

PREPARING STUDENTS TO SOLVE PROBLEMS

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Students often seem to pay little attention to the meaning of the words in story problems. Instead they apply the rule: "Take the two numbers and use the most recently studied operation." This comes less from laziness on the part of the student than from efficiency — much of the time, especially early in the learning process when the problem solving habits are first being developed, this rule works. Routine textbook problems can encourage the use of this rule as can teachers who want to make it "easier" for students to learn by giving them only simple story problems (Nesher, 1980; Lesh, 1981).

To help students decide which operation to use in a problem, teachers often resort to teaching them to look for "key words". Unfortunately, although this may help in the short term, it is harmful in the long term because (1) it does not work very well in more complex problems and (2) it discourages students from analyzing the problem as a whole.

As teachers we want to look beyond the immediate goal of getting students to solve routine one-step problems to the goal of teaching students to solve the types of problems the "real world" will present them, problems that typically are not neatly packaged with all the information, and only the information, necessary to solve them. Two things that teachers can do to help students develop problem solving skills are (1) help students to develop a feel for mathematics and (2) get students involved in the problems they are attempting to solve.

Developing a Feel for Mathematics

The best problem solvers are intuitive; they have developed an understanding of the interconnectedness of mathematical concepts that allows them to avoid many blind alleys in solving a problem and to make productive connections between what is known in a problem and what is wanted (Bruner, 1960; Westcott, 1968). Students who have learned their computational skills rote and at an abstract level develop little feeling for the operations. Students who have developed their computational skills through repeated experiences with concrete materials are more likely to develop an understanding of the operations and the relationships

among the operations (Suydam, 1986; Kennedy, 1986; Moser, 1986). To these students an addition problem will feel like an addition problem because they have gone through the physical actions of addition many times.

We need to help students make meaningful connections among mathematical ideas. If students understand, at a level appropriate to their intellectual development, the basic operations and the basic concepts of measurement and geometry that help them relate mathematics to the world around them, then they will have a better chance of applying mathematics successfully in their lives.

Involving Students in Problems

Assuming that our students have the prerequisite knowledge and skills in mathematics, we then want to actively engage them in applications of mathematics. Many of these applications can be presented in story problems, but the problems must be designed to capture the complete attention of the students. The following are a few of the many ways that we can encourage our students to become involved in the problem.

1. Develop classroom math projects. Planning a party, taking a poll, and running a classroom store all involve students in "real world" mathematics. Many holiday activities can be planned to incorporate as much math as possible; for example, Christmas preparations can include making and wrapping gifts, making ornaments and other decorations, and making Christmas cookies, all of which involve geometry and measurement.
2. Use story problems that relate to class activities like those described above or to activities the class is involved with in other areas of the curriculum. If students are studying dinosaurs, then create some story problems that involve dinosaurs.
3. Read a story that involves math problems.
4. Read any story and create math problems that use characters and ideas from the story.
5. Put up a picture or poster and create problems that relate to it. For example, put up a Garfield poster and create problems that have students calculate how much lasagne he eats per week, how many calories he burns

while chasing Odie, or how many hours of sleep he gets.

6. Show a brief excerpt from a movie or TV show, then have story problems that relate to the clip. Anything from the daily life math problems of Smurfs to problems inspired by the swashbuckling lifestyle of Teenage Mutant Ninja Turtles can add interest to a set of problems.
7. Create problems that relate to materials you bring into the classroom, such as restaurant menus (ask to take one home and then photocopy it for your class), empty food containers, newspaper ads, or old catalogs (have students bring them in or check stores at the end of a season for leftover catalogs).
8. Find several related pictures, create a problem based on each picture, and then have students match the problems with the appropriate pictures. Old calendars are a good source of pictures, as are all kinds of magazines. (Note that the picture does not even have to be closely related to the math content of the problem. What is important is that there be some information in the problem that connects it to the picture and therefore the student must read the problem. For example, you could have pictures of three different kinds of cars, or three pictures of Porsches with different backgrounds, and there would be some information in each problem that would connect it to one of the pictures.)
9. Have students draw pictures to illustrate problems. (Once again, the picture need not relate directly to the math content of the problem.)
10. Have students supply the question for a problem. For example, supply a question for the following: "Bill celebrated his twelfth birthday on February 6. His grandmother gave him \$20.00 for his birthday, and one of his aunts gave him \$10.00. Bill collects penguins; his sister gave him a penguin knickknack, but it was chipped, so he took it back to Lazarus. The store had several of the penguin knickknacks left; the price for each was \$6.95. Bill also saw a small Gund penguin for \$12.95 and a larger Gund penguin for \$19.95."
11. Have students act out problems. Bring in props to fit the problems or make up problems that use the classroom as the setting.
12. Have a "problem of the day". There are many books on problem solving that can provide challenging nonroutine problems for a variety of grade levels.

For example, the *Problem-Solving Skill Sheets* and other TOPS materials provide a variety of nonroutine problems that relate to many areas of the mathematics curriculum.

13. Frequently use problems that have extraneous information and occasionally use problems that have insufficient information.
14. Put the same numbers into multiple problems involving different operations.
15. Give students problems with blanks where the numbers would be. Give the students the numbers and have them put the numbers into the blanks. For example: "The Sears tower is _____ meters tall, and the Empire State Building is _____ meters tall. How much taller is the Sears tower than the Empire State Building? 381,442."
16. Have students write their own problems about themselves and their friends. Other students are likely to be interested in solving them, and students learn a great deal about problems as they try to write good ones.

For some of these suggestions (e.g., having students create their own problems or using problems with extraneous information) there is a temptation to say, "That is too difficult for my students." However, that is a good reason for students to do more of that kind of activity, not less. The real world will not hand our students neatly arranged textbook problems, so we must go beyond the use of such problems in our classes.

We must not lose sight of the reason we want our students to learn mathematics: so that they can apply it to the real world. What have we accomplished if our students are skilled in calculation, but cannot use those skills in their lives because they have not developed effective problem solving skills?

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Alaskan cryptarithms revisited:

Carl A. Bevington, Conneaut, OH, notes that the Alaskan cryptarithm $CAN + YOU = SEE$ (Winter, 1991) has 528 solutions, $HOW + COME = RICK$ has 122 and six if $RICK$ must be prime (the Editors certify that Rick is indeed prime.) Carl counts solutions for the others as well and adds some for you to try:

<u>WRONG</u>	<u>FINE</u>	<u>FLY</u>	<u>TELL</u>	<u>WAS</u>
<u>WRONG</u>	<u>TOYS</u>	<u>FOR</u>	<u>THE</u>	<u>THAT</u>
<u>RIGHT</u>	<u>TODAY</u>	<u>YOUR</u>	<u>WHOLE</u>	<u>ALL</u>
		<u>LIFE</u>	<u>TRUTH</u>	<u>RIGHT</u>

And a few more cryptarithms from the Anchorage, AK, teachers for Carl and others to ponder:

<u>CAN</u>	<u>CATS</u>	<u>SKI</u>	<u>NO</u>
<u>SHE</u>	<u>DOGS</u>	<u>TO</u>	<u>TO</u>
<u>DOIT</u>	<u>FIGHT</u>	<u>DIE</u>	<u>YOU</u>
Chuck Strauss	Judy Jeffrey	Linda Smith	Twyla Mundy