations ω, φ and κ.
- x and y are observed quantities which must be corrected for known errors due to lens distortion, film shrinkage, refraction, etc.

The equations (1) are partially differentiated with respect to the unknown parameters. The resulting linear equations (observation equations) are shown in their general form for picture i and point j:

$$\begin{aligned} v_{x_{ij}} = (&p_{11_{ij}} + p_{12_{ij}} d\omega_i + p_{13_{ij}} d\varphi_i + p_{14_{ij}} d\kappa_i \\ &- p_{15_{ij}} dX_{o_i} - p_{16_{ij}} dY_{o_i} - p_{17_{ij}} dZ_{o_i} \\ &+ p_{15_{ij}} dX_j + p_{16_{ij}} dY_j + p_{17_{ij}} dZ_j)/(A_3B)_{ij} \end{aligned} \qquad (2)$$

$$v_{y_{ij}} = (p_{21_{ij}} + p_{22_{ij}} d\omega_i + p_{23_{ij}} d\varphi_i + p_{24_{ij}} d\kappa_i$$
$$- p_{25_{ij}} dx_{o_i} - p_{26_{ij}} dy_{o_i} - p_{27_{ij}} dZ_{o_i}$$
$$+ p_{25_{ij}} dX_j + p_{26_{ij}} dY_j + p_{27_{ij}} dZ_j)/(A_3B)_{ij} \qquad (2) \text{ cont'd.}$$

Complete derivations and explanations of these expressions (1) and (2) are given in [1].

Expressions (2) are written for each image point. These are normalized so as to obtain a system of n equations with n unknowns. This process satisfies the condition of "least squares."

Finally, the system may be expressed in the following form:

$$[N] \cdot [X] + [U] = 0 \qquad (3)$$

where,

[N] is the coefficient matrix of the normal equations

[X] is the solution vector of the normal equations

[U] is the constant vector of the normal equations.

Equation (3) is solved. The solution vector [X] resulting from the inversion contains the corrections to be applied to the previously computed approximate values of the unknown parameters.

5. Block Adjustment Procedures

Based on the equations presented earlier a program was compiled to form the observation equations for each image point observed and then the contributions to the normal equations were computed.

In the tests, the "approximate" values of the unknowns used were (a) the ground and exposure station coordinates taken directly from the ISP data and (b) the rotation angles ω, φ and κ were computed by a space resection program compiled at the Ohic State University laboratory.

Ground points to be held as control were weighted by adding 1 to the appropriate diagonal elements of the normal equations so that very small corrections would be applied to these points during the adjustment.

No other weighting was included because fictitious data were used and weighting has no sense there.

The test was performed in the following cases:

Case 1A:

Adjustment of two strips of five photographs each, having 20% lateral overlap.

Case 1B:

Similar to Case 1A, but with the inclusion of the intermediate strip so that 60% sidelap is obtained.

Case 2A:

Identical with Case 1A; a different block.

Case 2B:

Identical with Case 1B; same block as 2A.

Case 3A:

Adjustment of three strips of five photographs each, with 20% lateral overlap.

Case 3B:

The same block as 3A, with the addition of the intermediate strips so that 60% sidelap is obtained and five strips are considered.

Note:

In each of the above six cases the four corner points of the block were held as plan and elevation control.

Case 3C:

Same as 3A except that a fifth point near the center of the block is held as an **additional** vertical control.

Case 4A:

Adjustment of four strips of seven photographs each, with 20% lateral overlap.

Case 4B:

The same block as 4A, with the addition of the intermediate strips so that 60% sidelap is obtained and seven strips are considered.

In both Cases 4A and 4B the four corner points of the block were held as control.

Case 4C:

Same as for 4A except that a fifth point near the center of the block is held as an additional control (both plan and elevation).

All the tests are summarized in Table I, and represented in the accompanying figures. (Appendix.)

TABLE I

PERTINENT INFORMATION ON ALL CASES

Pertinent Data	Case 1A 20% sidelap	Case 1B 60% sidelap	Case 2A 20% sidelap	Case 2B 60% sidelap	Case 3A 20% sidelap	Case 3B 60% sidelap	Case 3C 20% sidelap	Case 4A 20% sidelap	Case 4B 60% sidelap	Case 4C 20% sidelap
No. strips	2	3			3	5	same as	4	7	same as
Photos/strip	5	5			5	5	Case 3A	7	7	Case 4A
Total photos	10	15	same as	same as	15	25		28	49	
Control pts.	4	4	Case 1A	Case 1B	4	4	5	4	4	5
Check pts.	21	21	diff. area	diff. area	27	27	26	59	59	58
Obs. eqn's.	156	234			218	374		456	798	
Unknowns	135	165			183	243		357	483	
Redundant eqn's.	21	69			35	131		99	315	
σ_X in m & (max)±	0.6 (1.2)	0.6 (1.1)	0.9 (1.5)	0.9 (1.5)	0.7 (1.7)	0.6 (1.2)	0.6 (1.2)	0.9 (2.3)	0.8 (1.9)	0.9 (2.3)
σ_Y in m & (max)±	0.7 (1.4)	0.7 (1.4)	0.6 (1.6)	0.6 (1.5)	0.8 (1.8)	0.7 (1.8)	0.8 (1.6)	0.8 (2.4)	1.0 (2.2)	0.7 (2.0)
σ_Z in m & (max)±	0.6 (1.2)	0.5 (1.2)	0.6 (1.3)	0.6 (1.2)	3.3 (6.1)	0.5 (1.2)	1.3 (3.0)	11.9 (17.4)	1.2 (2.8)	0.7 (1.9)
σ_ω in 10^{-4} radians					±0.5 (0.7)	±0.6 (1.5)	±0.5 (1.3)	± 0.8 (1.7)	±1.0 (2.1)	±0.8 (1.8)
σ_ϕ in 10^{-4} radians					±5.6 (9.2)	±0.4 (0.9)	±2.3 (3.9)	± 8.9 (13.7)	±0.9 (1.8)	±1.1 (2.1)
σ_κ in 10^{-4} radians					±0.5 (1.0)	±0.4 (0.8)	±0.6 (1.1)	± 0.8 (1.9)	±0.6 (1.8)	±0.7 (1.5)
σ_{X_o} in meters					±5.7 (9.2)	±0.5 (1.1)	±2.3 (4.0)	± 9.3 (14.3)	±1.3 (3.6)	±1.6 (3.5)
σ_{Y_o} in meters					±0.8 (1.4)	±0.8 (1.8)	±0.8 (1.3)	± 1.2 (2.3)	±1.6 (3.3)	±1.0 (2.5)
σ_{Z_o} in meters					±2.1 (3.8)	±0.5 (1.1)	±0.6 (1.0)	±11.8 (16.0)	±1.2 (3.3)	±0.8 (1.9)

6. Analysis of Test Results

The entire test may be considered in two broad areas.

Small Blocks (two strips at 20% sidelap).

The accuracy of determining plan and elevation control is not improved by using 60% sidelap i.e., by adding extra strips. The standard errors in X, Y, Z are nearly identical. This may be due to the fact that error propagation is negligible in such small blocks.

Large Blocks (three or more strips at 20% sidelap).

The block adjustment in terms of planimetry is not improved by using 60% sidelap (as against 20%) regardless of the amount of control used. However, when ground control is present only near the corners of the block, the block adjustment in terms of Z coordinates is greatly improved when 60% sidelap is used. With an additional vertical control in the center of the block, the obtained standard error in Z using 20% sidelap is comparable to that using no additional control in the center but 60% sidelap. In short, analytical block triangulation with 60% sidelap photography requires less ground control than with the standard procedure using 20% sidelap, which with no control in the center indicates a cylindrical "error surface" (see Report No. 2).

7. Required Time (for the Various Phases of the Block Triangulation)

The figures are rounded up because often one operation can not be separated from the other.

	Man Hours
Examination and study of the ISP data and familiarizing with the problem	100
Programming, additional card punching, debugging and production runs at the Computer	300
Data Analysis	100
Representation of the test results (Cartographic representation not included)	100
Total	600

Actual Computer Time	14 Hours

8. Final Remarks and Recommendations

1) Tests should be conducted using real data so as to confirm (may be with modifications) the results obtained with the fictitious data.

2) With growing use of super wide angle photography, it is suggested that a block with this type of photographs should be tested, if possible with actual photography on a test area.

3) In enforcing control positions by adding numbers to appropriate diagonal elements, values smaller than 1.0 should be tried. In the reported test it was found that a value of 1.0 allowed control positions to be adjusted by amounts less than 1 mm which perhaps could distort the geometry of the block in some cases.

4) Image coordinates used in this test have inherent systematic errors which could not be removed because of insufficient information regarding these errors. It is suggested that, in future tests, if fictitious data are to be used the image coordinates be perturbed only randomly. In real block adjustment cases sometimes it is better if all systematic discrepancies are removed prior to the adjustment of the block.

5) To evaluate the efficiency of a system it is necessary to consider the attainable accuracy, the required time and the involving costs. It is hoped that in case comparisons are made (aimed at developing an optimum system) realistic criteria be used to consider the three basic factors.

6) This test has given valuable research experiences for which the Chairman of this Working Group in Commission III deserves extra thanks. The data generating team is also highly commended.

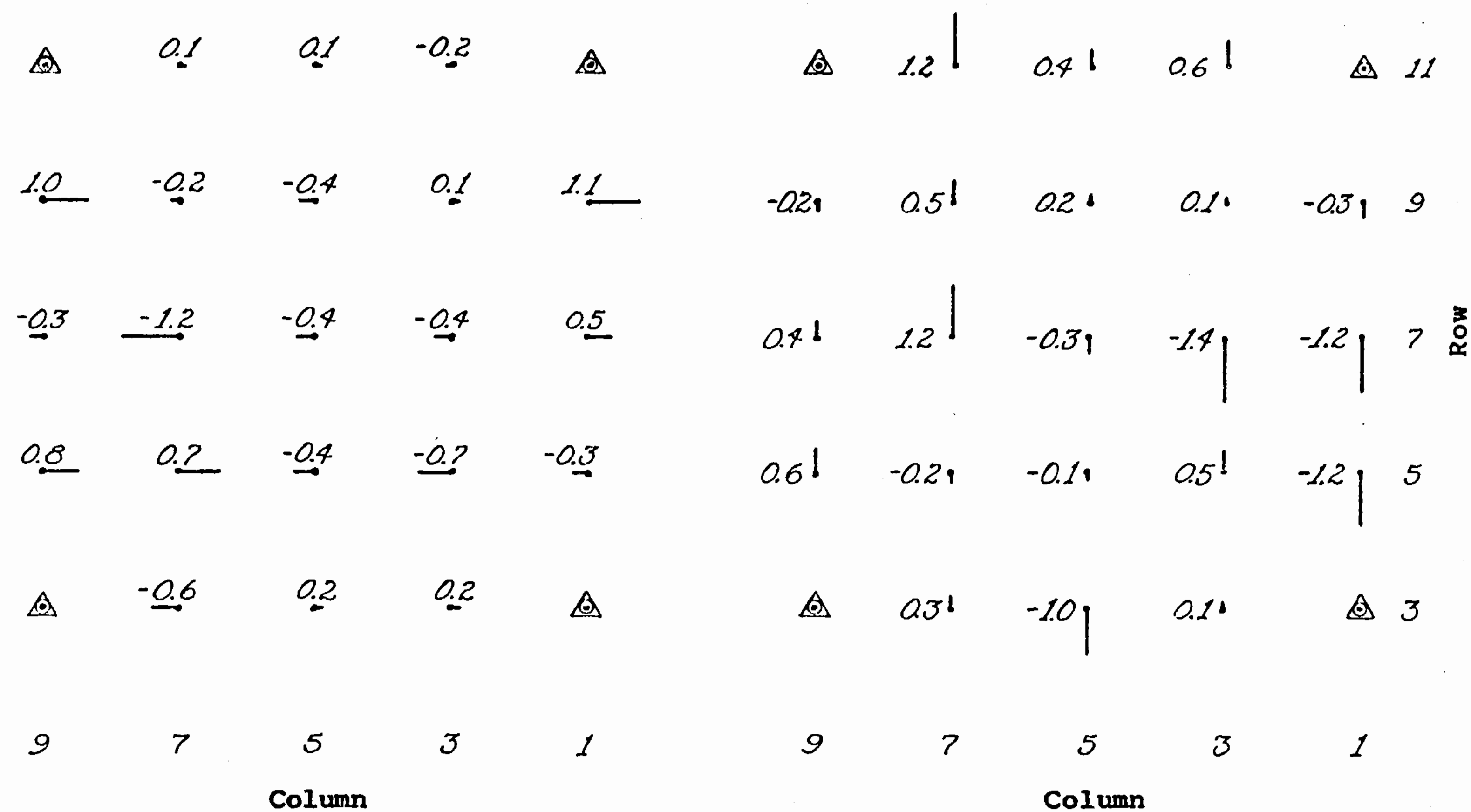
BIBLIOGRAPHY

1. Harris, W. D., etal; Analytic Aerotriangulation, Technical Bulletin No. 21, U. S. Coast and Geodetic Survey; July 1962, corrected July 1963.

APPENDIX I

DIAGRAMS SHOWING RESIDUAL ERRORS AFTER ADJUSTMENT FOR ALL TEST CASES

The diagrams shown herein depict the residual errors after adjustment for all test cases reported in the preceding text. The planimetric scale is such that one inch on the diagrams is equal to eight kilometers on the ground. The residual error vectors are drawn so that one inch on the diagrams is equal to four meters on the ground. All errors are in the local rectangular (secant plane) system. Row and column numbers are those given with the I.S.P. data. The coordinate system used is a right-handed one, Y coinciding with the direction of the strips and Z indicating elevations upwards from a datum plane.



TEST CASE 1A

Residual Errors in X

$\sigma_X = \pm 0.6$ m.

Residual Errors in Y

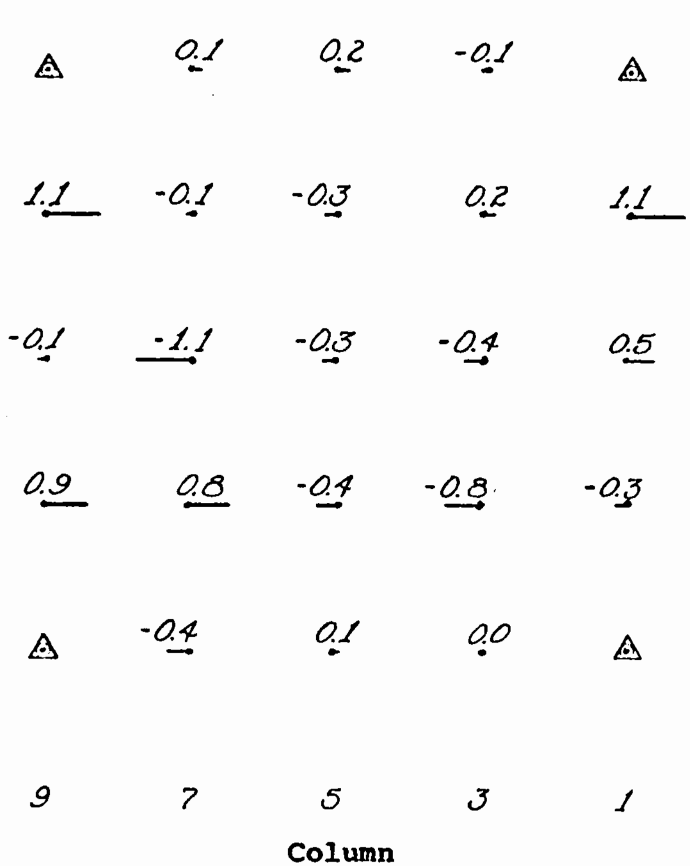
$\sigma_Y = \pm 0.7$ m.

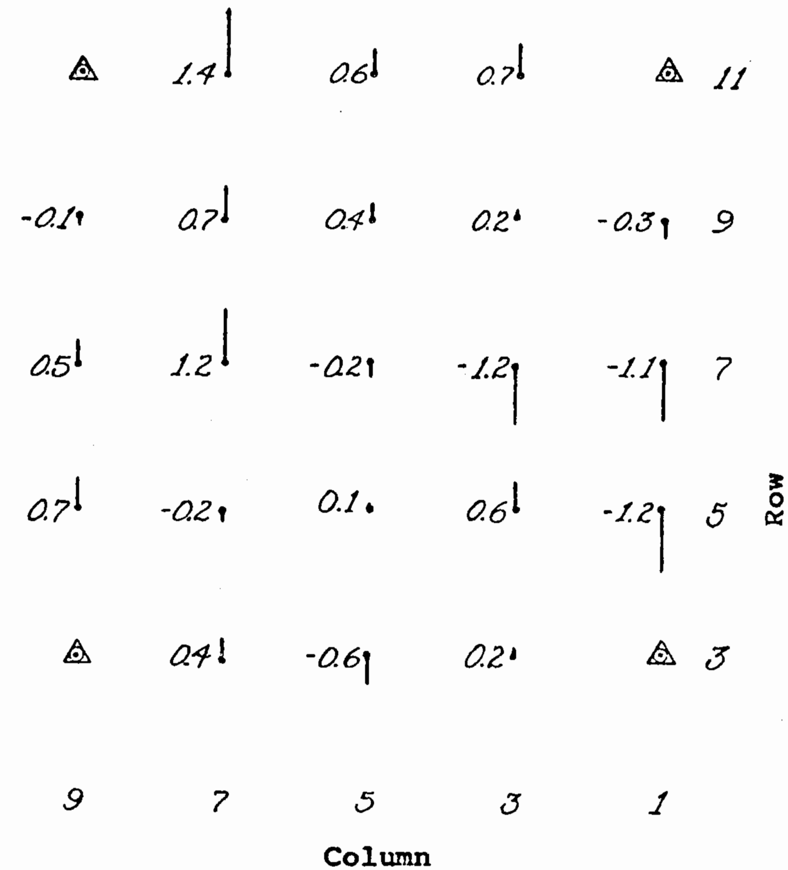
Row \ Column	9	7	5	3	1
11	△	0.2	0.7	0.7	△
9	0.3	0.4	0.4	1.1	0.6
7	-1.0	-0.4	0.9	0.6	0.4
5	-0.2	-0.3	1.0	0.5	1.2
3	△	0.0	0.4	0.0	△

TEST CASE 1A

Residual Errors in Z

$\sigma_Z = \pm 0.6$ m.





TEST CASE 1B

Residual Errors in X

$\sigma_X = \pm 0.6$ m.

Residual Errors in Y

$\sigma_Y = \pm 0.7$ m.

Row \ Column	9	7	5	3	1
11	△	-0.1	0.0	0.2	△
9	0.5	0.2	-0.1	0.6	0.5
7	-0.6	-0.2	0.5	0.3	0.4
5	0.0	-0.4	0.7	0.5	1.2
3	△	-0.2	0.3	0.5	△

Column

Row

TEST CASE 1B

Residual Errors in Z

$\sigma_Z = \pm 0.5$ m.

Row	13	11	9	7	5
11	△	-1.3	-1.2	-0.4	△
9	-0.7	-1.4	0.0	-0.7	-0.9
7	-0.2	0.8	-0.3	-1.5	-1.1
5	-0.5	1.1	1.2	0.8	-1.1
3	△	1.3	1.2	0.0	△

Column

Row	13	11	9	7	5
11	△	-0.1	-0.2	0.9	△
9	-0.4	-0.3	-0.7	0.0	-0.2
7	-0.3	0.1	-0.4	0.8	-0.5
5	-1.6	-0.4	-0.1	-0.5	0.2
3	△	-0.6	-0.4	0.5	△

Column

TEST CASE 2A

Residual Errors in X

$\sigma_X = \pm 0.9$ m.

Residual Errors in Y

$\sigma_Y = \pm 0.6$ m.

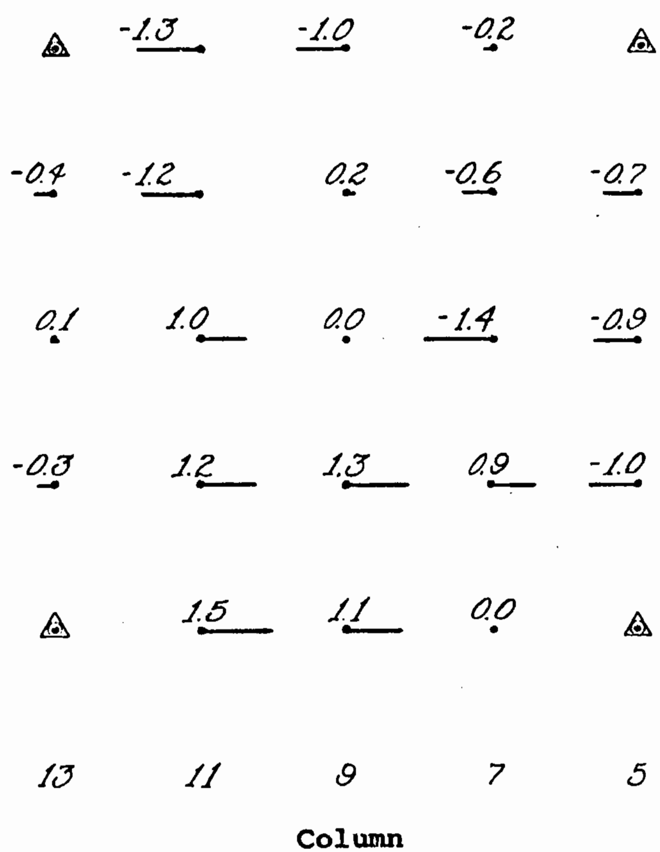
Row	13	11	9	7	5
11	△	0.3	-0.6	-0.2	△
9	-0.1	-1.2	-0.2	0.0	0.1
7	-0.1	-0.6	-0.5	-0.2	1.0
5	-0.4	-1.0	-1.0	-0.3	1.3
3	△	-0.1	-0.2	-0.5	△

Column

<u>TEST CASE 2A</u>

Residual Errors in Z

$\sigma_Z = \pm 0.6$ m.



TEST CASE 2B

Residual Errors in X

$\sigma_X = \pm 0.9$ m.

Residual Errors in Y

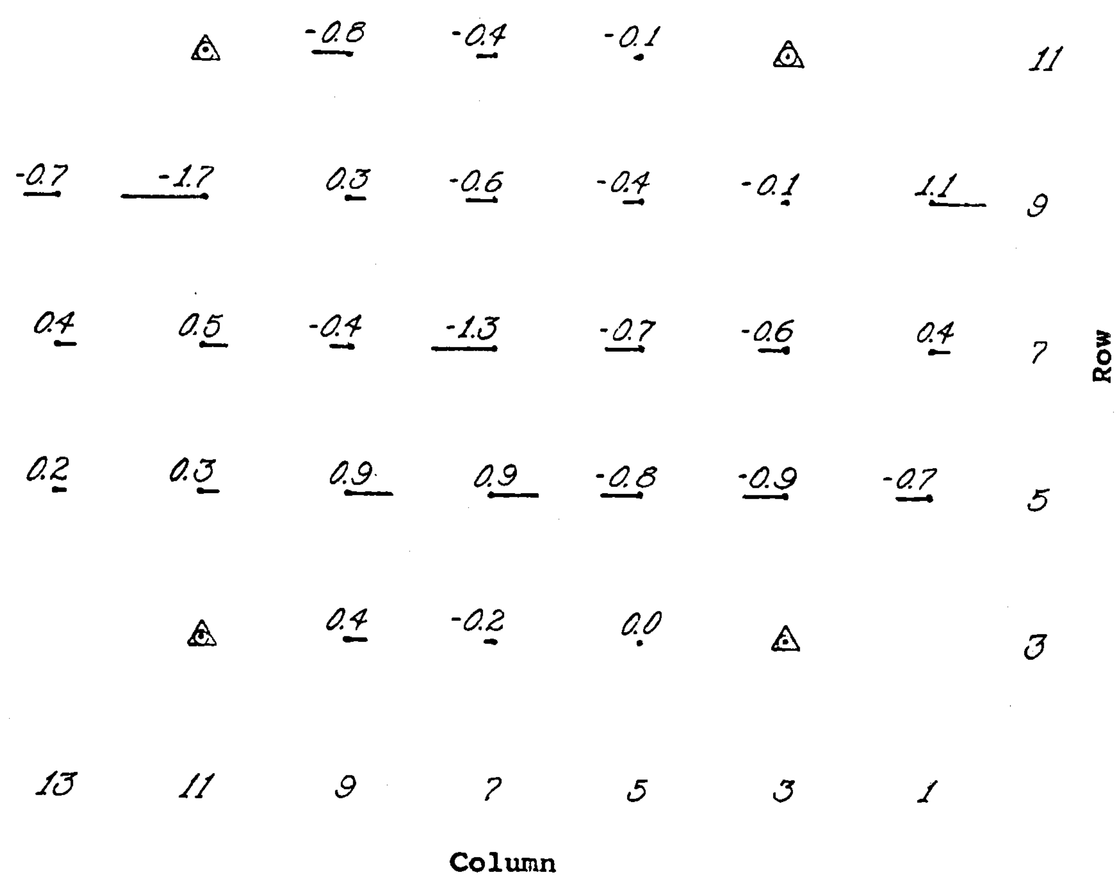
$\sigma_Y = \pm 0.6$ m.

Row \ Column	13	11	9	7	5
11		0.1	-0.8	-0.7	
9	0.5	-0.9	-0.3	-0.3	-0.1
7	0.7	-0.4	-0.5	-0.3	0.8
5	0.2	-0.7	-0.1	-0.3	1.2
3		-0.5	-0.3	0.0	

TEST CASE 2B

Residual Errors in Z

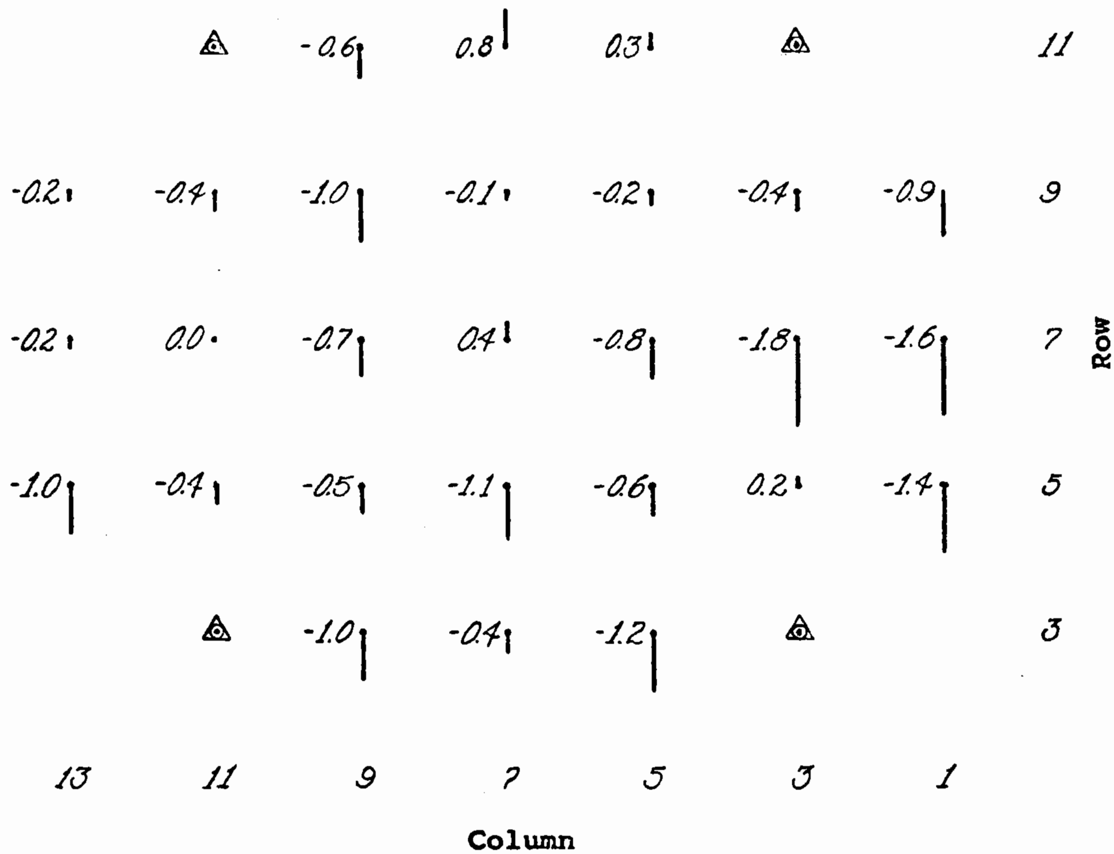
$\sigma_Z = \pm 0.6$ m.



TEST CASE 3A

Residual Errors in X

$\sigma_X = \pm 0.7$ m.



TEST CASE 3A

Residual Errors in Y

$\sigma_Y = \pm 0.8$ m.

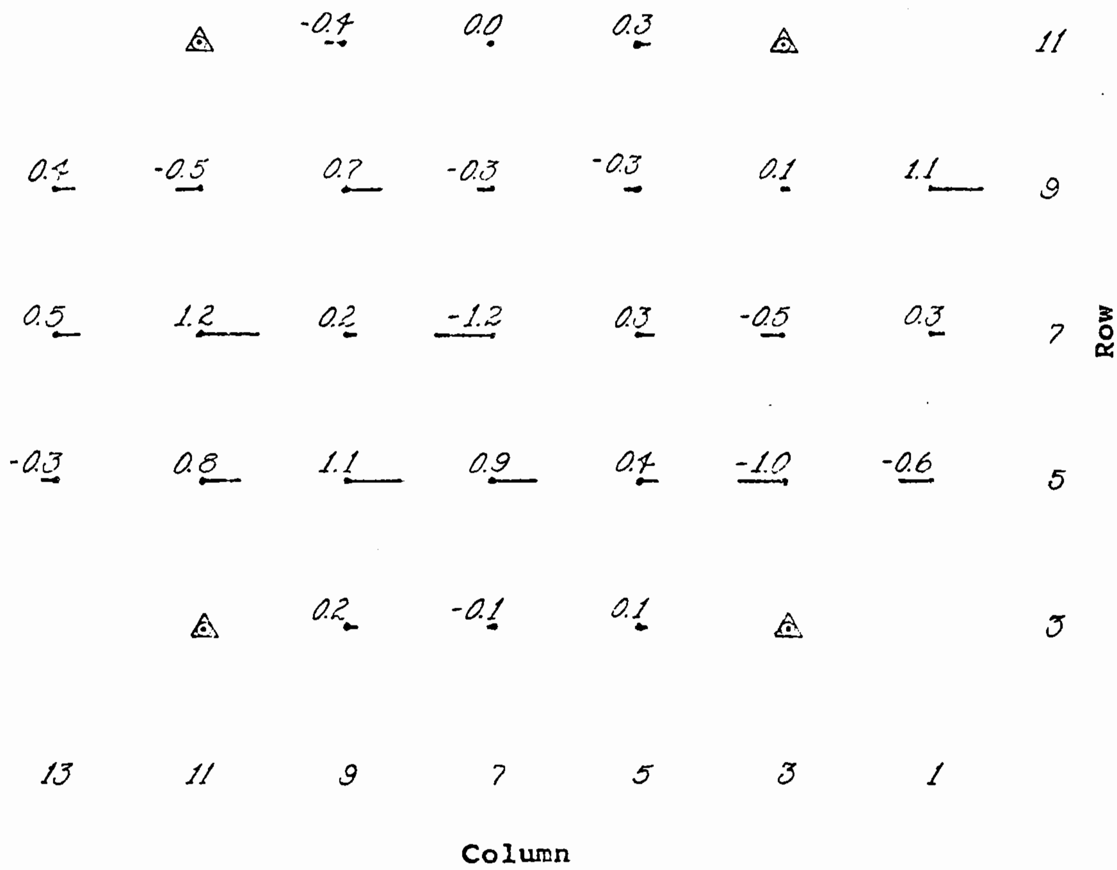
							Row
		4.8	3.2	1.7			11
-6.1	-1.6	5.0	3.5	1.8	0.5	-2.1	9
-5.7	-1.1	4.1	3.0	2.7	0.3	-2.0	7
-5.4	-1.3	4.6	3.0	2.9	0.4	-0.9	5
		4.3	3.0	2.2			3
13	11	9	7	5	3	1	

Column

TEST CASE 3A

Residual Errors in Z

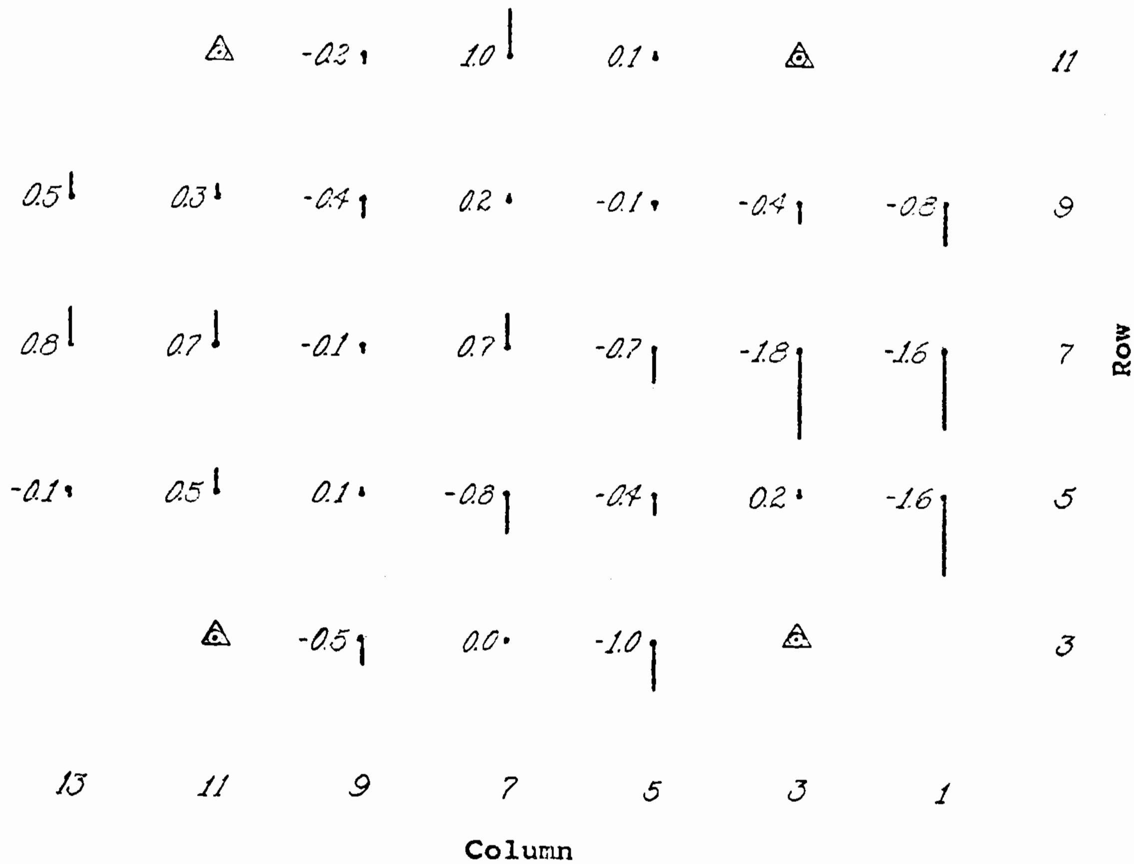
$\sigma_Z = \pm 3.3$ m.



TEST CASE 3B

Residual Errors in X

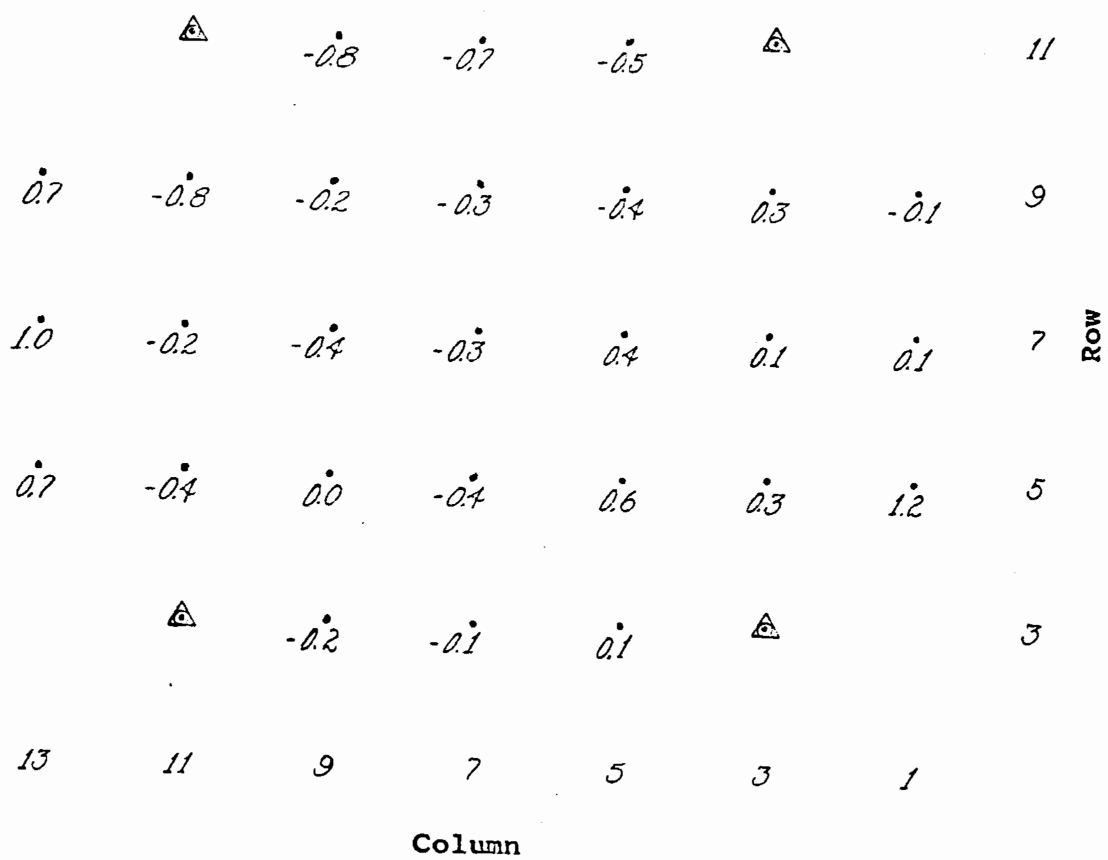
$\sigma_X = \pm 0.6$ m.



TEST CASE 3B

Residual Errors in Y

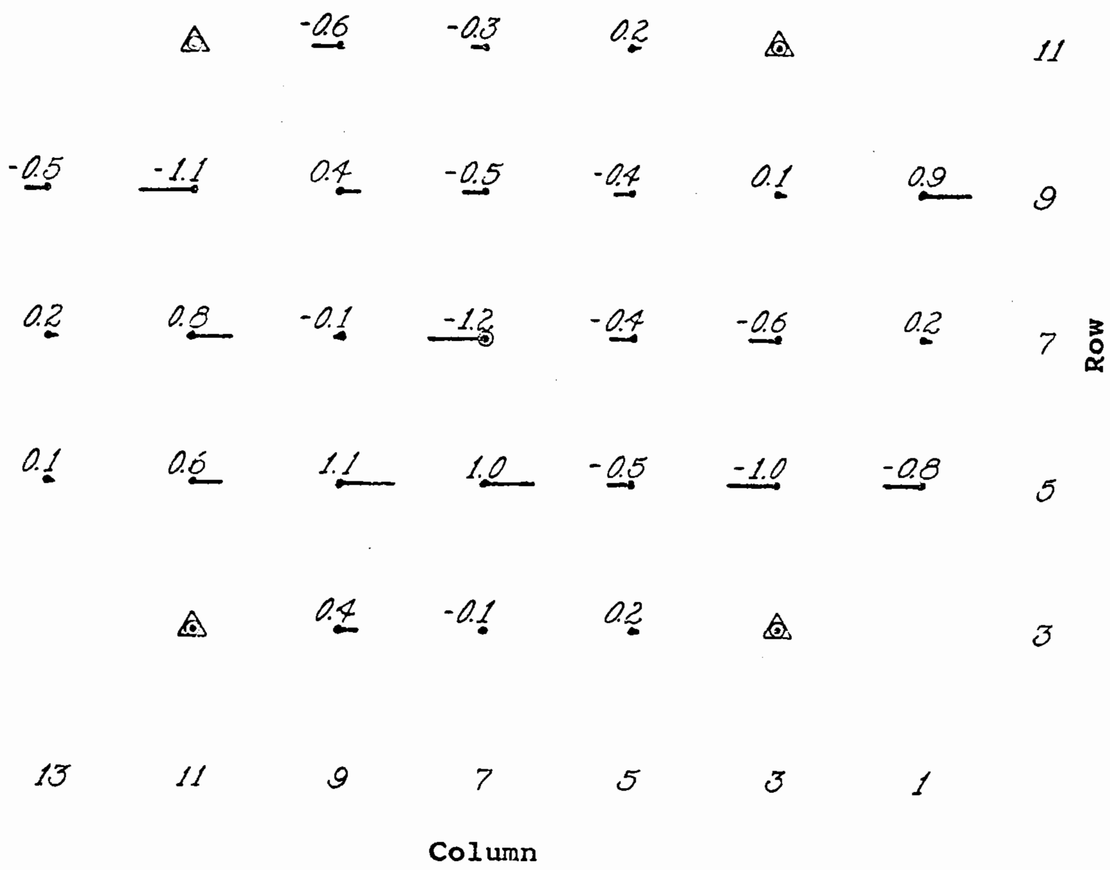
$\sigma_Y = \pm 0.7$ m.



TEST CASE 3B

Residual Errors in Z

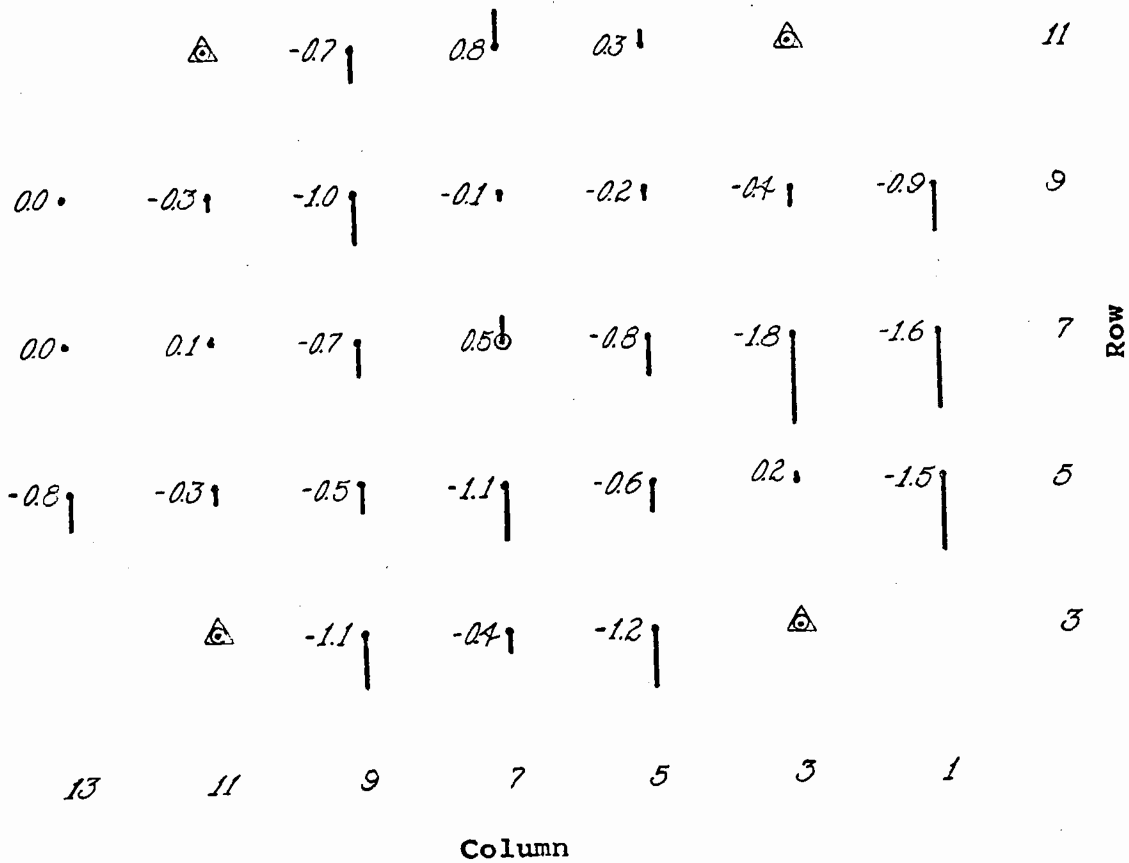
$\sigma_Z = \pm 0.5$ m.



TEST CASE 3C

Residual Errors in X

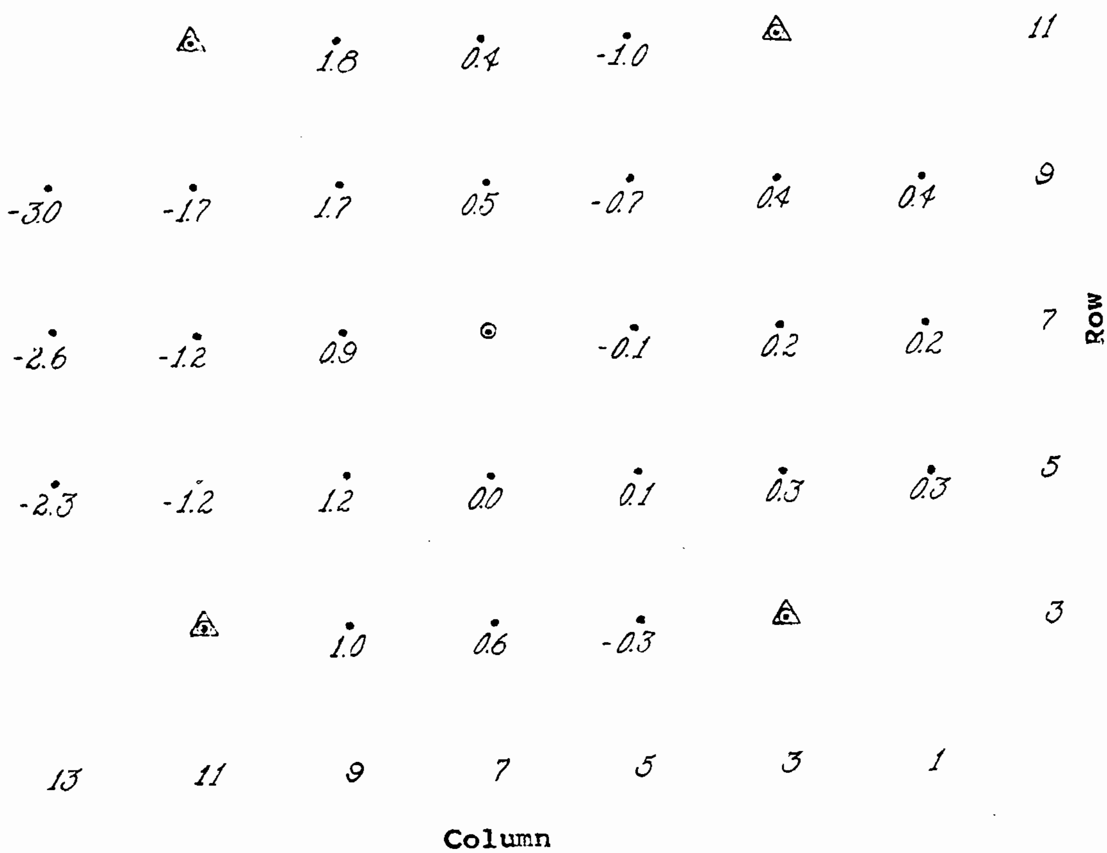
$\sigma_X = \pm 0.6$ m.



TEST CASE 3C

Residual Errors in Y

$\sigma_Y = \pm 0.8$ m.



TEST CASE 3C

Residual Errors in Z

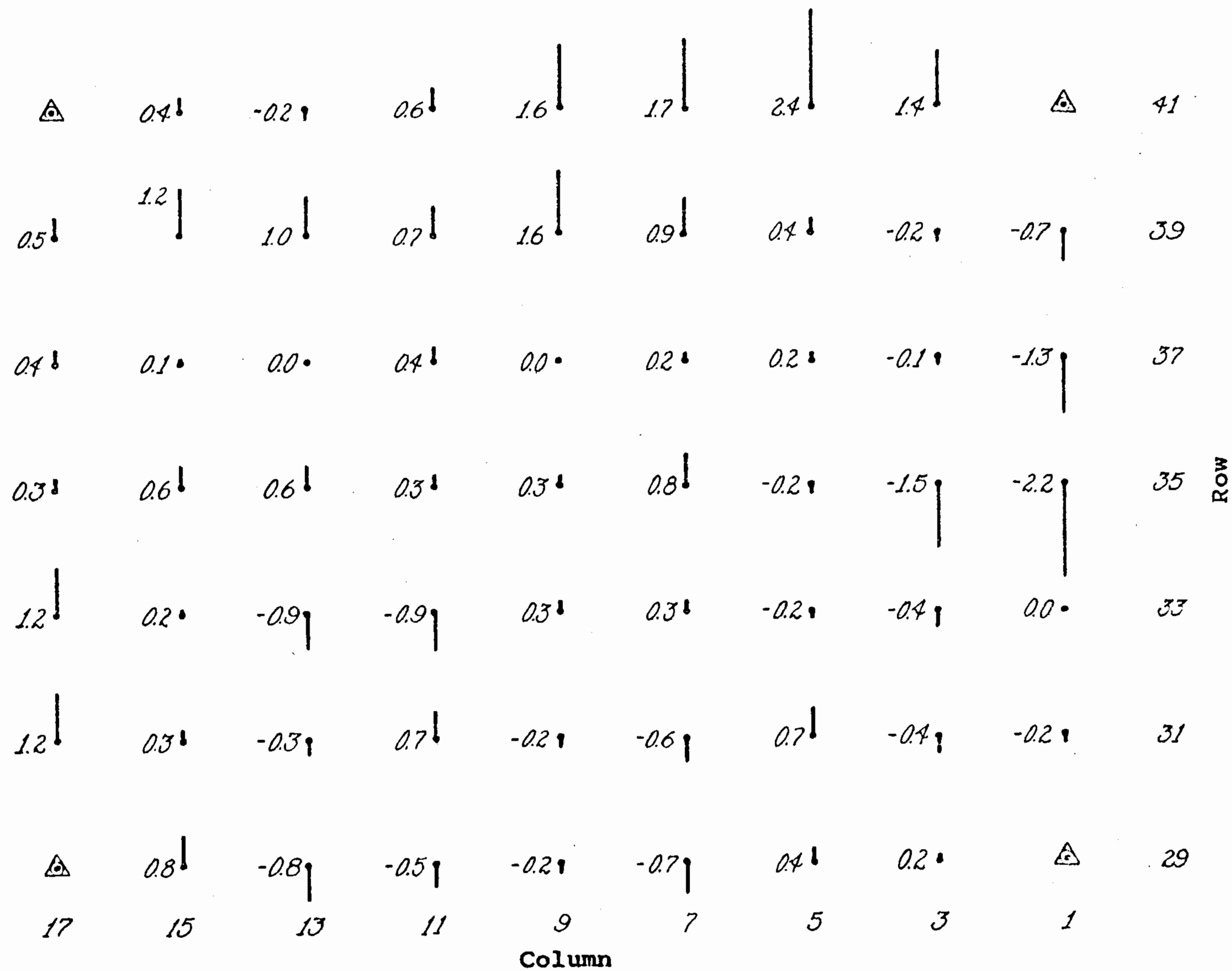
$\sigma_Z = \pm 1.3$ m.

Row \ Column	17	15	13	11	9	7	5	3	1
41	△	-1.4	-0.7	-2.3	-1.7	-1.2	0.2	1.5	△
39	-0.6	-1.3	0.3	-1.0	-1.1	-0.9	-0.4	0.6	0.1
37	-0.9	-0.9	0.2	-0.7	-0.9	0.2	-0.6	1.3	-0.5
35	0.9	0.4	0.0	-0.3	-0.8	0.4	-1.1	0.4	-0.5
33	0.7	-0.3	-0.3	-0.4	0.7	-0.2	-0.4	-0.2	-1.2
31	0.8	-0.2	0.6	-0.2	1.2	1.6	0.8	0.6	-1.8
29	△	0.1	0.8	0.4	0.9	1.6	1.7	-0.7	△

Column

Row

TEST CASE 4A Residual Errors in X $\sigma_X = \pm 0.9$ m.



<u>**TEST CASE 4A**</u> **Residual Errors in Y** $\sigma_Y = \pm 0.8$ m.

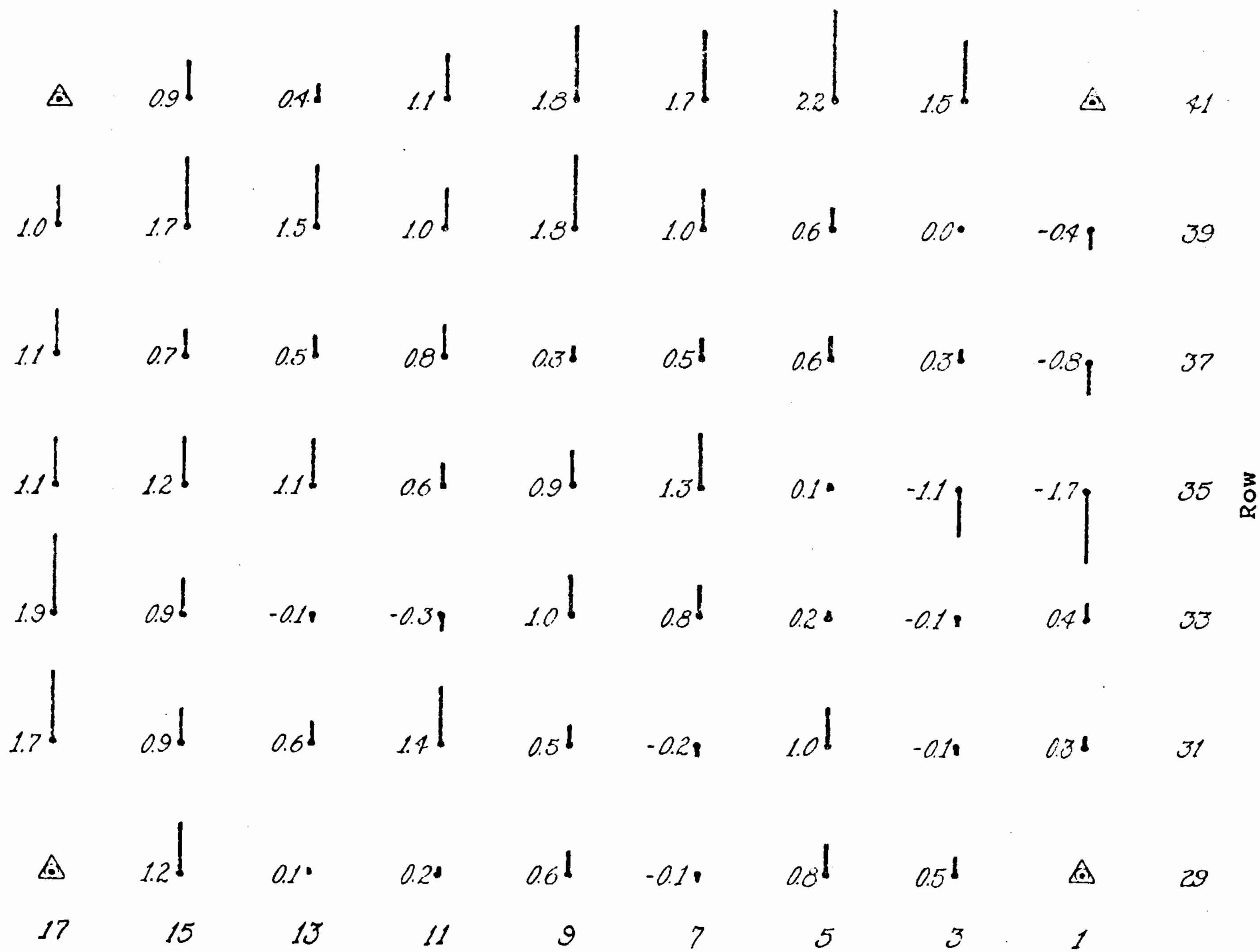
Row \ Column	17	15	13	11	9	7	5	3	1
41	△	6.8	13.9	15.0	16.0	13.9	11.2	5.6	△
39	-0.4	6.3	14.3	15.9	17.1	14.3	12.2	6.0	-0.4
37	0.7	6.6	14.4	15.3	16.2	14.8	13.1	5.9	0.1
35	-0.1	7.2	13.8	15.6	17.1	14.7	13.3	5.7	0.0
33	0.0	7.4	14.1	15.8	17.4	15.4	13.2	6.9	0.6
31	0.7	7.6	14.9	15.8	17.4	14.5	13.0	6.8	1.1
29	△	7.5	14.3	16.3	17.3	14.9	12.9	6.9	△

TEST CASE 4A Residual Errors in Z $\sigma_Z = \pm 11.9$ m.

17	15	13	11	9	7	5	3	1	Row
△	-1.5	-0.6	-1.9	-1.5	-1.0	-0.3	1.2	△	41
0.0	-0.3	0.3	-0.9	-1.0	-1.0	-0.7	0.4	-0.2	39
-0.1	-0.4	0.2	-0.6	-0.9	0.2	-0.4	0.3	-0.6	37
1.0	0.0	0.6	-0.4	-0.7	0.3	-1.0	-0.4	-0.7	35
0.9	-0.7	0.1	-0.2	0.6	-0.3	-0.6	-0.2	-1.4	33
-0.2	-0.2	0.5	0.0	1.1	0.9	0.4	0.3	-1.3	31
△	0.6	1.1	0.3	0.6	0.8	1.0	-1.0	△	29

Column

TEST CASE 4B Residual Errors in X $\sigma_X = \pm 0.8$ m.



TEST CASE 4B Residual Errors in Y $\sigma_Y = \pm 1.0$ m.

Row	17	15	13	11	9	7	5	3	1
41	△	-2.0	-2.5	-2.6	-2.6	-2.5	-2.8	-2.1	△
39	-0.4	-1.7	-1.4	-1.2	-1.4	-2.1	-1.9	-1.4	-0.8
37	-0.3	-0.8	-0.8	-1.2	-1.7	-1.3	-1.1	-1.3	-0.2
35	0.4	0.1	-0.9	-0.5	-0.8	-1.2	-0.5	-1.0	0.0
33	0.4	0.4	-0.5	-0.2	-0.2	-0.1	-0.2	0.3	0.7
31	1.0	0.4	0.2	-0.1	0.1	-0.5	-0.1	0.3	1.3
29	△	0.6	0.5	0.6	0.4	0.2	0.1	0.9	△

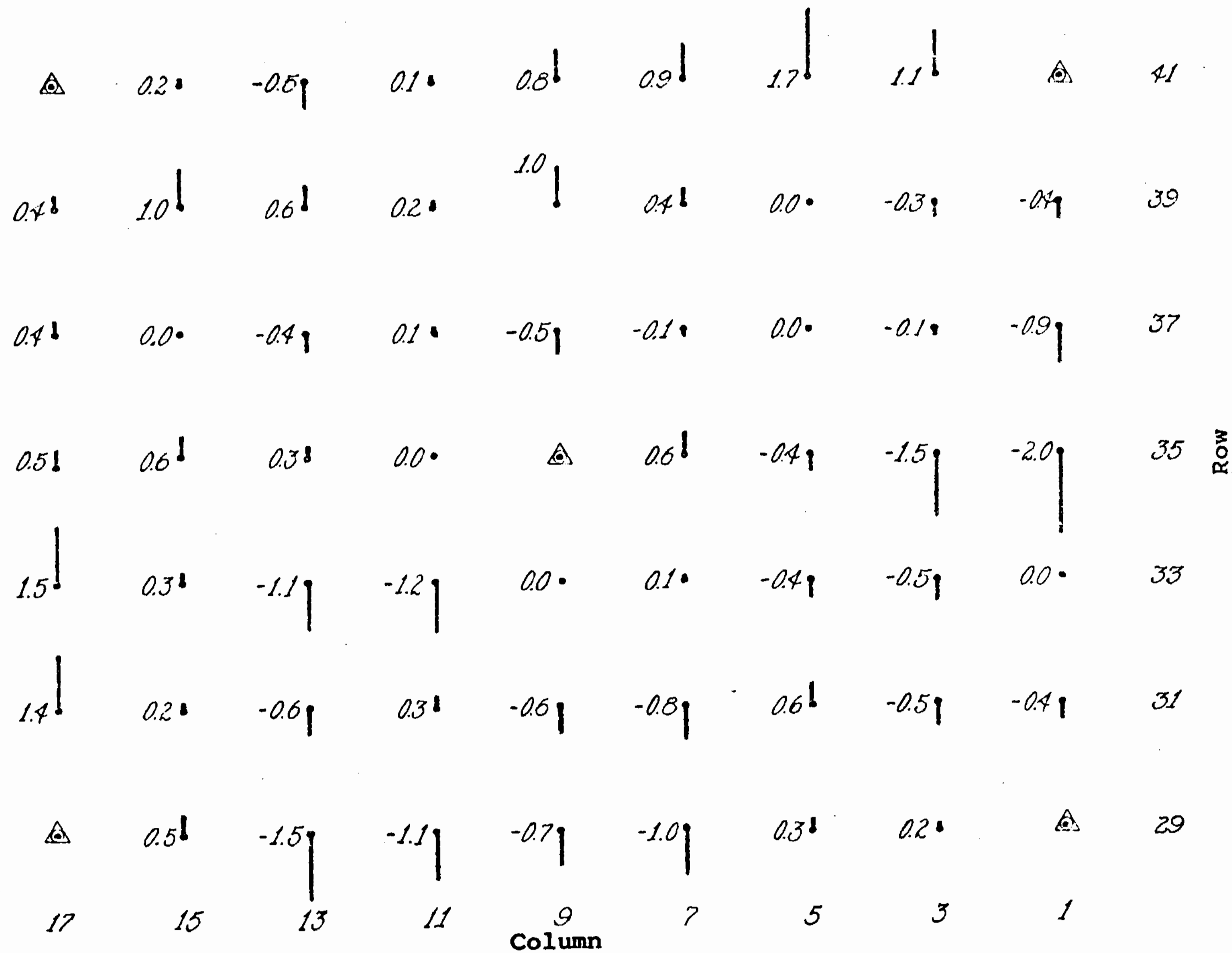
Column

TEST CASE 4B Residual Errors in Z $\sigma_Z = \pm 1.2$ m.

Row \ Column	17	15	13	11	9	7	5	3	1
41	△	-0.8	0.2	-1.3	-1.1	-0.9	-0.2	1.3	△
39	0.4	0.2	1.0	-0.4	-0.5	-0.6	-0.3	0.7	0.1
37	0.3	0.1	0.8	0.0	-0.2	0.8	0.1	1.0	0.1
35	1.6	0.5	1.1	0.4	△	1.0	-0.4	0.4	0.2
33	1.4	-0.2	0.6	0.4	1.4	0.6	0.2	0.6	-0.4
31	0.2	-0.1	0.9	0.5	1.9	2.1	1.4	1.1	-0.7
29	△	0.1	1.2	1.0	1.6	2.3	2.3	0.0	△

Column

TEST CASE 4C Residual Errors in X $\sigma_X = \pm 0.9$ m.



<u>**TEST CASE 4C**</u> **Residual Errors in Y** $\sigma_Y = \pm 0.7$ m.

Row	17	15	13	11	9	7	5	3	1
41	△	-0.1	0.0	-0.3	-0.7	-1.1	-1.9	-1.2	△
39	-0.1	-0.5	0.4	0.5	0.0	-1.1	-1.3	-1.1	-1.0
37	-0.2	-0.1	0.6	-0.1	-0.7	-0.6	-0.8	-1.6	-1.0
35	0.5	0.7	0.3	0.5	△	-0.8	-0.6	-1.8	-1.1
33	0.4	0.8	0.5	0.4	0.3	-0.1	-0.6	-0.5	-0.4
31	1.0	0.8	0.8	0.2	0.2	-0.8	-0.5	-0.2	0.5
29	△	0.6	0.4	0.8	0.3	-0.2	-0.3	0.2	△

Column

TEST CASE 4C Residual Errors in Z $\sigma_Z = \pm 0.7$ m.

International Society of Photogrammetry

Group Study for Experimental Researches
on
Analytical Block Triangulation

Center of Work
Department of Geodetic Science
The Ohio State University
Columbus, Ohio 43210 U.S.A.

Report No. 2
on
Analytical Block Triangulation,
Improvement with Horizon Control

1. Material and Equipment

IBM readouts containing all information on the fictitious data generated for this experimental research were obtained from Mr. Charles M. Theurer, Chairman of this working group. Computations and data analyses were performed at the facilities of The Ohio State University, Department of Geodetic Science, and the OSU Computer Center (IBM 7094).

2. Personnel

The adjustment and programming were primarily performed by Captain Arthur S. Brown, Graduate Research Assistant under the advice of Dr. Sanjib K. Ghosh. Mr. Fletcher E. Twitty assisted in the cartographic representations. Mrs. Vera N. Hoff assisted in typing. Dr. Sanjib K. Ghosh supervised the entire project.

3. Characteristics of the Test

In the research reported earlier by S. K. Ghosh (Report No. 1 from the Department of Geodetic Science, The Ohio State University, Columbus, Ohio) we found several unexplained discrepancies in the solution for 20% sidelap. A very large "error surface" for the elevations of all points in the block was noticed, also the coordinates of the exposure stations and φ rotations at the exposure stations appeared to be systematically distorted.

In an effort to find out why the above discrepancies occured and how they could be corrected, the following studies were made:

1. Investigation of the discrepancies in the Z-coordinates of ground points and the apparent systematic distortions of exposure station coordinates and φ rotations in the 20% sidelap photography.
2. Determining the validity of the addition of weights directly to the normal equations.
3. Determining the effects that auxiliary data, obtained from horizon controlled photography, have on the discrepancies obtained in the 20% sidelap case.

It was noticed (Cases 3A and 4A) in Report 1 in two different blocks, one consisting of 3 strips of 5 photographs each and the other consisting of 4 strips of 7 photographs each (both having 20% sidelap), that a Z error surface in clearly noticeable. The surface in each case appears almost cylindrical (close to linear in Y and of a quadratic form in X, see Figures 1 and 2; Y axis coincides with strip directions).

On a close inspection of the cases and after a careful study, it was decided that the situation could be considerably improved by one of the following approaches (apart from many other approaches):

a) Using 60% sidelap and control at the 4 corners of the block.

b) Using 20% sidelap but an additional control in the center of the block, or

c) Using 20% sidelap with auxiliary data, with control only in the 4 corners of the block.

The first two approaches were investigated as reported earlier (Report No. 1, Refer to Cases 3B, 4B, 3C and 4C). It is the third approach that is being reported on here. From the fictitious data one could easily obtain information on the ω and φ rotations of each photograph, which were used in this investigation. In doing this and in performing analytical block triangulations, the problem of weighting was carefully considered. The basic mathematical approach and programs as reported earlier (Report No. 1) were used in this case also.

For further details, the reader is referred to [1] i.e., M.S. thesis of Captain Arthur S. Brown entitled "The Effects of Horizon Control on 20% Sidelap Photography for Analytical Aerotriangulation Block Adjustment."

4. Mathematical Basis and Weighting

The mathematical basis and the computer programs are the same as reported in Report 1.

It was assumed that after all the influences had been taken into consideration, there was no correlation between the images and observations selected and that each observation should be assigned a weight of unity. The weight matrix [W] for the observations will be an identity matrix and does not affect the process of formation of the normal equations through the least squares adjustment.

As far as the weighting of the control is concerned, the position data are enforced in the adjustment phase (as against the formation phase). This was done for the sake of simplicity. This application of the weights in the adjustment phase allows for any changes which one might want to make at a later time because of new information concerning the control data in question.

The actual weights of the control data were determined from the relation:

$$W^2_{X_k} = \frac{m_o^2}{m^2_{X_k}}$$

where

W_{x_k} is the weight of diagonal element x_k of the normal equation matrix.

m_o is the variance of unit weight.

m_{X_k} is the estimated variance of the "Control Parameter" in question.

X_k is one of the "Control Parameters" corresponding to X_j, Y_j, Z_j (ground coordinates of a control point j), or ω_i, φ_i (horizon controlled orientation angles of exposure station i).

The weight is considered inversely proportional to the variance.

From a purely practical point of view, arbitrarily, weights were assigned to the approximate diagonal elements of the normal equation matrix as follows (also see [1]):

$$W_{X_j} = 0.01 \qquad W_{Y_j} = 0.01 \qquad W_{Z_j} = 1.0$$

where $(W_X, W_Y, W_Z)_j$ are the weights corresponding to ground point j which is to be used as control. No covariances are involved because any correlation between the coordinates has been neglected. Admitted that this is theoretically incorrect, but the overall effect will still be valid from a practical standpoint.

For the horizon data the standard errors of φ and ω as determined by Kaasila [2] and by Korhonen [3] was taken as good representative values, i.e.,

$$m_\varphi = 3\overset{c}{.}0 \qquad \text{and} \qquad m_\omega = 2\overset{c}{.}5$$

The weights derived from the above values were used in this study and assigned to the exposure stations. No correlation between ω and φ are considered. No weights are assigned to the other elements of exterior orientation because no control is to be provided for them, other than the resection values.

5. Test Procedure

In order to obtain good approximations for the orientation angles ω, φ and κ of each exposure station, a resection was performed using the unperturbed data. This was done because a good resection program was available at The Ohio State University, Department of Geodetic Science and it was more practical than trying to convert the non-sequential X-tilt, Y-tilt heading system given in the ISP data, to the sequential ω, φ, κ system.

The solution of the normal equations was accomplished by two methods which served as a check on each other [1].

CASE 1A

The block 3A of Report No. 1 was studied first. The case is much the same as reported earlier (Report 1) except that more realistic weights were used for the survey control points in planimetry and elevation. The control points were located in the four corners of the block and no auxiliary control was used. The results show that the planimetric values were not significantly affected. However, the differences in elevation values before and after the adjustment i.e., the true errors, formed a highly warped surface. Similarly, the values for the ω, κ, and Y_o elements of exterior orientation were not significantly affected, whereas the errors in the φ, X_o and Z_o elements also formed warped surfaces.

CASE 1B

Case 1A was used. However, the values of ω and φ determined by the resection were treated as the horizon data. Since they were considered to be more accurate than the other elements, they were weighted more heavily. The results of Case 1B indicate a great improvement. (See Table II, Figures 3 and 4.)

CASE 2A

This block corresponds to Case 4A of earlier Report No. 1 with a change in the weighting of the ground control. The results show effects similar to Case 1A but this time the block is larger (one additional strip with two additional photographs in each) and all errors increased substantially in those elements affected previously.

CASE 2B

Enforcement of the horizon data to Case 2A was made in the same manner as in Case 1B. The results (see Table II, Figures 5 and 6) again showed improvements similar to Case 1B (with respect to Case 1A). All values now fall within reasonable limits.

6. Analysis of Test Results and Recommendations

Table II provides a rapid means of comparing the results of the four tests and allows one to see at a glance that 20% sidelap horizon controlled block triangulation provides a solution which differs very little from either (a) the 60% sidelap block triangulation case with minimum control (i.e., one point at each of the 4 corners of the block), or (b) the 20% sidelap block triangulation case with additional control in the center of the block.

NOTE:

The 60% sidelap case involves more strips i.e., more photographs than the 20% sidelap case. This means considerably more flying, more data handling and more cost.

Also, 20% sidelap with additional control in the middle of the block means more work on the ground, i.e., more cost involved.

The study indicates that horizon data would not require additional control and would provide the highest accuracy.

The method of weighting the normal equations in the adjustment process was proven valid and provided a means of checking a large loss of significance in the results.

The methods of determining the specific weights of the controlling parameters, although valid, provided only relative weights because the accuracies of the control parameters were not as high as we would have liked. Nevertheless, this was more than sufficient for the purpose at hand.

The horizon data were found to have certain advantages over the additional ground control with respect to time, effort and financial economy. This means the use of horizon controlled photography is expected to provide a powerful alternative for controlling the unavoidable error propagation in strip or block triangulation using only 20% sidelap and only one row of tie (common) points between strips.

Based on the above the following are recommended:

a) Future research should be conducted using real data in lieu of fictitious data, so that the results obtained with fictitious data can be verified or refuted.

b) Several different test areas of known and high precision should be flown and analyzed.

c) The accuracies of control determination should be improved. Earnest effort should be made to develop horizon photographs of larger formats with more sensitive emulsions or other means of defining the horizon more clearly.

d) The correlations between control elements should be established and taken into consideration in further investigations.

7. Required Time (for Various Phases)

	Man Hours
Examination and study of the ISP data and familiarizing with the problem	50
Programming, additional card punching, production run (also debugging) at the Computer	75
Data analysis	75
Representation of the test results	50
Total	250

Actual Computer time	4 hours

TABLE II

PERTINENT INFORMATION ON ALL CASES

	CASE 1A 20% Sidelap NO Horizon Control	CASE 1B 20% Sidelap Horizon Control	CASE 2A NO Horizon Control	CASE 2B Horizon Control
Strips	3	3	4	4
Photos/Strip	5	5	7	7
Total Photos	15	15	28	28
Control Points	4	4	4	4
Check Points	27	27	59	59
Observation Equations	218	218	456	456
Unknowns	183	183	357	357
Degrees of Freedom	35	35	99	99
MAXIMUM ERRORS APPEAR IN PARENTHESES BELOW				
σ_X in meters	$\pm$ 0.7(1.7)	$\pm$ 0.6(1.2)	$\pm$ 0.9(2.3)	$\pm$ 0.8(2.2)
σ_Y in meters	0.8(1.8)	0.8(1.9)	0.8(2.4)	0.8(2.1)
σ_Z in meters	3.3(6.1)	0.6(1.5)	11.9(17.4)	1.3(2.7)
σ_ω in 10^{-4} radians	0.5(0.7)	0.3(0.7)	0.8(1.7)	0.7(1.6)
σ_ϕ in 10^{-4} radians	5.7(9.2)	0.3(0.6)	8.9(13.7)	0.6(1.3)
σ_κ in 10^{-4} radians	0.5(1.0)	0.5(1.0)	0.8(1.9)	0.7(1.6)
σ_{X_o} in meters	5.7(9.2)	0.5(1.3)	9.3(14.3)	1.2(3.2)
σ_{Y_o} in meters	0.8(1.4)	0.7(1.3)	1.2(2.3)	0.9(2.2)
σ_{Z_o} in meters	2.1(3.8)	0.4(0.7)	11.8(16.0)	1.3(2.7)

Bibliography

1. Brown, Arthur S., "The Effects of Horizon Control on 20% Sidelap Photography for Analytical Aerotriangulation Block Adjustment." M.Sc. Thesis, The Ohio State University, March 1968.

2. Kaasila, P., "Investigation of the Accuracy of Wild HC-1 Horizon Camera." International Archives of Photogrammetry, Vol. XV, Part 5, Commission III, 1965.

3. Korhonen, U., "Horizon Controlled Analytical Block Triangulation for Extending Ground Control for Small-Scale Mapping." Department of Geodetic Science, The Ohio State University, 1966.

4. Kenefick, John F., "A Comparison of 60% and 20% Sidelap Photography for Analytical Aerotriangulation Block Adjustment." M.Sc. Thesis, The Ohio State University, March 1968.

APPENDIX II

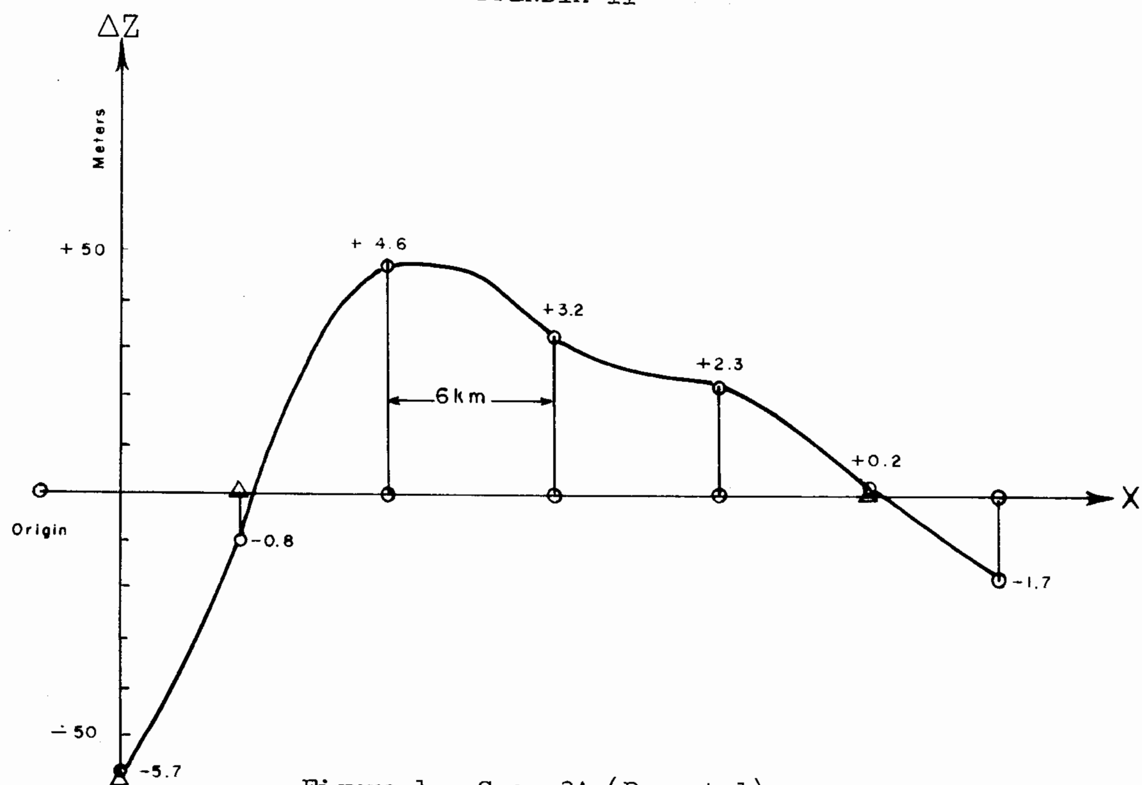


Figure 1. Case 3A (Report 1)
Residual Errors in Z

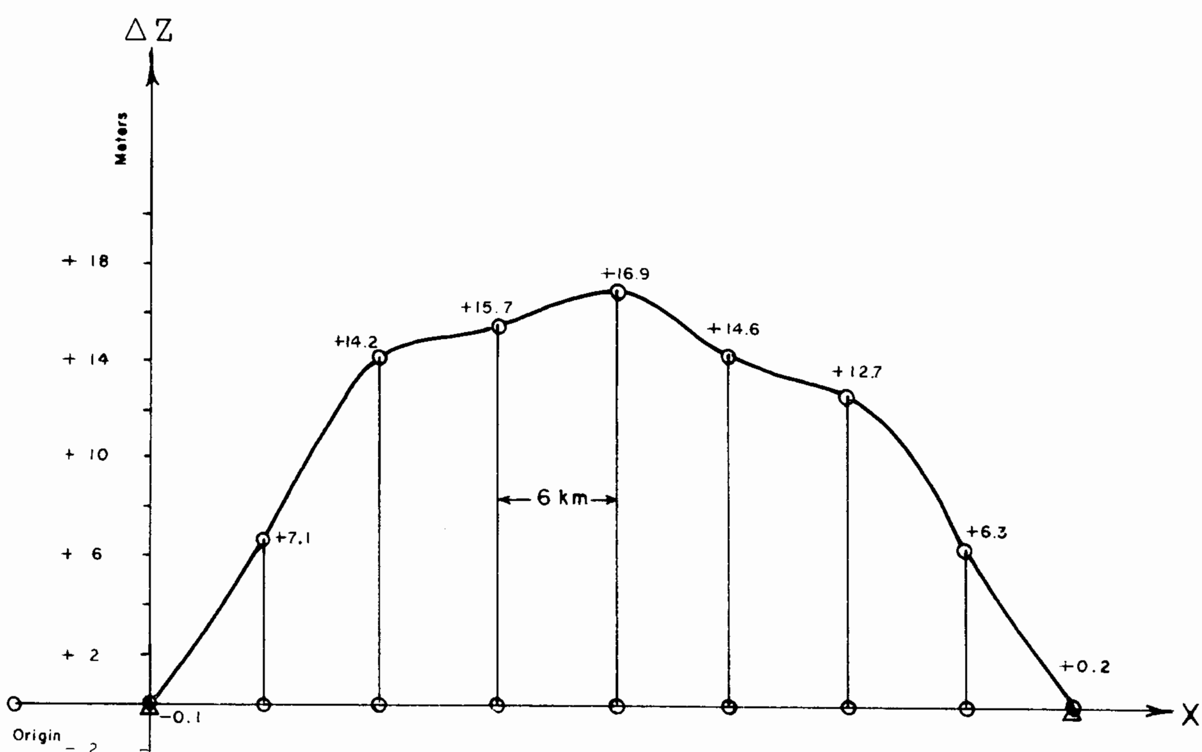


Figure 2. Case 4A (Report 1)
Residual Errors in Z

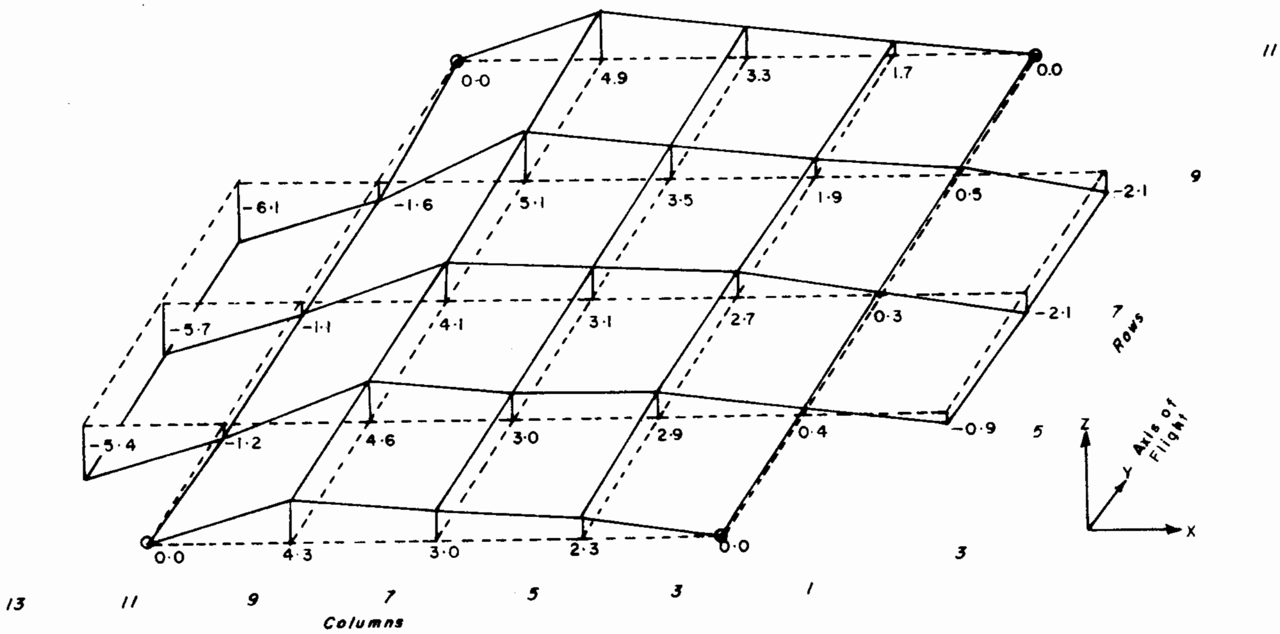


Figure 3. Case 1A
20% Sidelap/No Horizon Control

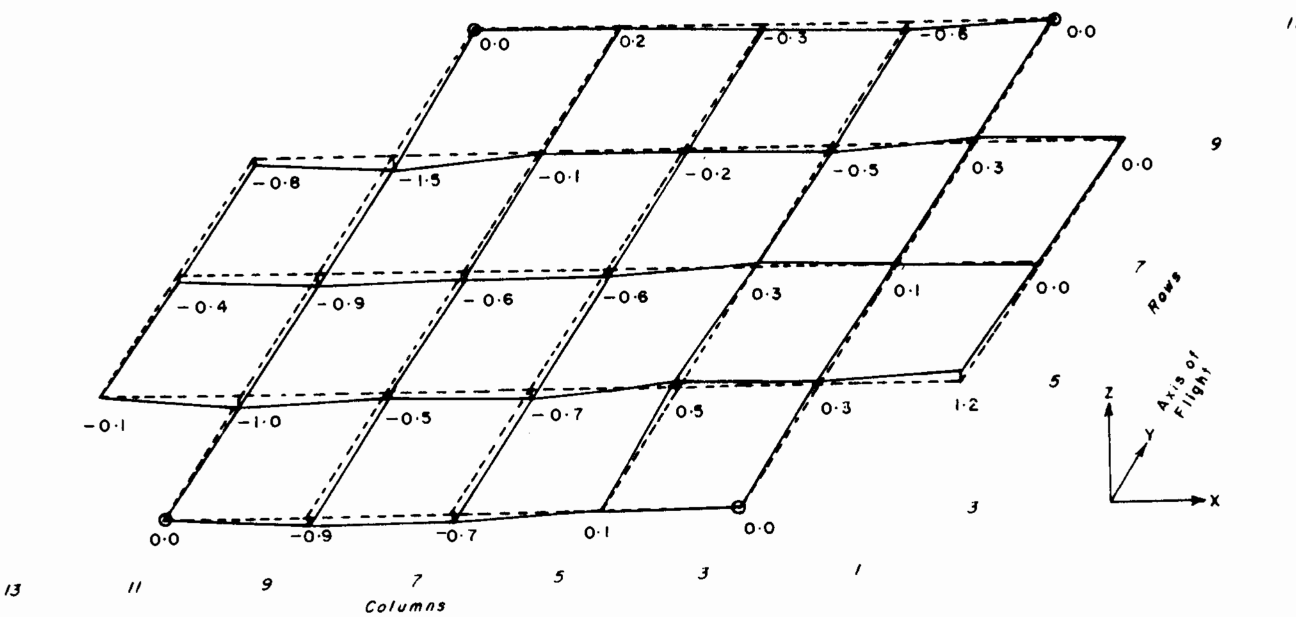


Figure 4. Case 1B
20% Sidelap/Horizon Control
Residual Errors in Z after Adjustment
Strips 1, 3, 5 : $M_B = 1/66,000$

Distance between points:
6 kilometers (1" = 8 km)

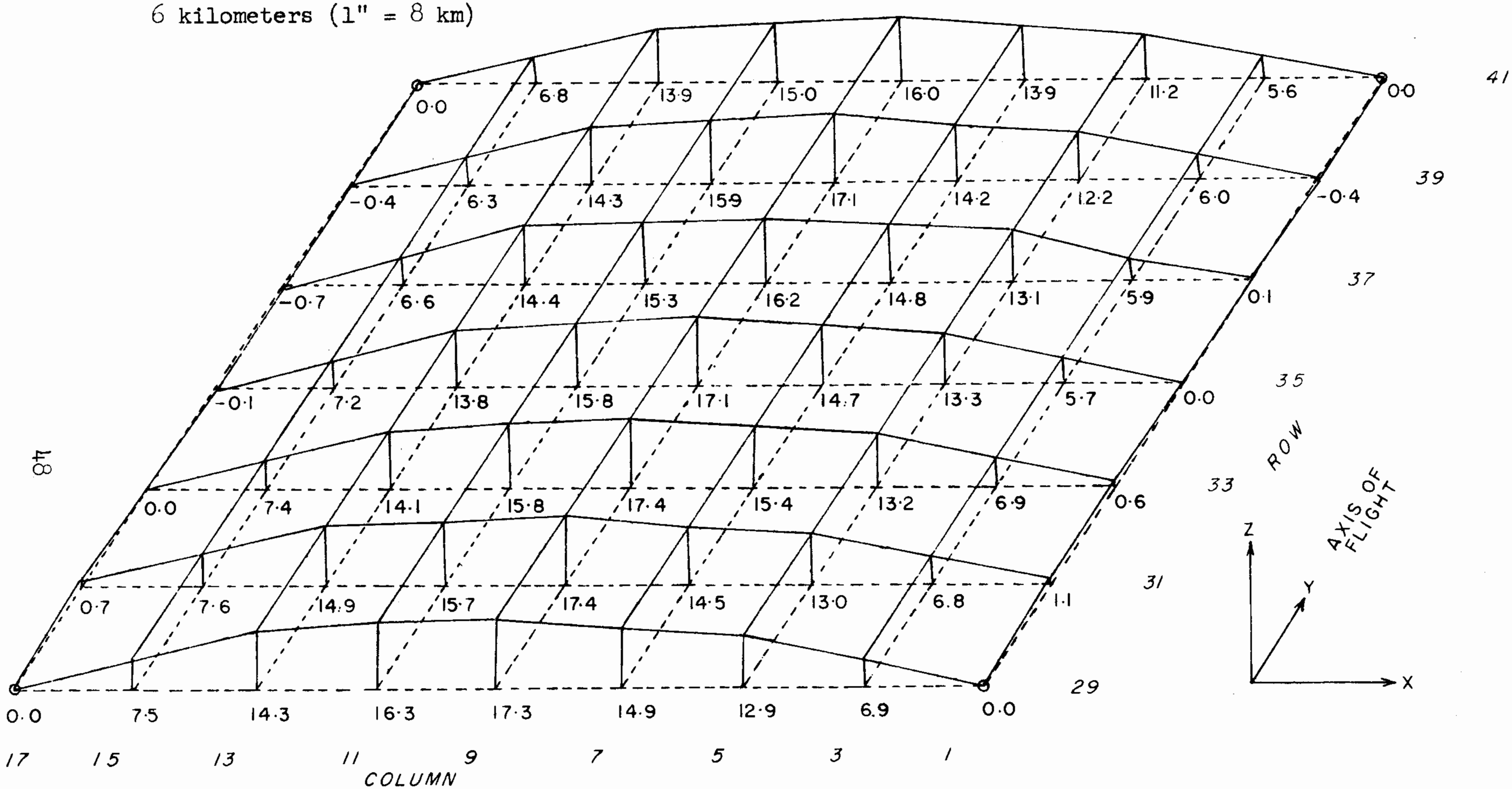


Figure 5. Case 2A
Residual Errors in Z after Adjustment

20% Sidelap/No Horizon Control
Strips 1, 3, 5, 7

$\sigma_Z = \pm 11.9$ meters
$M_B = 1/66,000$

Distance between points:
6 kilometers (1" = 8 km)

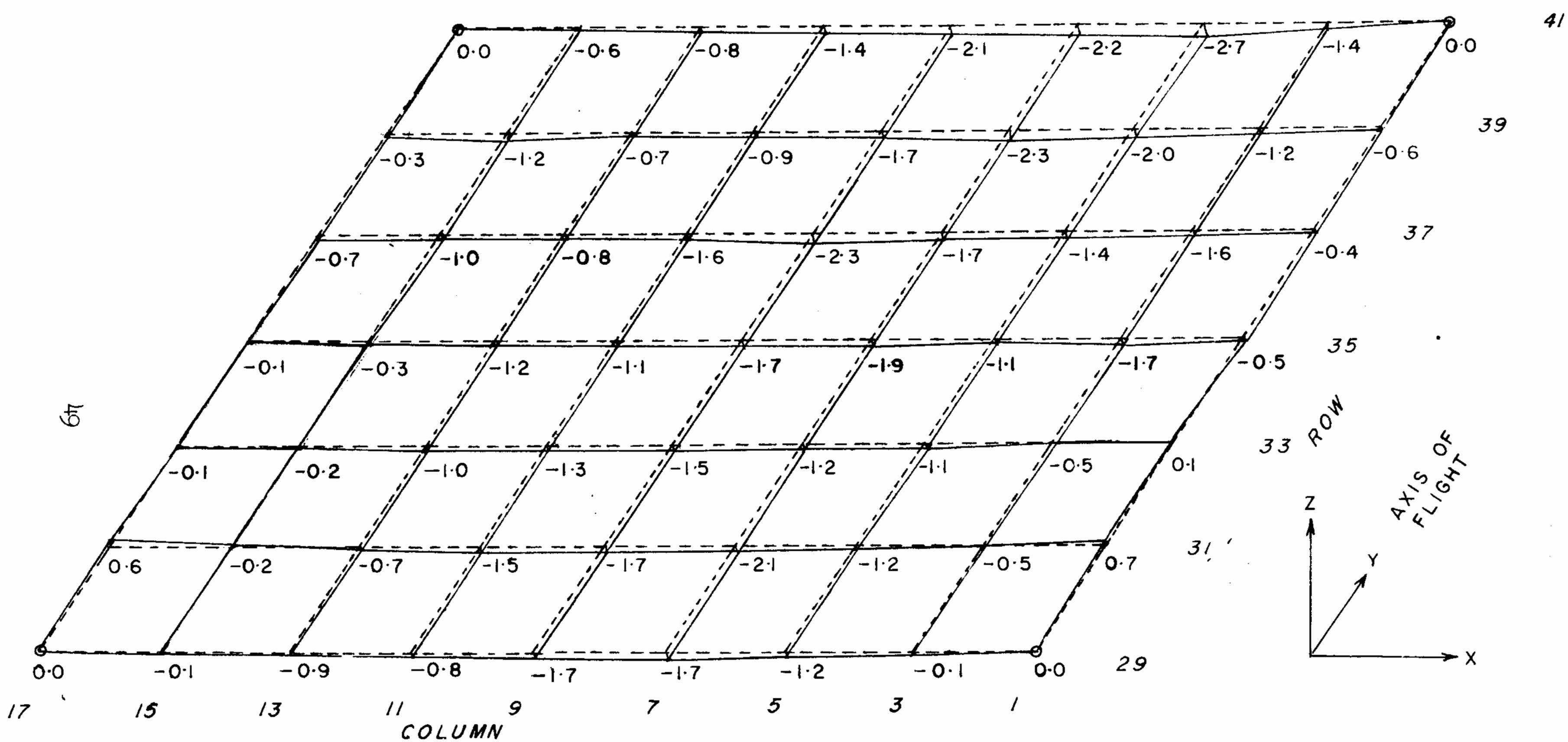


Figure 6. Case 2B
Residual Errors in Z after Adjustment

20% Sidelap/Horizon Control
Strips 1, 3, 5, 7

$\sigma_Z = \pm 1.3$ meters
$M_B = 1/66,000$

Associate Professor
and Participant
from
The Ohio State University

Date Feb. 15, 1968