TRIANGULAR ARRAYS:
ACTIVITIES AND NUMBER PATTERNS

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Teachers are always seeking activities that provide practice in computational skills (paper–and–pencil or calculator). When these activities lead to a search for number relationships, the student is doubly blessed. A series of such activities related to triangular arrays follows:

Activity 1

This triangular array was constructed using the consecutive natural numbers. The center or non–boundary number has been circled.

```
         1
       2   3
     4  (5)  6
   7   8   9   10
```

1) Find the sum of the nine numbers on the boundary of the triangular array; call this sum S. In this case the sum is 50.
2) The circled number for this triangular array is 5. Call the circled number I and find S ÷ I. Here, 50 ÷ 5 = 10.

Activity 2

Redo Activity 1 for a triangular array using the natural number multiples of 4.

```
          4
       8    12
    16  (20)  24
  28   32   36   40
```

1) The sum of the boundary numbers is 200.
2) The circled number is 20 and S ÷ I = 200 ÷ 20 or 10.

A teacher will understand her students well enough to know just how much to tell and how much to ask as she leads them to look for the ratio of outside to inside.

Activity 3

Redo Activity 2 for triangular arrays of the natural number multiples of 2, 5, and 3.

$S = 100$
$I = 10$
$S \div I = 10$

2
4 6
8 (10) 12
14 16 18 20

$S = 250$
$I = 25$
$S \div I = 250 \div 25 = 10$

5
10 15
20 (25) 30
35 40 45 50

$S = 150$
$I = 15$
$S \div I = 150 \div 15 = 10$

3
6 9
12 (15) 18
21 24 27 30

Activity 4

Construct other triangular arrays of the first ten multiples of other natural numbers. Is $S \div I$ always 10?

Activity 5

Construct a triangular array of the first 15 natural numbers and circle the three numbers which are not on the boundary.

1

2 3

4 5 6

7 8 9 10

11 12 13 14 15

1) Find the sum of the 12 numbers which lie on the boundary of the triangular array. Again call this sum S. In this case S = 98.

2) Find the sum of the three circled numbers; call this sum I. In this case, I = 22.

3) Find the ratio $\frac{S}{I}$. In this case $\frac{S}{I} = \frac{98}{22} = \frac{49}{11}$.

<u>Activity 6</u>

Redo Activity 5 using the triangular arrays formed by using the first 15 natural number multiples of 2, 3, and 5.

S = 196
I = 44
$\frac{S}{I} = \frac{196}{44} = \frac{49}{11}$

2

4 6

8 10 12

4 16 18 20

22 24 26 28 30

S = 294
I = 66
$\frac{S}{I} = \frac{294}{66} = \frac{49}{11}$

3

6 9

12 15 18

21 24 27 30

33 36 39 42 45

S = 490
I = 110
$\frac{S}{I} = \frac{490}{110} = \frac{49}{11}$

5

10 15

20 25 30

35 40 45 50

55 60 65 70 75

Activity 7

Construct other triangular arrays of the first 15 natural number multiples of 6, 7, 8, and 9. Is $\frac{S}{I}$ always $\frac{49}{11}$?

Challenge:

Make larger triangular arrays of the multiples of the natural numbers. Circle the non–boundary numbers and find S ÷ I. Does a constant ratio emerge for all triangular arrays of a given size?

Note to Algebra Teachers:

A triangular array of ten consecutive integers may be written as follows:

$$\begin{array}{ccccccc} & & & x & & & \\ & & x+1 & & x+2 & & \\ & x+3 & & x+4 & & x+5 & \\ x+6 & & x+7 & & x+8 & & x+9 \end{array}$$

S = sum of boundary numbers = 9x+41
I = interior number = x+4

The ratio of S to I is (9x+41)/(x+4). When x = 1, the result is 50/5 = 10. If every number in the array is multiplied by y, the ratio becomes 50y/5y = 10, and so the ratio remains constant. Notice that a triangular array of consecutive integers beginning with a number other than 1 will have a ratio different from 50/5 (e.g. if x = 2, (9x+41)/(x+4) = 59/6), and that ratio will be constant for arrays which are multiples of this one.