A STATISTICAL INVESTIGATION OF CONSECUTIVE SHOTS IN BASKETBALL

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The NCTM *Curriculum and Evaluation Standards for School Mathematics* emphasizes the importance of connections between mathematics and the "real world". Statistics supplies a diverse collection of such connections, many of great interest to students and teachers. We shall present one from the sports world, namely basketball.

Basketball announcers are fond of describing "hot shooting streaks". If a player makes several consecutive field goals, these announcers are apt to describe this player as having a "hot hand" and to urge that he take several more shots before he "cools off". Many coaches and fans agree with this analysis.

There are, no doubt, instances in which a player's coordination, adrenaline, or motivation are at a particularly high level for a span of time. At these times it is possible that a player's shooting accuracy may be enhanced and that a wise team would let this player take most of the shots. However, at other times a succession of successful shots may be due to chance alone and is not an indication of an unusual physical acuity at that moment. Our purpose is to identify the likelihood of "hot streaks" that are due to chance alone.

Suppose that Russ is an NBA player whose probability of success for any given shot is 50%. Specifically let us arbitrarily assume that a series of shots taken by Russ are independent; that is, the result of one shot has no effect whatsoever on the probability for the next shot.

Suppose that Russ takes ten shots in a game. What is the likelihood that Russ will make at least three in a row, at least four in a row, or at least five in a row during these ten shots?

To answer this question, we generated all possible 10-shot games for Russ. Since each of the ten shots can result in a success or a failure, there are altogether 2^{10} or 1024 possible 10-shot outcome sequences. The extreme cases would be ten consecutive successes (SSSSSSSSSS) and ten consecutive failures (FFFFFFFFFF). One of the 1022 "intermediate" cases is SFFSSSFFSS.

Since Russ has a 50% chance of success with each shot, all 1024 of these possible outcomes are equally likely. The question then becomes: In how many of the 1024 outcomes are there given numbers of consecutive successes? When we examined all 1024 possible sequences, we found the following:

–In 520 (50.8%) of the 1024 possible sequences, Russ would make at least three shots in a row at least once.

–In 251 (24.5%) of the 1024 possible sequences, Russ would make at least four in a row at least once.

–In 112 (10.9%) of the 1024 possible sequences, Russ would make at least five in a row.

We conclude that at least three consecutive successful shots, an occurrence which would get announcers very excited, will happen in over one-half of the games for Russ by chance alone. Streaks of at least four shots occur in almost one-fourth of Russ' games by chance alone. Even a five or more shot streak is not that surprising in terms of chance alone (10.8%).

Announcers also become excited about "cold spells" in which Russ might miss several consecutive shots. Because we assumed the probability of success was 50% on each shot, the probability of a cold streak of a given length is the same as the probability of a hot streak of the same given length.

<u>Challenges for the reader:</u>

1. Increase the number of shots that Russ takes from 10 to 11, 12, 13... and calculate again the probabilities of streaks of different lengths.

2. Change the probabilities of Russ making a given shot to an arbitrary p; the probability of failure on a given shot is then $1-p$. The 1024 separate outcomes are no longer equally likely. The probability, for instance, of the outcome SSSFFSFSFS is $ppp(1-p)(1-p)p(1-p)p(1-p)p$ or $p^6(1-p)^4$; in fact, the probability of any series of ten shots involving 6 successes and 4 failures is $p^6(1-p)^4$. Use this new pattern to analyze again the "streak" question.

3. Apply this same process to compute the probability that a baseball player has various numbers of consecutive hits by chance alone.