Classroom Activity

Problems to Ponder while on a Car-Trip

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Introduction

Since adolescents are interested in cars and driving, it is natural to try to engage them in learning mathematics by using questions about driving. But we have found in writing mathematics questions that if we begin this process with a mathematical skill in mind, and then seek problems with which students can hone the skill, the questions turn out to seem artificial and uninteresting. Instead, we have started with questions that have occurred to us over the years as drivers, passengers, and thinkers.

This real-world use of mathematics is what we hope for from our students, after all. But in order to make the most meaningful use of these questions, we have found that we cannot simply assign them at the appropriate times to our students. Students will probably need help while attempting these, from choosing the appropriate mathematics to approach the situation, to checking on the reasonableness of their answers. Hopefully, students will see mathematics as a process and a fruitful (but not always easy) way of looking at the world. The questions, suitable for grades 7-12, vary in difficulty and range of topics. While we hope the use of these questions helps students appreciate the usefulness of mathematics and to practice some specific skills, more important to us is how we have been able to encourage students, through modeling, to ask questions and to seek their answers.

Rich Tasks

1. Systems of Equations Task

On a road with a speed limit of 55 mph, what distance must you travel so that speeding by 10 miles per hour gets you to your destination 1 minute early? 5 minutes early? Round to the nearest tenth of a mile.

2. Unit Conversions Task

Some drivers change the size of their car tires and rims without realizing that this makes their speedometers inaccurate. (Speeding tickets have been given for this reason!) Suppose your car is meant to have wheels with a radius of 13 inches, and you put on wheels with a radius of 14 inches. When the speedometer reads 60 mph, how fast are you really going?

3. Triangle Congruence Task

Suppose you are travelling on a two-lane highway with posts placed on the median. Another car is on the opposite lane traveling in the opposite direction, and you notice that your car, a certain post, and the other car remain collinear as you pass by. Assuming you both stay in the center of your respective lanes, prove that you and the other car are going the same speed (Refer to Figure 1).

4. Spatial Reasoning Task

Ashley and her family are about to take a trip to the Rock and Roll Hall of Fame in Cleveland. Their van has a storage area that is 6 feet 6 inches wide, 4 feet 8 inches long,

and 3 feet tall. They have multiple pieces of luggage. The extra-large suitcase is 36" × 24" × 48". There are two large suitcases that are 24" × 20" × 40". The two medium suitcases are 20" × 16" × 30". Three small suitcases are 12" × 18" × 20".

(a) Find the most numerous set of luggage which can fit into the storage area.

(b) Draw a sketch showing how that set of luggage fits, and calculate its volume in cubic feet.

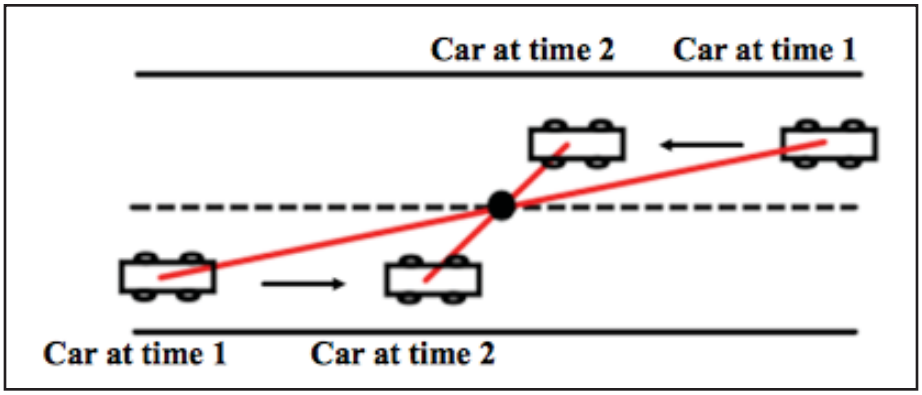

Fig 1 Triangle congruence scenario

5. Trigonometry Task

How fast do you have to travel west to keep the sun in the same spot overhead? Assume the earth is a perfect sphere of radius 3955 miles. This problem will depend on your latitude; solve it for latitude 0° (the equator) and latitude 42° (Toledo). Give your answers in miles per hour.

6. Arc Length Task

Imagine your car and another car in the next lane are traveling side by side on a flat highway. You both have the cruise control set to 60 mph, and you both stay in the center of your 12-feet wide lanes. What happens when you finish traversing the following curve, which is the union of 2 identical 45° circular arcs, each of radius 50 feet? Who is ahead, and by how many feet? In the diagram below, assume traffic is moving to the right, and you are in the "bottom" lane. (Refer to Figure 2).

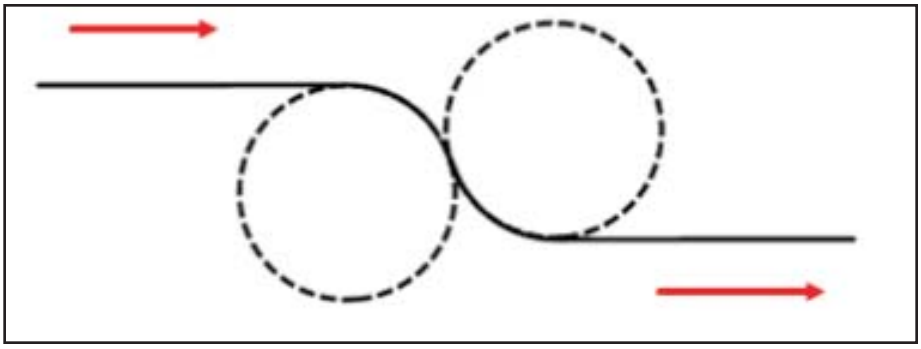

Fig 2 Arc Length scenario

7. Algebra Task

Suppose your family and your grandparents are in two cars traveling together. While you continue down the highway at 70 mph, your grandparents stop at a rest stop. After 10

minutes, they resume traveling at 70 mph and call your car on their cell phone. You then slow down to 60 mph, allowing them to catch up. How long will it take for them to catch up after they leave the rest stop? Give your answer in minutes.

8. Trigonometry/Vectors Task

While driving in the rain with no wind, you notice that the back window remains dry. Only when slowing down upon arriving at your destination does the back window get hit by raindrops. Suppose the back window makes an angle of 50° with the horizontal and that the rain is falling at 12 mph. Calculate how fast you have to travel to insure that no rain hits the back window. (Refer to Figure 3).

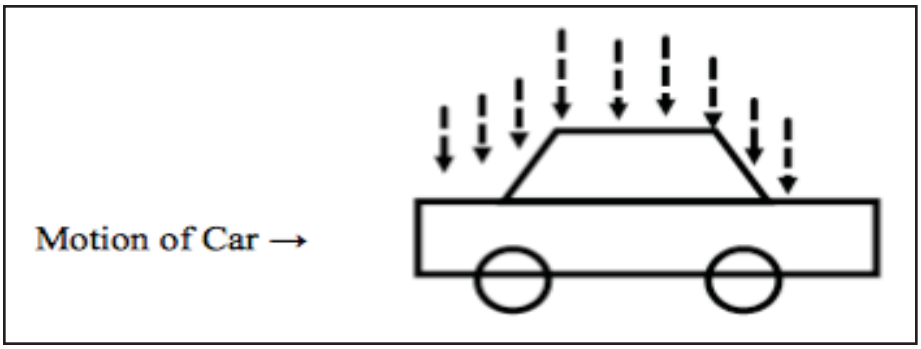

Fig 3 Trigonometry / Vectors scenario

9. A Fixed-Point Theorem Task

On Monday you travel from Cleveland to Cincinnati from 10am to 2pm. On Tuesday you travel back, on the exact same roads, from 10am to 2pm. Your speed varied on both trips, and you even stopped occasionally. Prove that once you were in the exact same spot (although in different lanes) at the exact same time of day!

10. Combinatorics / Classification Task

Nathan W. Mills noticed that new Ohio license plates are of the form xxx9999, with three letters followed by four numbers.

(a) Find the chance of randomly getting a license plate with Nathan's initials in order.

(b) After seeing such a plate, Nathan realizes his initials are difficult to read at a distance because the letters look too similar. Your job is to make improvements to the license plate lettering combinations. Classify the letters into typographically-similar classes (for instance, Nate would argue that N, M, and W are in the same class and so should not be used together). Defend your reasoning by describing the classes, perhaps geometrically. Offer some criteria the license bureau could use in deciding what letter combinations to print.

(c) Research the topological classification of the letters. How many classes are there?

11. The Touring Colleges Task

As you approach your final year of high school, you and your family are scheduling a trip to visit Ohio colleges. You would like to visit Ohio University, The Ohio State University, University of Toledo, Xavier University, Kenyon College, and Case Western Reserve University. The mileage between selected schools is shown on the map. Starting

and ending at Ohio University in Athens, in what order should you visit the schools so that your total traveling distance is as small as possible?

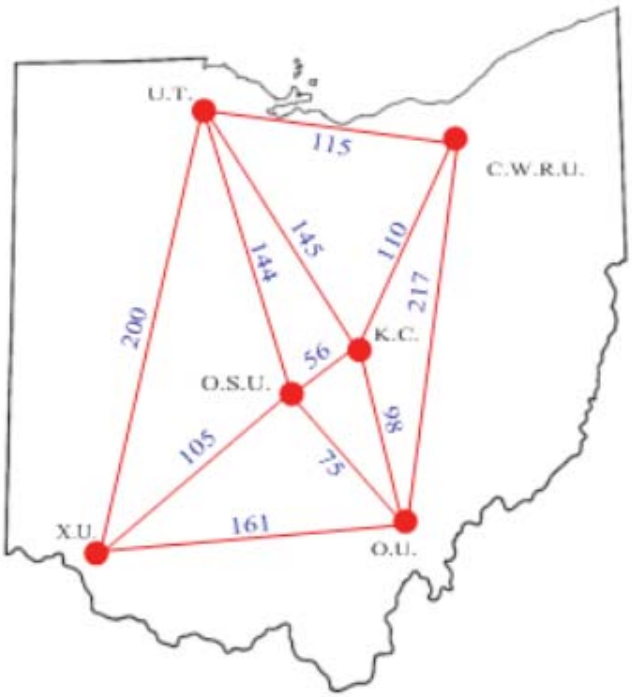

Fig 4 Touring Colleges scenario

12. Harmonic Series

Suppose you are traveling 60 miles to Columbus, and your speed is 60 mph. At the exact moment you pass a sign which says "Columbus: 30 miles", you have to slow to 30 mph, for the remainder of the trip, due to major road construction. You notice the total trip takes 1½ hours.

(a) Now instead suppose there is a sign at the 40 mile point, and you have to travel 40 mph, until you pass a sign at the 20 mile point, when you have to travel 20 mph for the remainder of the trip. What is the total time of the trip in hours?

(b) Now instead suppose there are n such signs, equally spaced on the roadway and you have to reduce your speed accordingly. What is the total time of the trip?

13. A Net Change Problem

Why is the use of cruise control not recommended in hilly terrain? This question will help to answer that! The two graphs below represent two different ways to drive over a hill.

In Figure 5, the car's cruise control is engaged; therefore, the car's speed remains constant. At minute 1, the car begins going uphill, and the engine's RPM rises from 2000 to 3000. At minute 3, the top of the hill is reached and the car begins descending; the engine's RPM falls to 1500 because of the downhill trajectory. At minute 4, the bottom of the hill is reached and the RPM rises to 2000 for the flat road.

In Figure 6, the car's cruise control is off, and the speed is allowed to drop a little while climbing. At minute 1 the car begins going uphill, and the engine's RPM stays at 2000. At minute 2.5, the engine's RPM rises to 2500. At minute 3.5, the top of the hill is reached and the car begins descending; the engine's RPM falls to 1500. At minute 4.5, the bottom of the hill is reached and the RPM rises to 2000.

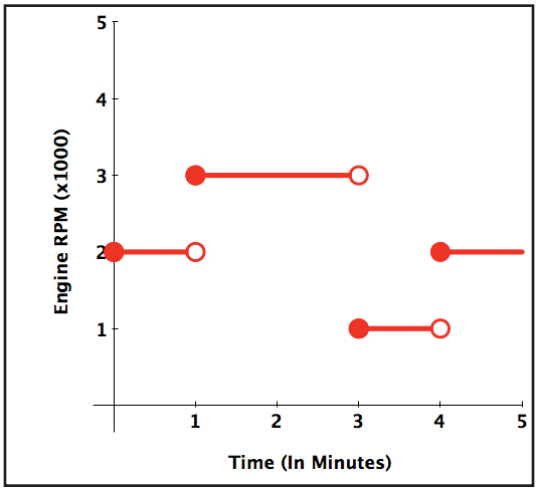

Fig 5 Cruise control on

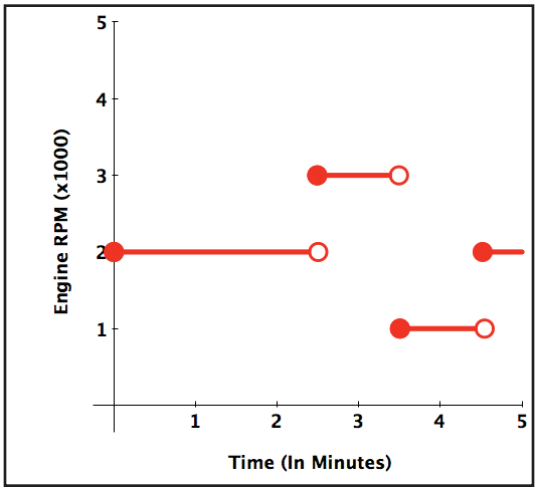

Fig 6 Cruise control off

The amount of a gasoline a car uses is directly proportional to the number of rotations of the engine's driveshaft, as measured in the engine's RPMs. For the trip up and down the hill, calculate the total rotations for the two methods of driving. Which version uses the least amount of gasoline? Which version takes longer time? Justify your answers.

Brief Solutions

Note: Detailed Solutions are available upon email request of one of the authors.

1. 1 minute early: 6.0 miles; 5 minutes early: 29.8 miles
2. 64.62 mph
3. Use SAS triangle congruence
4. a. All of Ashley's luggage fits; b. Diagrams may vary; V = 389/6 cubit feet.
5. For latitude 0°, ≈ 1035.4 mph.; For latitude 42°, ≈ 769.5 mph.
6. The cars are even once again.
7. 70 minutes.
8. at least 10 mph.
9. Answers vary.
10. a. 1/10000; b. Answers Vary; c. There are 3 topologically distinct classes.
11. 717 miles
12. a. 1 + 1/2 + 1/3 miles; b. 1+ 1/2 + 1/3 + … + 1/n miles
13. With cruise control: 7500 total revolutions, 3 seconds; Without cruise control: 7000 total revolutions, 3.5 seconds.

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