Reports of the Department of Geodetic Science

Report No. 95

COMPUTATION OF MEAN ANOMALIES OF 1°x1° BLOCKS

by

Urho A. Uotila

Prepared for

Chart Research Division
Aeronautical Chart and Information Center
Second and Arsenal Streets
St. Louis, Missouri 63118

Contract No. F23601-67-C-0125
OSURF Project No. 2353

The Ohio State University
Research Foundation
Columbus, Ohio 43212

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(This series is a continuation of the reports of the
Institute of Geodesy, Photogrammetry, and Cartography)

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FOREWORD

This report was prepared by Dr. Urho A. Uotila, Professor, Department of Geodetic Science, The Ohio State University, under Air Force Contract No. F23601-67-C-0125, OSURF Project No. 2353, which is supervised by Dr. Urho A. Uotila. The contract covering this work is administered by Aeronautical Chart and Information Center, Second and Arsenal Streets, St. Louis, Missouri 63118.

ABSTRACT

In this investigation 15 different methods of the computation of mean anomalies of 1° x 1° blocks were examined and compared in a limited test area. The purpose of the study was to find an economical method of the computation of mean anomalies for the blocks in which some observed gravity values were given with corresponding location and elevation information. The mean elevation of the 1° x 1° block was also assumed to be known. As a result of the tests, five out of 15 techniques were found to be usable for the prediction and they were about equal in accuracy. The economy of the computations was also considered in the final recommendation about the best technique to be used.

The effect of uncertainty in b-coefficient on predicted anomalies was examined. A simple formula for estimating the effect was given.

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COMPUTATION OF MEAN ANOMALIES OF 1° x 1° BLOCKS

1. Introduction

As it is well known, gravity anomalies are input data for many kinds of computations and analyses. Depending on our task, we are interested in a local gravity field or a more general field. Numerous studies have been made about the interpolation techniques in local fields as well as in more general fields. There are three important factors in selecting an approach to be used in the interpolation:

(a) the size of an area to be analyzed,

(b) the density of the observed gravity points in the area,

(c) other information available in the area.

In the case that we are interested in a relatively small area, usually we have much gravity data as well as supporting information available; and we can select a quite sophisticated method of interpolation. The particular method might not be feasible for utilization over a large area because nonavailability of the necessary extensive gravity data and supporting information or because the method itself is not economical for large-scale computations. In this case we must try to find a method which does not necessarily give the same theoretical accuracy, but which is a reasonable method for large-scale computations.

In this investigation we are limiting ourselves to the problem of computation of mean anomalies of 1° x 1° blocks in the world-wide scale. It means

that we do not have extensive additional information available. We assume that we have available observed gravity values, corresponding locations and elevations. We also assume that we know the mean elevation of $1^\circ \times 1^\circ$ and $5^\circ \times 5^\circ$ blocks where necessary.

2. Computational Methods

2.1 Average Values of Point Anomalies

The simplest method of obtaining a free-air anomaly for the area is to compute an arithmetic mean value of free-air anomalies. It is natural that the value which we obtain through averaging is a representative value for the area only if we have a dense network of evenly-distributed free-air anomalies over the whole area. It is difficult to define what is meant with a dense network in general terms. Through knowledge that the free-air anomalies are correlated with the local topography and the rock density contrasts, the sufficient dense network can be determined only after an examination of the local situation. In the majority of the cases we do not have sufficient coverage of observed values for an area and we do not have a nice, even distribution of the points; therefore, the usefulness of this method is almost zero. For identification purposes, let's call this technique "Method A."

In Method A we did not consider the well-known correlation between free-air anomalies and local topography at all. The next method, which is one step better than Method A is to reduce all free-air anomalies to the mean elevation of the block for which a mean anomaly is sought using either the

standard or a regional or a local b-coefficient and then computing average value of these reduced values. The corresponding formula is:

$$\overline{\Delta g} = \frac{1}{n} \Sigma [\Delta g_i - b (h_i - h_m)]$$

where

Δg_i = observed free-air anomaly at point i,

h_i = elevation of point i,

i = 1, 2, $\cdots$, n,

h_m = mean elevation of the block in question,

$\overline{\Delta g}$ = mean anomaly of the block.

If we want a mean anomaly for 1° x 1° block, we use the mean elevation of the 1° x 1° block in the above formula. Let's call this technique "Method B" for identification purposes.

2.2 Average Values of Subblocks

In Methods A and B we did not consider at all the distribution of the horizontal locations of the points. This can be considered in some sense very easily by computing average values for the subblocks, such as 0°.5 x 0°.5, 10' x 10', or 5' x 5' blocks and then using Method B for the computation of a mean anomaly. For h_m in this case we must use the mean elevation of 1° x 1° block. After we have mean values of subblocks, we average them in order to get a mean value of the 1° x 1° block in question. Let's call this technique "Method C."

Under Method C we can use several different techniques. First of all, we can use different subblock sizes in the computations. We can also divide Method C into two different groups depending on what type b-coefficient was used in the computations:

(a) We preselect b-coefficient for the area,

(b) We use the least-squares solution for solving b-coefficient for the $1^\circ \times 1^\circ$ square and mean values for subsquares simultaneously as explained in the other report [Uotila, 1967].

2.3 Surface-Fitting Techniques

All of the methods described above have one common deficiency. It is clear that in Methods A and B the horizontal locations of the points are not taken into consideration; therefore, some local areas usually get too much weight in the representation of the mean anomaly of the block. In Method C some of this weakness has been removed; however, if we do not have observed gravity values in all subblocks, empty cells do not have any representation in the mean value. In order to improve Method C we must predict gravity anomalies for the empty subblocks and then compute average value for the whole block utilizing predicted values and computed ones.

It is a well-known fact that simple surfaces, such as a plane, can be used very well in the prediction, if the points are close to the point for which gravity anomaly is to be predicted [Moritz, 1962]. However, it is a tremendous task to have a separate surface fitting for each area where the anomaly is needed.

In order to avoid extensive computation, the point values might be replaced by average Bouguer anomalies of subblocks. This means that we compute the average Bouguer anomalies for each subblock for which we have observations (b-value can have the standard value or any other adopted value). With some loss in accuracy these average values of subblocks can be considered as point values. Instead of predicting the average anomaly for each subblock separately through surface fitting, a surface can be fitted through existing mean anomalies in a $1^\circ \times 1^\circ$ block using a least-squares solution. By utilizing numerical values and functional relations used in the solution, the missing mean anomalies of subblocks in the $1^\circ \times 1^\circ$ block can be predicted. The mean anomaly of the $1^\circ \times 1^\circ$ block is then computed using predicted mean anomalies of the subblocks and the computed ones.

We selected four mathematical models for the prediction of missing average anomalies of subblocks. They are as follows:

$$(1)\quad \underline{\Delta g}_j^s = \underline{x}_1 + \Delta\varphi_j \underline{x}_2 + \Delta\lambda_j \underline{x}_3 + \Delta\varphi_j \Delta\lambda_j \underline{x}_4 + \Delta\varphi_j^2 \underline{x}_5 + \Delta\lambda_j^2 \underline{x}_6 + (h_j - h_m)\, \underline{x}_7$$

$$(2)\quad \underline{\Delta g}_j^s = \underline{x}_1 + \Delta\varphi_j \underline{x}_2 + \Delta\lambda_j \underline{x}_3 + \Delta\varphi_j \Delta\lambda_j \underline{x}_4 + \Delta\varphi_j^2 \underline{x}_5 + \Delta\lambda_j^2 \underline{x}_6$$

$$(3)\quad \underline{\Delta g}_j^s = \underline{x}_1 + \Delta\varphi_j \underline{x}_2 + \Delta\lambda_j \underline{x}_3 + \Delta\varphi_j \Delta\lambda_j \underline{x}_4$$

$$(4)\quad \underline{\Delta g}_j^s = \underline{x}_1 + \Delta\varphi_j \underline{x}_2 + \Delta\lambda \underline{x}_3$$

For the practical least-squares computation, we used the following steps and formulas:

(a) Compute mean Bouguer anomalies using preselected b-values for each subblock with any gravity observations in the 1° x 1° block

$$\hat{\Delta g}_j^B = \frac{1}{n} \sum_{i=1}^{n} \Delta g_i^B \quad , \quad \hat{h}_j = \frac{1}{n} \sum_{i=1}^{n} h_i$$

where

$i = 1, 2, 3, \cdots, n,$

n = number of points in subblock j,

h_i = elevation of point i in subblock j .

(b) Compute for each subblock j the values for $\Delta\varphi_j$ and $\Delta\lambda_j$ using the following formulas:

$$\Delta\varphi_j = \varphi_m - \varphi_j$$

$$\Delta\lambda_j = \lambda_m - \lambda_j$$

where

φ_m = the latitude of the central parallel of the 1° x 1° block in question,

λ_m = the longitude of the central meridian of the 1° x 1° block,

φ_j = the latitude of the central parallel of the subblock j,

λ_j = the longitude of the central meridian of the subblock j.

(c) Compute:

$$\Delta g_m = \frac{1}{k} \sum_{j=1}^{k} \hat{\Delta g}_j^B$$

where

$j = 1, 2, \cdots, k$,

k = number of subblocks having gravity observations in the $1^\circ \times 1^\circ$ block .

The given mean elevation of the $1^\circ \times 1^\circ$ block is h_m .

(d) Form the following matrices:

$$A = \begin{bmatrix} 1 & \Delta\varphi_1 & \Delta\lambda_1 & \Delta\varphi_1\Delta\lambda_1 & \Delta\varphi_1^2 & \Delta\lambda_1^2 & h_m - \hat{h}_1 \\ 1 & \Delta\varphi_2 & \Delta\lambda_2 & \Delta\varphi_2\Delta\lambda_2 & \Delta\varphi_2^2 & \Delta\lambda_2^2 & h_m - \hat{h}_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \Delta\varphi_j & \Delta\lambda_j & \Delta\varphi_j\Delta\lambda_j & \Delta\varphi_j^2 & \Delta\lambda_j^2 & h_m - \hat{h}_j \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \Delta\varphi_k & \Delta\lambda_k & \Delta\varphi_k\Delta\lambda_k & \Delta\varphi_k^2 & \Delta\lambda_k^2 & h_m - \hat{h}_k \end{bmatrix}$$

$$L = \begin{bmatrix} \Delta g_m - \hat{\Delta g}_1^B \\ \Delta g_m - \hat{\Delta g}_2^B \\ \vdots \\ \Delta g_m - \hat{\Delta g}_j^B \\ \vdots \\ \Delta g_m - \hat{\Delta g}_k^B \end{bmatrix} \qquad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix}$$

$$N = A'A \quad , \quad U = A'L$$

(e) The solution vector is obtained from the following relation:

$$X = -N^{-1} U$$

and other usable quantities as follows:

$$V'V = L'L + X'U ,$$

$$m_o^2 = \frac{V'V}{k-u}$$

$$\Sigma_x = m_o^2 N^{-1} ,$$

where

u = number of unknown or number of elements in X vector,

Σ_x = variance-covariance matrix of unknown parameters.

(f) The mean anomaly of the 1° x 1° block is obtained from the following formula:

$$\overline{\Delta g_M} = \frac{1}{s} GX + \Delta g_m + b h_m$$

where

$$G = \left[k ; \sum_{j=1}^{k} \Delta\varphi_j ; \sum_{j=1}^{k} \Delta\lambda_j ; \sum_{j=1}^{k} \Delta\varphi_j \Delta\lambda_j ; \right.$$

$$\left. \sum_{j=1}^{k} \Delta\varphi_j^2 ; \sum_{j=1}^{k} \Delta\lambda_j^2 ; \sum_{j=1}^{k} (h_m - \hat{h}_j) \right]$$

s = total number of subblocks in the 1° x 1° block .

Sample variance of the predicted mean anomaly can be obtained from the following formula:

$$m^2_{\overline{\Delta g_M}} = G \Sigma_x G' .$$

We realize that m_o^2 obtained through the least-squares solution is a function of known mean anomalies and number of the subblocks with the known mean anomalies. If we had only seven subblocks with known anomalies, m_o^2 would be zero as a result of the computations. Therefore, a more realistic value for the sample variance of a predicted mean anomaly can be obtained by computing new Σ_x^* which is based on sample variance of unit weight computed from a large number of samples. The variance-covariance matrix for unknown parameters is computed through the following formula:

$$\Sigma_x^* = m_o^{*2} N^{-1} \quad .$$

Now $m^2_{\overline{\Delta g}_M} = G\Sigma_x^* G'$. The sample variance of unit weight m_o^{*2} is, of course, a function of the area in question and it should be determined separately for each area.

The computational formulas given above have been derived for our model No. 1. It is obvious that the same formulas can be used for the other cases omitting the unknowns which do not exist in the system and making proper modifications in the dimensions of the matrices in question. For identification purposes, let's call this technique "Method D" with the subscripts 1, 2, 3, and 4 identifying the respective mathematical models given above.

2.4 Least-Squares Prediction

Moritz has derived an excellent theory for prediction of point and mean anomalies utilizing correlation matrices [Moritz, 1962]. Rapp has made fast computer programs to utilize Moritz's formulas [Rapp, 1964] and he has made

predictions for mean free-air anomalies of 1° x 1° blocks for the large areas [Rapp, 1966]. The basic formulas for the least-squares predictions are [Moritz, 1962].

$$\Delta g_p = [C_{p1}, C_{p2}, C_{p3}, \cdots, C_{pn}] \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1n} \\ C_{21} & C_{22} & \cdots & C_{2n} \\ \vdots & \vdots & & \vdots \\ C_{n1} & C_{n2} & \cdots & C_{nn} \end{bmatrix}^{-1} \begin{bmatrix} \Delta g_1 \\ \Delta g_2 \\ \vdots \\ \Delta g_n \end{bmatrix}$$

$$m_p^2 = C_o - [C_{p1}, C_{p2}, \cdots C_{pn}] \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1n} \\ C_{21} & C_{22} & \cdots & C_{2n} \\ \vdots & \vdots & & \vdots \\ C_{n1} & C_{n2} & \cdots & C_{nn} \end{bmatrix}^{-1} \begin{bmatrix} C_{p1} \\ C_{p2} \\ \vdots \\ C_{pn} \end{bmatrix}$$

where

Δg_i = given gravity anomaly ,

$C_o = C_{11} = C_{22}$ = variance of gravity anomalies ,

C_{ik} = covariance of gravity anomalies located at points i and k .

The above formulas are very good when used with anomalies which are not correlated with elevations. Moritz [1962] also developed procedures which include elevation correlations; however, practical computations are not economical for the purpose discussed here. Even the method without elevation correlations requires the computation of distances between the given points and the predicted points in all combinations as well as the evaluation of correlation functions for each distance. In order to make the program more economical,

we felt that mean anomalies of 5' x 5' subblocks (or of another size) can be used. In this case, we must compute the distances and evaluate a correlation function only once for a block in a latitudinal zone. We might modify our procedure one more step without losing much more accuracy by assuming that the 1° x 1° blocks are rectangular instead of trapezoidal. This way we decrease the number of covariances considerably. For example, if we use 10' x 10' blocks and 80' x 80' area, we have a maximum of 64 covariances. Prediction is done only for those squares without any observations. A mean anomaly of the 1° x 1° block is computed as an average of predicted anomalies and given subblock anomalies at the mean elevation.

If we assume that mean anomalies of subblocks computed from observed values are without errors, we get a variance-covariance matrix of predicted mean anomalies of subblocks from the following formula:

$$\Sigma_{\Delta g_p} = \begin{bmatrix} C_0 & C_{P_1P_2} & C_{P_1P_3} & \cdots & C_{P_1P_k} \\ C_{P_2P_1} & C_0 & C_{P_2P_3} & \cdots & C_{P_2P_k} \\ \vdots & \vdots & \vdots & & \vdots \\ C_{P_kP_1} & C_{P_kP_2} & C_{P_kP_3} & \cdots & C_0 \end{bmatrix} -$$

$$\begin{bmatrix} C_{P_11} & C_{P_12} & \cdots & C_{P_1n} \\ C_{P_21} & C_{P_22} & \cdots & C_{P_2n} \\ \vdots & \vdots & & \vdots \\ C_{P_k1} & C_{P_k2} & \cdots & C_{P_kn} \end{bmatrix} \begin{bmatrix} C_0 & C_{12} & C_{13} & \cdots & C_{1n} \\ C_{21} & C_0 & C_{23} & \cdots & C_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ C_{n1} & C_{n2} & C_{n3} & \cdots & C_0 \end{bmatrix}^{-1} \begin{bmatrix} C_{P_11} & C_{P_21} & \cdots & C_{P_k1} \\ C_{P_12} & C_{P_22} & \cdots & C_{P_k2} \\ \vdots & \vdots & & \vdots \\ C_{P_1n} & C_{P_2n} & \cdots & C_{P_kn} \end{bmatrix}$$

where C_{ij} are covariance between points i and j, k is total number of subblocks for which mean anomaly has been predicted, and n is the number of subblocks for which we had the mean anomaly in the 1° x 1° block. The total number of the subblocks in the block for which mean anomaly is predicted is n + k and the variance of the mean anomaly is:

$$m^2_{\Delta g} = \frac{1}{(n+k)^2} \sum_{i=1}^{k} \sum_{j=1}^{k} \sigma_{ij} \quad .$$

where σ_{ij} are the element of $\Sigma_{\Delta g_p}$ - matrix. These formulas as well as the earlier ones can be reformulated utilizing correlation coefficients instead of covariances. As the result of our experience, we consider it preferable to use correlation coefficients and scale the results with the proper variance. In the final accuracy evaluation, the inaccuracies of the given mean anomalies of the subblocks must be taken into consideration.

When 5' x 5' subblocks are used, the mean anomaly is computed for each 0°.5 x 0°.5 subblock and the average of four of these gives the mean anomaly of 1° x 1° block. When 10' x 10' subblocks are used, 1° x 1° mean anomaly is predicted directly. For identification purposes, let's call the method in which 5' x 5' mean Bouguer anomalies are used "Method E_1" and the corresponding method using 10' x 10' subblocks "Method E_2" and the method used by Rapp [1966] "Method E_3."

3. Test Area

In order to compare the results of the different methods, a sample 5° x 5° block was selected for the test computations. The test area was

selected from a location where there was much gravity information available and considerable elevation differences existed in each 1° x 1° block. The selection of the test area was otherwise more or less random. The 1° x 1° blocks in the area were numbered from 1 to 25 for identification purposes. In Table 1 the number of the observed gravity points in each 1° x 1° block is given with minimum and maximum elevations of the observed points, average elevation of the observed points, and estimated mean elevation of 1° x 1° block. In order to obtain a better understanding about the effect of the distribution of observed points on the mean anomalies, we decided to form two subsets of this data. The first one which we call "60% data" was formed by dividing observed gravity points according to their horizontal location into 5' x 5' block subsets. In each 1° x 1° block we had 144 of this kind of subsets if we had observations in each 5' x 5' block. In case we did not have observed gravity values in each 5' x 5' block, we had less than 144 subsets. From these 144 or less subsets we selected randomly 60% of the 5' x 5' blocks. The observed gravity values which fell into the selected 5' x 5' blocks formed the so-called "60% data" set. We prepared also the so-called "30% data" set which was selected similarly as the "60% data" set except first we randomly selected one 0°.5 x 0°.5 block in each 1° x 1° block from which we did not take any data. Care was taken that the "30% data" set was a subset of the "60% data" set. Similar information as given in Table 1 for the 100% data is given for the subsets "60% data" and "30% data" in Tables 2 and 3.

Table 1

Information About Test Area, 100% Data

1° x 1° Block No.	Number of Points	Elevation of the Points in Meters			Estimated Mean Elevation in Meters
		Minimum	Maximum	Average	
1	49	1417	2529	1817	2234
2	63	1295	2535	1847	2265
3	121	1462	2308	1740	2097
4	259	1544	2911	2014	2111
5	296	1716	3150	2287	2469
6	228	996	2415	1529	1973
7	280	1429	2650	1620	1823
8	702	1328	2034	1477	1501
9	222	1434	2148	1660	1889
10	280	1714	2914	2105	2470
11	322	860	2171	1147	1400
12	166	1259	2136	1369	1497
13	172	1318	1825	1456	1673
14	556	1372	2794	1784	1977
15	70	1802	2723	2156	2361
16	174	1461	2091	1723	1847
17	42	1285	1852	1417	1540
18	15	1287	1469	1351	1388
19	21	1312	2269	1718	2013
20	89	1890	2781	2202	2171
21	108	1275	1973	1750	1829
22	15	1239	1622	1291	1348
23	300	1241	1998	1500	1563
24	705	1282	2914	1512	2072
25	228	1550	3581	2448	2558

Table 2

Information About Test Area, 60% Data

1° x 1° Block No.	Number of Points	Elevation of the Points in Meters			Estimated Mean Elevation in Meters
		Minimum	Maximum	Average	
1	41	1417	2529	1789	2234
2	27	1326	2163	1799	2265
3	88	1465	2308	1726	2097
4	167	1544	2911	2046	2111
5	117	1911	2622	2306	2469
6	172	996	2415	1549	1973
7	167	1452	2519	1623	1823
8	481	1328	2034	1478	1501
9	119	1434	2148	1661	1889
10	152	1714	2914	2138	2470
11	187	860	2055	1131	1400
12	79	1259	1744	1353	1497
13	69	1318	1825	1455	1673
14	270	1372	2794	1781	1977
15	27	1829	2723	2184	2361
16	112	1465	2091	1742	1847
17	11	1286	1519	1372	1540
18	1	1300	1300	1300	1388
19	8	1312	2269	1846	2013
20	55	1890	2781	2169	2171
21	47	1649	1961	1796	1829
22	8	1239	1362	1276	1348
23	177	1241	1925	1515	1563
24	459	1282	2914	1491	2072
25	110	1289	3581	2396	2558

Table 3

Information About Test Area, 30% Data

1° x 1° Block No.	Number of Points	Elevation of the Points in Meters			Estimated Mean Elevation in Meters
		Minimum	Maximum	Average	
1	19	1417	2529	1794	2234
2	12	1703	2159	1967	2265
3	43	1465	2308	1716	2097
4	63	1544	2569	1984	2111
5	84	1911	2622	2320	2469
6	119	996	2236	1476	1973
7	65	1452	1812	1604	1823
8	204	1334	2034	1512	1501
9	81	1434	2148	1683	1889
10	81	1900	2914	2183	2470
11	51	1125	2055	1275	1400
12	42	1259	1609	1316	1497
13	44	1333	1635	1437	1673
14	62	1493	2248	1910	1977
15	24	1829	2723	2187	2361
16	64	1516	1841	1754	1847
17	7	1286	1519	1402	1540
18	1	1300	1300	1300	1388
19	2	1816	1817	1816	2013
20	26	1930	2330	2145	2171
21	22	1702	1961	1797	1829
22	4	1244	1362	1302	1348
23	82	1282	1790	1439	1563
24	223	1286	2298	1490	2072
25	63	1289	3086	2089	2558

4. Results of Predictions

Mean free-air anomalies were predicted for all 25 1° x 1° blocks in the test area using "100%, 60%, and 30% data" sets utilizing 15 different methods. Predictions were done only in the blocks where there was sufficient material available for the particular method and for comparison purposes. Wherever it was necessary to use b-coefficient in the prediction method we selected b-value = 0.1119 mgal/m for simplicity. The results of the computations are given in Table 4. The methods used in the computations are identified as follows:

Method A: Mean free-air anomalies computed as an average of free-air anomalies of points as explained in Section 2.1.

Method B: Mean free-air anomalies computed as an average of Bouguer anomalies of points at mean elevation of the block as explained in Section 2.1.

Method C_1: Mean free-air anomalies computed as an average value of mean Bouguer anomalies of 5' x 5' subblocks at mean elevation as explained in Section 2.2.

Method C_2: As C_1, but using 10' x 10' subblocks as explained in Section 2.2.

Method D_1^5: Mean free-air anomalies using the mathematical model 1 and 5' x 5' mean anomalies as explained in Section 2.3.

Method D_2^5: Mean free-air anomalies using the mathematical model 2 and 5' x 5' mean anomalies as explained in Section 2.3.

Table 4

Results From Test Computations

Block No.		1			2			3			4			5		
Method		100%	60%	30%	100%	60%	30%	100%	60%	30%	100%	60%	30%	100%	60%	30%
A	$\overline{\Delta g}$	-14.9	-18.0	-21.5	-11.0	-14.7	0.4	17.1	17.3	17.6	30.6	33.1	27.2	34.6	36.2	37.5
B	$\overline{\Delta g}$	31.8	31.7	27.8	35.8	37.4	33.7	57.0	58.8	60.2	41.4	40.3	41.5	54.9	54.4	54.2
C_1	$\overline{\Delta g}$	32.4	32.5	27.8	36.1	37.0	34.5	48.8	49.3	52.7	44.3	44.2	45.2	56.0	38.9	40.1
C_2	$\overline{\Delta g}$	32.2	31.5	29.1	35.9	36.9	34.5	45.2	44.6	49.5	46.2	44.4	44.7	56.6	38.7	40.4
D_1^5	$\overline{\Delta g}$	25.2			37.7	43.7	70.3	42.5	42.5	37.6	48.0	47.9	38.2	63.4		
D_2^5	$\overline{\Delta g}$	25.7			35.4	36.4	32.3	45.7	45.6	39.3	46.9	46.8	40.2	64.1		
D_3^5	$\overline{\Delta g}$	27.8			36.2	37.0	32.5	46.2	44.9	39.6	46.9	46.7	44.2	56.0		
D_4^5	$\overline{\Delta g}$	27.9			36.2	37.0	35.0	46.5	44.8	40.4	45.9	46.2	46.6	56.5		
	$m_{\overline{\Delta g}}$	1.5			1.8	2.5	4.1	2.0	2.1	3.2	1.7	1.9	2.7	2.0		
D_1^{10}	$\overline{\Delta g}$	25.4			37.0	39.2	61.5	42.4	41.2	37.3	46.9	46.4	37.6	61.8		
D_2^{10}	$\overline{\Delta g}$	25.4			35.4	36.9	32.9	45.3	44.2	39.0	46.6	45.8	42.0	62.1		
D_3^{10}	$\overline{\Delta g}$	27.1			35.8	36.9	33.4	44.5	43.5	40.1	46.6	45.7	42.2	57.0		
D_4^{10}	$\overline{\Delta g}$	28.3			35.9	36.9	35.5	44.6	43.0	48.9	46.7	45.5	45.9	57.2		
	$m_{\overline{\Delta g}}$	1.7			2.1	2.6	3.8	2.9	3.1	4.3	2.3	2.6	3.5	3.0		
E_1	$\overline{\Delta g}$				36.0	37.2		44.9	44.6		46.3	46.3	46.3	58.2		
	$m_{\overline{\Delta g}}$				0.8	1.3		0.6	0.9		0.6	0.7	1.5	1.0		
E_2	$\overline{\Delta g}$	29.7			35.7	37.7	36.1	44.6	43.6	43.5	46.5	45.8	44.7	58.2		
	$m_{\overline{\Delta g}}$	2.6			0.8	1.1	1.9	0.6	0.8	1.9	0.5	0.5	1.6	1.0		
E_3	$\overline{\Delta g}$	28.7			34.9	36.3	35.4	45.0	45.0	44.5	46.7	46.4	45.4	58.8		
	$m_{\overline{\Delta g}}$	5.4			2.2	2.8	3.7	1.9	2.0	3.9	1.7	1.7	3.0	2.6		

Table 4 (Continued)

Results From Test Computations

Block No.		6			7			8			9			10		
Method		100%	60%	30%	100%	60%	30%	100%	60%	30%	100%	60%	30%	100%	60%	30%
A	$\overline{\Delta g}$	8.6	8.5	9.0	14.9	13.4	16.1	12.0	12.3	17.2	13.6	12.5	14.0	14.5	21.9	24.4
B	$\overline{\Delta g}$	58.3	56.0	64.6	37.6	35.7	40.6	14.7	14.8	15.9	39.2	38.0	37.0	55.3	59.0	56.5
C_1	$\overline{\Delta g}$	49.1	44.6	57.5	31.2	29.7	40.0	15.7	16.1	17.9	34.2	13.3	16.7	48.8	51.5	45.2
C_2	$\overline{\Delta g}$	42.9	43.0	56.1	26.0	27.3	38.2	15.4	15.3	17.2	31.9	13.6	16.2	47.3	51.2	46.5
D_1^5	$\overline{\Delta g}$	36.4	35.9	40.6	28.8	28.5	10.7	15.7	16.7	17.1	29.9			41.2	42.4	31.4
D_2^5	$\overline{\Delta g}$	36.7	36.6	40.9	30.0	29.0	19.3	15.7	16.6	17.4	30.0			43.5	45.5	31.3
D_3^5	$\overline{\Delta g}$	36.3	35.8	36.5	31.9	31.8	31.9	15.8	16.8	18.0	30.2			44.5	47.5	48.5
D_4^5	$\overline{\Delta g}$	36.5	35.9	37.8	31.3	31.5	34.8	15.4	15.4	18.7	30.3			44.0	47.2	45.9
	$m_{\overline{\Delta g}}$	1.1	1.2	2.0	1.8	2.2	3.5	1.0	1.3	2.3	1.0			2.0	2.4	4.6
D_1^{10}	$\overline{\Delta g}$	37.2	37.4	42.9	28.4	29.6	14.4	15.5	15.4	16.6	30.0			42.8	43.9	38.5
D_2^{10}	$\overline{\Delta g}$	36.9	37.0	42.9	30.9	29.9	26.2	15.5	15.4	16.2	30.0			45.1	47.0	37.6
D_3^{10}	$\overline{\Delta g}$	36.2	36.2	38.5	32.2	32.3	32.1	15.6	15.5	17.0	30.0			44.6	47.4	48.7
D_4^{10}	$\overline{\Delta g}$	36.5	36.5	40.1	30.7	30.7	34.6	15.2	14.9	18.7	30.0			44.5	47.0	44.3
	$m_{\overline{\Delta g}}$	1.8	2.0	2.4	2.5	2.8	4.7	1.7	1.8	2.4	1.4			3.3	3.4	5.2
E_1	$\overline{\Delta g}$	36.3	36.4		30.9	30.9		15.0	15.4	15.9	30.3			46.0	47.5	
	$m_{\overline{\Delta g}}$	0.6	0.7		0.9	1.0		0.2	0.5	1.7	0.5			0.9	1.4	
E_2	$\overline{\Delta g}$	37.0	37.2	38.4	32.3	32.0	34.6	15.3	15.1	15.7	30.0			46.3	48.4	48.6
	$m_{\overline{\Delta g}}$	0.5	0.5	2.1	0.9	1.0	2.3	0.2	0.4	1.7	0.5			0.9	1.2	2.8
E_3	$\overline{\Delta g}$	36.0	36.2	38.3	31.2	31.2	33.7	14.6	15.2	16.0	30.1			44.9	47.0	49.7
	$m_{\overline{\Delta g}}$	1.9	1.6	3.5	2.2	2.2	4.7	1.3	1.3	3.3	1.6			2.2	2.9	5.4

Table 4 (Continued)

Results From Test Computations

Block No.		11			12			13			14			15		
Method		100%	60%	30%	100%	60%	30%	100%	60%	30%	100%	60%	30%	100%	60%	30%
A	$\overline{\Delta g}$	6.2	6.2	14.1	6.3	4.8	2.3	2.7	0.9	-2.7	9.1	9.3	8.7	13.0	16.8	16.2
B	$\overline{\Delta g}$	34.4	36.3	28.0	20.6	20.9	22.2	27.1	25.3	23.7	30.8	31.3	16.2	35.9	36.6	35.7
C_1	$\overline{\Delta g}$	25.1	28.1	19.6	17.2	4.1	3.3	23.2	2.0	-0.9	25.5	26.8	16.8	34.9	37.8	36.3
C_2	$\overline{\Delta g}$	18.0	22.3	14.8	16.8	5.2	5.1	21.2	3.1	-0.4	21.4	24.0	16.5	33.9	36.8	36.3
D_1^5	$\overline{\Delta g}$	19.0	18.9	20.2	18.6			22.8			23.2	24.2	23.8	33.5	26.1	32.7
D_2^5	$\overline{\Delta g}$	18.2	19.0	18.3	18.0			21.9			23.0	23.8	21.7	33.2	28.0	34.2
D_3^5	$\overline{\Delta g}$	19.2	20.0	20.7	17.9			22.1			22.8	24.0	20.7	34.3	33.1	32.6
D_4^5	$\overline{\Delta g}$	19.3	19.6	18.9	17.7			22.2			22.8	23.7	20.8	33.5	32.7	32.0
	$m_{\overline{\Delta g}}$	1.2	1.7	2.3	1.4			1.3			1.0	1.5	1.7	1.6	2.3	2.4
D_1^{10}	$\overline{\Delta g}$	17.5	18.3	20.6	17.9			21.4			21.9	23.3	22.5	34.2	36.1	38.8
D_2^{10}	$\overline{\Delta g}$	17.4	17.9	19.3	17.8			21.1			21.9	23.6	21.2	34.1	37.0	39.4
D_3^{10}	$\overline{\Delta g}$	17.4	18.3	20.4	17.6			21.2			21.8	23.5	20.3	34.6	33.3	33.0
D_4^{10}	$\overline{\Delta g}$	17.4	17.5	17.6	17.6			21.2			21.8	23.2	20.5	33.9	32.2	31.7
	$m_{\overline{\Delta g}}$	1.6	2.0	2.9	1.6			2.0			1.5	1.8	1.8	1.6	2.4	2.6
E_1	$\overline{\Delta g}$	18.5	18.1	17.3	17.9			21.3			23.0	22.9	22.6	35.1		
	$m_{\overline{\Delta g}}$	0.4	0.8	1.5	0.6			0.4			0.3	0.9	2.3	2.2		
E_2	$\overline{\Delta g}$	17.6	18.1	18.1	17.1			21.0			21.7	22.6	19.6	35.4	35.9	38.8
	$m_{\overline{\Delta g}}$	0.3	0.8	1.7	0.6			0.3			0.2	0.8	2.0	1.8	3.6	4.2
E_3	$\overline{\Delta g}$	18.2	18.1	17.3	17.9			21.0			23.1	22.9	22.4	34.9	32.1	33.5
	$m_{\overline{\Delta g}}$	1.5	1.8	2.9	1.8			1.6			1.5	1.8	3.8	4.6	7.1	7.9

Table 4 (Continued)

Results From Test Computations

Block No.		16			17			18			19			20		
Method		100%	60%	30%	100%	60%	30%	100%	60%	30%	100%	60%	30%	100%	60%	30%
A	$\overline{\Delta g}$	10.1	10.8	12.3	0.0	-1.0	-2.7	-10.7	-39.1	-39.1	-32.2	-24.3	-15.0	11.5	8.4	2.4
B	$\overline{\Delta g}$	23.9	22.6	22.6	13.8	17.8	12.7	- 6.6	-29.3	-29.3	0.7	- 5.6	7.0	8.0	8.7	5.3
C_1	$\overline{\Delta g}$	25.4	24.4	25.0	13.8	-1.7	-4.2	-12.2	-39.1	-39.1	- 0.7	-25.6	-15.0	9.3	3.4	0.3
C_2	$\overline{\Delta g}$	25.4	25.0	25.3	11.6	-2.3	-3.5	-16.6	-39.1	-39.1	3.8	-23.2	-15.0	10.6	4.7	4.0
D_1^5	$\overline{\Delta g}$	26.2	25.6	28.6	14.4			32.7			-10.6			7.6		
D_2^5	$\overline{\Delta g}$	25.7	25.3	28.9	16.6			33.1			- 7.9			10.6		
D_3^5	$\overline{\Delta g}$	25.6	25.1	27.3	14.0			34.7			- 1.2			11.0		
D_4^5	$\overline{\Delta g}$	25.6	25.3	25.8	15.5			-6.1			- 2.0			11.8		
	$m_{\overline{\Delta g}}$	1.0	1.4	1.8	2.0			9.2			2.8			2.1		
D_1^{10}	$\overline{\Delta g}$	25.5	25.1	28.3	13.3			73.9			- 9.0			8.0		
D_2^{10}	$\overline{\Delta g}$	25.4	25.3	28.8	13.9			80.3			- 3.0			10.0		
D_3^{10}	$\overline{\Delta g}$	25.4	25.3	27.3	12.8			50.6			0.1			10.7		
D_4^{10}	$\overline{\Delta g}$	25.4	25.3	25.8	14.0			-4.3			- 0.6			11.7		
	$m_{\overline{\Delta g}}$	1.3	1.5	2.0	2.5			12.2			3.9			2.3		
E_1	$\overline{\Delta g}$	25.5	25.4		10.6						5.0			11.2		
	$m_{\overline{\Delta g}}$	0.4	0.7		2.1						3.2			1.7		
E_2	$\overline{\Delta g}$	25.3	25.3	28.6	10.3			-3.9			4.0			10.6		
	$m_{\overline{\Delta g}}$	0.3	0.5	1.9	2.0			5.7			3.2			1.8		
E_3	$\overline{\Delta g}$	24.7	24.6	27.7	11.3			-8.7			6.0			11.5		
	$m_{\overline{\Delta g}}$	1.4	1.5	3.7	4.3			9.8			6.8			4.0		

Table 4 (Continued)

Results From Test Computations

Block No.		21			22			23			24			25		
Method		100%	60%	30%	100%	60%	30%	100%	60%	30%	100%	60%	30%	100%	60%	30%
A	$\overline{\Delta g}$	0.3	-1.4	-1.6	-18.0	-18.7	- 9.0	-10.6	-12.0	-22.0	-31.5	-31.3	-28.2	42.7	36.8	-1.2
B	$\overline{\Delta g}$	9.2	2.3	1.9	-11.5	-10.6	- 3.8	- 3.3	- 6.6	- 8.1	31.2	33.7	36.9	53.5	54.9	51.3
C_1	$\overline{\Delta g}$	11.3	-1.1	-0.8	-13.1	-19.8	- 8.7	- 0.1	- 2.0	- 0.9	21.1	25.1	26.0	52.3	52.1	46.3
C_2	$\overline{\Delta g}$	11.8	-0.2	0.5	-12.7	-23.8	- 8.9	1.1	0.0	4.0	17.4	20.4	25.4	50.8	48.4	45.7
D_1^5	$\overline{\Delta g}$	12.8			39.4			- 5.8	- 8.8	-139.8	16.4	19.7	24.1	53.2	55.6	41.8
D_2^5	$\overline{\Delta g}$	12.6			2.7			- 6.2	- 9.8	-104.6	16.2	18.5	23.3	47.6	49.0	45.7
D_3^5	$\overline{\Delta g}$	12.6			10.3			- 4.5	- 2.2	-28.9	15.7	17.4	20.2	48.7	49.6	44.6
D_4^5	$\overline{\Delta g}$	12.7			-11.3			1.9	1.8	-14.7	15.5	16.6	19.7	48.7	49.5	47.6
	$m_{\overline{\Delta g}}$	1.5			5.2			3.6	3.5	11.4	1.5	2.2	2.8	2.3	3.5	3.6
D_1^{10}	$\overline{\Delta g}$	11.7			-42.0			- 6.4	- 6.5	100.7	15.4	16.8	23.3	52.2	52.7	39.9
D_2^{10}	$\overline{\Delta g}$	11.8			-22.1			- 6.4	- 6.7	-211.3	15.5	16.9	21.7	48.2	46.9	43.6
D_3^{10}	$\overline{\Delta g}$	11.9			-14.7			- 7.9	- 5.9	-13.2	15.3	15.4	18.7	48.7	47.4	43.9
D_4^{10}	$\overline{\Delta g}$	12.0			-12.0			0.9	1.0	- 8.2	15.2	14.9	17.3	48.6	47.4	46.5
	$m_{\overline{\Delta g}}$	1.7			3.6			5.0	4.9	14.0	2.2	2.7	4.0	2.7	3.4	3.7
E_1	$\overline{\Delta g}$	12.6									15.2	17.6	23.2	50.5	50.8	
	$m_{\overline{\Delta g}}$	0.7									0.6	1.6	2.9	0.9	1.8	
E_2	$\overline{\Delta g}$	11.8			-13.8			0.0	0.1	2.5	15.4	15.4	17.1	50.5	49.1	47.2
	$m_{\overline{\Delta g}}$	0.5			5.0			3.7	3.7	6.4	0.5	0.9	1.8	1.0	1.4	3.2
E_3	$\overline{\Delta g}$	12.6			-12.6			- 7.3	3.5	1.2	15.2	17.6	22.7	50.4	49.6	52.8
	$m_{\overline{\Delta g}}$	2.0			10.1			8.4	10.4	13.3	1.8	2.6	4.8	2.3	3.7	6.5

Method D_3^5: Mean free-air anomalies using the mathematical model 3 and 5' x 5' mean anomalies as explained in Section 2.3.

Method D_4^5: Mean free-air anomalies using the mathematical model 4 and 5' x 5' mean anomalies as explained in Section 2.3.

Method D_1^{10}: As D_1^5, except 10' x 10' subblocks were used.

Method D_2^{10}: As D_2^5, except 10' x10' subblocks were used.

Method D_3^{10}: As D_3^5, except 10' x 10' subblocks were used.

Method D_4^{10}: As D_4^5, except 10' x 10' subblocks were used.

Method E_1: Mean free-air anomalies computed using Moritz's theories but 5' x 5' mean Bouguer anomalies as explained in Section 2.4.

Method E_2: As E_1, but 10' x 10' subblocks were used.

Method E_3: Mean free-air anomalies computed using Rapp's program as explained in Section 2.4.

All anomalies and standard deviations are given in milligals.

5. Analyses of the Results

In order to make a decision about which model we should use in Method D, we made statistical test of validity of the terms in the models. We computed the square sum of the residuals after adjustment in each case and also the square sum of the Bouguer anomalies from the mean Bouguer anomaly of the 1° x 1° block in question. We started from Model 1 and proceeded toward Model 4 step by step computing

$$\frac{V_1'PV_1 - V_0'PV_0}{V_0'PV_0}$$

which is distributed as $\frac{b}{n-u} F_{b,n-u}$ where n = number of subblocks with observations in the 1° x 1° block, u = number of unknown parameters in the model which has square sum of the residuals as $V_0'PV_0$, and b = unknown parameters fixed to zero in the model which has square sum of the residuals as $V_1'PV_1$ as compared to model with $V_0'PV_0$ square sum of the residuals [Hamilton, 1964]. We tested the models in 5% and 1% significance level. The summary of the tests is given in Table 5 in the form of percentage of cases from total number of the cases where the H_o hypothesis was not rejected. In the "100% data" set we had 25 1° x 1° blocks in the test. The corresponding number in the "60% and 30% data" set was 14 in each. From Table 5 we can see very clearly that Model 1 is hardly any better than Model 2. Model 2 is the one which could be considered in the case we have much data, but not in a case where we have little data. Model 3 should be ruled out as compared with Model 4. Model 4 is definitely much better than the method of computing mean free-air anomalies through mean Bouguer anomalies. Comparing also the results given in Table 4, we rule out all other methods in the method group D except Model 4. The variances of mean Bouguer anomalies and of unit weight given in Table 6 also support the above statistical results.

For further analyses we assumed that the value computed with Rapp's method (Method E_3) and using "100% data" set are the most probable mean anomalies for the blocks. In order to have a better understanding of what kind

Table 5

Summary of Statistical Tests (Method D)

	100% Data				60% Data				30% Data			
	5' x 5'		10' x 10'		5' x 5'		10' x 10'		5' x 5'		10' x 10'	
	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01
$H_o: X_7 = 0$	76*	88	76	92	93	100	100	100	79	100	79	100
$H_o: X_7 = X_6 = X_5 = 0$	40	52	52	76	50	71	64	79	64	64	64	93
$H_o: X_7 = X_6 = X_5 = X_4 = 0$	68	76	72	84	79	86	71	93	86	100	93	100
$H_o: X_7 = X_6 = X_5 = X_4 = X_3 = X_2 = 0$	8	16	20	24	7	7	7	21	7	14	21	21

*percent of cases, where H_o not rejected

Table 6

Variances of Mean Bouguer Anomalies and of Unit Weights (100% Data)

1° x 1° Block No.	Number of Subblocks With Observations		Variance of Mean Bouguer Anomalies		Variance of Unit Weight From Least Squares							
					Model D_4		Model D_3		Model D_2		Model D_1	
	5' x 5'	10' x 10'	5' x 5'	10' x 10'	5' x 5'	10' x 10'	5' x 5'	10' x 10'	5' x 5'	10' x 10'	5' x 5'	10' x 10'
1	36	21	76.4	71.5	32.5	30.6	33.5	30.8	23.3	24.0	23.6	25.6
2	48	26	85.8	70.1	89.1	79.0	91.0	81.5	79.1	65.6	78.9	63.9
3	58	26	352.2	294.4	137.1	145.8	117.0	126.5	65.2	77.2	59.8	64.7
4	81	33	212.3	204.9	132.2	117.9	115.5	104.1	115.1	99.3	111.0	97.9
5	74	26	156.3	132.9	158.8	142.3	155.7	146.9	53.7	37.8	53.3	38.2
6	82	31	986.8	1064.4	50.4	61.6	48.4	59.4	35.1	42.8	34.6	40.2
7	65	27	508.5	594.6	113.9	102.5	72.7	58.6	48.6	47.8	44.1	37.6
8	126	35	82.2	66.1	73.9	63.3	50.0	40.0	48.2	41.2	48.6	42.7
9	86	31	264.7	230.3	50.7	42.0	50.6	40.4	34.4	26.1	34.6	27.1
10	60	26	474.1	507.9	135.1	178.2	133.2	174.6	94.0	141.3	92.5	129.0
11	88	33	426.5	512.8	70.8	59.2	71.3	61.2	52.2	44.8	46.9	32.4
12	74	30	326.4	303.6	81.0	47.0	63.8	29.0	55.3	24.2	54.9	25.0
13	75	33	150.8	150.8	77.5	84.5	78.2	86.1	69.3	66.7	68.7	65.4
14	99	35	216.1	193.8	54.7	51.7	52.7	50.3	50.9	47.9	51.3	49.5
15	44	21	243.7	182.0	39.3	27.1	24.0	8.4	23.8	8.4	24.4	9.5
16	87	34	62.5	44.6	54.2	39.4	53.9	39.9	54.1	42.1	53.6	41.1
17	39	20	178.5	175.0	76.1	70.4	70.4	67.7	68.2	74.0	62.0	64.2
18	10	7	363.3	415.1	291.3	374.2	209.3	59.1	205.8	5.6	-	-
19	17	12	389.9	401.4	82.3	114.9	87.9	128.6	21.5	82.7	16.3	41.2
20	30	16	65.0	52.0	53.6	51.6	50.3	45.6	45.0	42.1	34.4	36.6
21	65	31	227.0	216.7	80.3	56.9	81.0	58.0	73.9	52.5	75.0	54.2
22	12	7	114.1	59.5	88.4	39.2	62.1	36.6	69.3	173.7	-	-
23	44	17	188.8	205.0	139.1	147.8	135.2	134.7	59.9	36.3	57.9	39.9
24	92	32	391.6	390.7	112.6	95.1	113.3	98.0	111.4	102.0	109.9	100.3
25	65	26	300.1	239.3	182.3	124.0	180.2	128.4	169.3	129.1	132.2	74.6

of agreement we had between the results obtained through various methods, we computed the root mean square of differences between Rapp's values (Method E_3) with the "100% data" set and the other values with the "100%, 60%, and 30% data" set. The results are given in Table 7. The root mean squares are not very comparable between classes because of the different number of samples in the "100%, 60%, and 30% data" set; however, in the same data set they are giving a good indication of the agreements between different methods. By examining the results in Table 7, we can eliminite immediately as unacceptable the following methods: A, B, D_1^5, D_2^5, D_3^5, D_1^{10}, D_2^{10}, and D_3^{10}. From the seven methods which are left, Methods C_1 and C_2 are the simplest ones; however, the computed root mean squares are also considerably higher than in the other methods in the case of little data. We believe that these methods can be satisfactorily employed if we have a good coverage of the gravity observations in the block.

It was very interesting to find out that Method D_4^{10} gave comparable or even somewhat better results than Method E_3 in the "30% data" case. Of course, our sample size is relatively small and because of it we can not make a definite decision about which one of the five methods left is the best. If we assume that the computed root mean squares are representative values of accuracies, we can draw the conclusion that it does not make very much difference which one of the methods left is used. If we accept that, then the next step is to make an analysis of which method involves the least computations. The simplest and fastest to apply is, of course, Method D_4^{10}. The

Table 7

Root Mean Squares of Differences Between Anomalies Computed With "Method E_3" + 100% Data and With Other Methods and All Data Sets

Method	100% Data RMS	60% Data RMS**	30% Data RMS**
A	23.1	24.3	25.6
B	8.1	10.0	11.7
C_1	4.3	5.3	7.7
C_2	3.2	4.0	7.5
D_1^5	13.9 (2.5)*	4.1	13.0
D_2^5	9.5 (1.9)*	2.4	6.5
D_3^5	10.0 (1.4)*	2.0	3.6
D_4^5	2.8	2.7	3.3
D_1^{10}	17.9 (2.1)*	1.9	10.4
D_2^{10}	18.0 (0.6)*	1.6	5.0
D_3^{10}	12.0 (1.4)*	1.5	3.4
D_4^{10}	2.5	2.7	3.0
E_1	0.5	1.3	3.7
E_2	1.9	2.4	3.7
E_3	-	3.1	3.6

*Block Nos. 18, 19, and 22 omitted.

**Block Nos. 1, 5, 9, 12, 13, 17-22 omitted.

rest of the methods would be considered in the following order: D_4^5, E_2, E_1, and E_3. The last method is the most sophisticated and theoretically correct.

In all the above methods we should use anomalies which are not correlated with the topography. However, we have quite a problem finding the best b-value for the computations. As we stated earlier in these test computations, we have used the standard Bouguer coefficient, b = 0.1119 mgal/meter. We might be correct in assuming that the uncertainty in the b-coefficient will have about the same effect in all five methods mentioned above; and, therefore, this does not change the order of the preference about which method should be adopted.

6. Effect of b-Coefficient on Mean Anomalies

As it was stated earlier, all predicted values given in the previous chapters were computed utilizing b-value = 0.1119 wherever it was necessary to use it. The analysis of the localized b-coefficient for the test area has been done in the other report [Uotila, 1967]. In that research a common b-value was computed for the 5° x 5° test area. Using the reported value, b = 0.09909, we made predictions of mean anomalies for the test areas with the "100% data" set and by the Methods E_1 and E_2, and also by the Method E_2 with the "30% data."

From the results given in Table 8 we see that the effects of the change in b-value on mean anomalies are about the same in each method and the differences between the anomalies computed with the two b-values are in some cases over 5 mgal. From the practical point of view, we must realize

Table 8

Mean Anomalies Computed With b = 0.1119 and b = 0.09909

1° x 1° Block No.	Method E_1 5' x 5' 100% Data				Method E_2 10' x 10' 100% Data				Method E_2 10' x 10' 30% Data			
	b = 0.1119	b = 0.09909	Diff.	$m_{\overline{\Delta g}}(b)$	b = 0.1119	b = 0.09909	Diff.	$m_{\overline{\Delta g}}(b)$	b = 0.1119	b = 0.09909	Diff.	$m_{\overline{\Delta g}}(b)$
1	-	-	-	-	29.7	24.6	5.1	4.8	26.5	21.7	4.8	5.9
2	36.0	31.0	5.0	5.3	35.7	30.3	5.4	5.5	36.1	30.8	5.3	5.8
3	44.9	42.0	2.9	3.4	44.6	41.6	3.0	3.2	43.5	40.9	2.6	3.4
4	46.3	44.6	1.7	2.0	46.5	44.9	1.6	1.4	44.7	42.5	2.2	2.5
5	58.2	55.7	2.5	2.3	58.2	55.9	2.3	3.2	58.4	56.2	2.2	2.7
6	36.3	33.3	3.0	3.7	37.0	33.7	3.3	4.5	38.4	34.4	4.0	5.9
7	30.9	29.4	1.5	1.8	32.3	30.1	2.2	2.0	34.6	32.0	2.6	3.1
8	15.0	14.7	0.3	0.1	15.3	14.9	0.4	0.1	15.7	15.6	0.1	0.2
9	30.3	28.7	1.6	1.6	30.0	28.4	1.6	1.6	30.0	28.7	1.3	1.3
10	46.0	42.1	3.9	3.9	46.3	42.2	4.1	4.3	48.6	45.5	3.1	3.4
11	18.5	17.9	0.6	0.8	17.6	17.2	0.4	0.2	18.1	17.4	0.7	0.6
12	17.9	16.8	1.1	1.3	17.1	15.9	1.2	0.7	12.5	11.1	1.4	1.9
13	21.3	19.1	2.2	2.1	21.0	18.4	2.6	2.3	21.8	19.7	2.1	2.8
14	23.0	21.2	1.8	1.8	21.7	19.9	1.8	2.2	19.6	17.8	1.8	1.1
15	35.1	32.7	2.4	2.2	35.4	33.1	2.3	3.2	38.8	37.5	1.3	2.4
16	25.5	24.2	1.3	1.4	25.3	23.8	1.5	1.7	28.6	27.2	1.4	1.8
17	10.6	9.9	0.7	1.3	10.3	9.8	0.5	0.3	2.4	1.6	0.8	0.4
18	-	-	-	-	-3.9	-3.7	-0.2	0.2	-23.1	-22.4	-0.7	0.8
19	5.0	1.8	3.2	3.6	4.0	0.3	3.7	3.4	9.9	6.4	3.5	2.1
20	11.2	10.3	0.9	0.1	10.6	9.5	1.1	0.7	15.2	14.3	0.9	0.6
21	12.6	11.2	1.4	1.2	11.8	10.3	1.5	1.3	9.1	8.2	0.9	0.5
22	-	-	-	-	-13.8	-13.0	-0.8	0.5	-17.4	-16.0	-1.4	1.9
23	-	-	-	-	0.0	-1.2	1.2	1.7	2.5	0.0	2.5	2.5
24	15.2	12.2	3.0	3.2	15.4	12.0	3.4	3.2	17.1	12.8	4.3	4.5
25	50.5	47.4	3.1	2.7	50.5	46.9	3.6	3.6	47.2	41.9	5.3	5.8

that the effect on mean anomalies is more or less systematic as we could have expected. This is very important because usually if we do not have a good local b-value, we use the standard value. We also know that very frequently mean elevation of the stations is less than the mean elevation of the 1° x 1° block. These factors together are thus usually giving us too high mean anomalies in areas where we have considerable elevation differences. In order to avoid this systematic effect, we suggest that a common localized b-coefficient be computed for a large area, say 5° x 5° block, and this value be used for the squares for which a localized b-coefficient can not be computed.

Moritz has also derived formulas to be used in the prediction where the elevation correlation is taken into account [Moritz, 1962]. The application of these formulas for the prediction of anomalies uses a considerable amount of computer time [Rapp, 1964]; and, therefore, the formulas might not be practical for the computation of mean anomalies. In the same paper Moritz [1962] gave the following theoretical formula for the computation of b-coefficient:

$$b = \frac{B_o - 2\sum_i \alpha_i B_i + \sum\sum_{i\,k} \alpha_i \alpha_k B_{i\,k}}{A_o - 2\sum_i \alpha_i A_i + \sum\sum_{i\,k} \alpha_i \alpha_k A_{i\,k}}$$

where

$$A(r) = \text{Cov.}(\Delta h_i \Delta h_k) \quad , \quad r = \overline{P_i P_k}$$

$$B(r) = \text{Cov.}(\Delta g_i \Delta h_k) = \text{Cov.}(\Delta g_k \Delta h_i) \quad , \quad r = \overline{P_i P_k} \quad .$$

In addition, $\Delta h_i = h_i - h_o$ where h_o is the mean elevation of the whole area considered so that mean of Δh is equal to zero.

The limited time available under this contract did not allow us to go into application of these formulas or into research of other computational techniques, but much more work should be done in this area. The above formulas should be used and the results should be compared to the b-value computed through the least-squares solution.

As far as the effect of the uncertainty in b-coefficient on predicted anomalies is concerned, Moritz [1962] has derived the following formula for the computation of the combined standard error of predicted anomaly caused by an error in prediction and in b-coefficient:

$$m = \varkappa\, m_o$$

where

$$\varkappa = \sqrt{1 + (b-b')^2 \frac{A_o}{Z_o}}$$

m_o = standard error of prediction if $b = b'$, i.e., if the correct b-value has been used,

A_o = the mean square of elevation differences,

Z_o = the mean square of the "correct" Bouguer anomalies,

$b-b'$ = deviation of the coefficient b' from its correct value b.

We applied this formula and separated the effect of the error in b-coefficient on anomalies and compared the results with the values given in Table 8. Correlation between the numbers mentioned above was not good. We have to realize that this formula is valid only if

$$\frac{A(r)}{A_o} = \frac{C(r)}{C_o}$$

where

$$A(r) = \text{Cov.}(\Delta h_i \, \Delta h_k) \quad , \quad r = \overline{P_i P_k}$$

$$C(r) = \text{Cov.}(\Delta g_i \, \Delta g_k) \quad , \quad r = \overline{P_i P_k}$$

$$A_o = \text{Var.}(\Delta h_i \, \Delta h_i)$$

$$C_o = \text{Var.}(\Delta g_i \, \Delta g_i) \quad .$$

The condition stated above was not fulfilled in the test area and therefore our results were not good.

In order to get some formula which would give a reasonable estimate for the error caused by the error in b-coefficient, we performed several correlation studies. It is obvious that the differences in mean anomalies caused by different b-values must be a function of elevation differences in the block. In Table 9 the mean elevations of the blocks and mean elevations of the points in the block are given. Because in Methods E_1 and E_2 mean anomalies of subblocks were used, elevation differences between estimated mean elevation of block and the average elevation of the block derived from mean point elevations of the subblocks are given in Table 9. Also, the mean square differences of same quantities are given in the same table.

On the basis of various correlation studies, we conclude that the following simple formula is a rough estimate of the variance caused by an error in b-coefficient on predicted mean anomaly:

$$m^2_{\overline{\Delta g}}(b) = m_b^2 \, (h_m - \overline{h})^2$$

Table 9

Mean Elevations of the Blocks and Points and Elevation Differences

1° x 1° Block No.	Estimated Mean Elevation h_m	Mean Elevations of the Points			Elevation Differences $h_m - \frac{1}{n}\sum_{i=1}^{n} h_i$			Root Mean Squares of Elevation Differences $\frac{1}{n}\sum_{i=1}^{n}(h_m - h_i)^2$		
		100%	60%	30%	100%		30%	100%		30%
					5' x 5'	10' x 10'	10' x 10'	5' x 5'	10' x 10'	10' x 10'
1	2234	1817	1789	1794	-	377	459	-	229,625	310,836
2	2265	1847	1799	1967	417	426	456	241,468	252,573	277,018
3	2097	1740	1726	1716	266	248	267	129,337	120,358	120,393
4	2111	2014	2046	1984	160	112	193	89,711	97,350	82,476
5	2469	2287	2306	2320	182	252	208	74,780	125,738	70,519
6	1973	1529	1549	1476	290	349	462	184,972	254,810	287,132
7	1823	1620	1623	1604	143	157	240	73,283	92,391	102,067
8	1501	1477	1478	1512	10	- 6	-17	9,552	11,217	12,501
9	1889	1660	1661	1683	122	122	99	35,180	61,206	78,873
10	2470	2105	2138	2183	306	337	267	157,803	169,772	124,009
11	1400	1147	1131	1275	65	14	43	49,150	92,369	54,970
12	1497	1369	1353	1316	98	51	151	29,170	45,328	30,471
13	1673	1456	1455	1436	167	180	221	50,284	47,154	57,746
14	1977	1784	1781	1910	140	169	89	61,030	66,701	55,384
15	2361	2156	2184	2187	168	253	189	76,492	86,877	101,233
16	1847	1723	1742	1754	109	133	138	27,943	40,866	35,727
17	1540	1417	1372	1402	101	20	33	29,380	36,039	17,836
18	1388	1351	1300	1300	-	-16	65	-	9,984	6,331
19	2013	1718	1846	1816	285	269	165	175,922	179,179	177,703
20	2171	2202	2169	2145	7	-51	-49	79,119	98,075	75,274
21	1829	1750	1796	1797	90	105	42	27,369	37,060	7,579
22	1348	1291	1276	1302	-	-42	-152	-	27,048	62,798
23	1563	1500	1515	1439	-	133	195	-	39,444	44,216
24	2072	1512	1491	1490	251	248	348	228,534	206,514	248,114
25	2558	2448	2396	2089	211	279	451	242,377	260,792	326,224

where

$m^2_{\overline{\Delta g}}(b)$ = variance of predicted mean anomaly caused by the variance of b-coefficient,

m_b^2 = variance of b-coefficient,

h_m = estimated mean elevation of 1° x 1° block,

$\overline{h}$ = average elevation of point values or of mean values of point values in subblocks used in the prediction.

When we take $m_b = 0.0128$, we get the $m_{\overline{\Delta g}}(b)$-values given in Table 8. If we compare these values with the differences obtained by predicting gravity anomalies using two different b-values, we can see a very good agreement.

7. Conclusions

Only on the basis of the results in the test area of 5° x 5°, we have shown that there are several methods which should be considered as usable methods in the prediction of mean anomalies of 1° x 1° blocks:

(1) The simplest and least time-consuming method is to compute average Bouguer anomalies (using the standard or localized b-coefficient) of 10' x 10' subblocks in the 1° x 1° block. Then we predict the mean anomalies of 10' x 10' subblocks for which we did not have any observations by fitting a plane through computed average anomalies of subblocks. After the prediction, an average value of computed and predicted mean anomalies of 10' x 10' subblocks is computed. This average Bouguer anomaly is brought to

the estimated mean elevation of the 1° x 1° block using the same b-coefficient as earlier. The value obtained through these steps is a predicted mean free-air anomaly for the 1° x 1° block. This method was identified earlier in the text as "Method D_4^{10}."

The other acceptable methods are as follows:

(2) The surface-fitting method which is similar to the method described above except 5' x 5' subblocks are used ("Method D_4^5").

(3) The least-squares prediction method using correlation between mean Bouguer anomalies of 10' x 10' subblocks ("Method E_2").

(4) The least-squares prediction method similar to the above method, but using 5' x 5' subblocks ("Method E_1").

(5) The least-squares prediction method using correlation between gravity anomalies and point values ("Method E_3").

We must remember that the above-drawn conclusions have been made from a limited sample data and the results should be accepted only with reservations. Additional tests should be conducted and more methods should be tested before the final decision can be made about which method should be used for the prediction of 1° x 1° mean anomalies in a larger scale.

We found that the effect of uncertainty in b-coefficient on predicted mean anomalies is approximately a function of the difference between estimated mean elevation of the block for which mean anomaly is being predicted and average elevation of the points used in the prediction. However, additional tests should also be done in this area before adopting the result of the limited tests for the general application.

ACKNOWLEDGMENT

The programming for the computations was done by Richard Henning and David Robinson. Several calculations and preparations of the tables were done by Pentti Karki.

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