The Book as a Hook: Literature and Exponential Functions

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Playing with rice provides a concrete representation of exponential functions. The math-literature connection provides a "hook" to generate interest in exponential functions. Subsequent investigations provide the opportunity to analyze and understand how the functions can be used to represent real world situations. Students create their own story containing a situation whose resolution requires exponentials. The classroom comes alive as students actively pursue mathematical understanding.

Introduction

The mathematician pushed and pulled at the ebony beads of his abacus and he scribbled with his brush on a paper scroll. "Imperial Majesty, at this rate, in ten days there will be no rice left in the palace!"...On the twenty-fifth day, sixteen-million-seven-hundred-seventy-seven-thousand-two-hundred-and-sixteen grains of rice were delivered in brocaded sacks (pp. 43-4, 53).

This passage is from a Chinese folk tale, *A Grain of Rice,* written by Helena Claire Pittman. The story begins on the single day each year when the Emperor opens his court to allow anyone to come before him. Pong Lo, a mere peasant, asks for the hand of the Emperor's daughter, Princess Chang Wu, in marriage. After the request is denied, Pong Lo uses his clever mind to serve the Emperor. When the princess becomes ill, Pong Lo is the only one able to heal her. The king states he will grant Pong Lo any reward, and of course, the peasant once more asks to marry the princess. The request is yet again denied; however, then Pong Lo asks for a grain of rice which is to be doubled each day for one hundred days. Believing it to be a ridiculous request, the Emperor quickly grants it. The exponential function enters the scene. Is the kingdom doomed to die?

Literature as a "Hook"

Literature is frequently used to explore mathematical concepts at the elementary level. Young students love to listen as a story is read and enthusiasm rises as mathematics enters the story. The math-literature connection is a "hook" to generate student interest in a particular mathematical topic for the secondary level student as well. "Playing with Rice," the project described in this article, provides students with the opportunity to explore the exponential function in an experiential fashion. *Principles and Standards for School Mathematics* states, "The kinds of experiences teachers provide clearly play a major role in determining the extent and quality of students' learning. Students' understanding of mathematical ideas can be built throughout their school year if they actively engage in tasks and experiences designed to deepen and connect their knowledge" (p. 21). In this article, we describe an unconventional lesson that weaves reading, writing, and critical thinking skills to explore the exponential function. In order to complete this lesson successfully students need to be familiar with polynomial functions, but not exponential functions. The lesson is intended to introduce exponential functions.

Introducing the Lesson

Students begin the lesson by "playing with rice" in small groups. Containers are distributed to members of the group. The student with the zero container which holds one grain of rice, writes the ordered pair (0, 1) on a sticker to represent the number of grains of rice in container 0 and places the sticker at the point (0, 1) on the graph. For round 1, the banker gives the person with container 1 double the number of grains of rice that are in container 0. The person with container 1 writes the ordered pair (1,2) on a sticker to represent that there are 2 grains of rice on round 1, and then places the sticker at the appropriate point on the graph. For each subsequent round, the banker gives the person double the grains of rice that are in the previous container and that person writes the ordered pair (# of container, # of grains of rice in that container) on a sticker and places it at the appropriate point on the graph. This continues, ending with the recorder drawing a smooth curve to connect the points. See Figure 1.

Classrooms should be "places where interesting problems are regularly explored using important mathematical ideas."

Students then respond to questions leading them to analyze the ideas, moving from the concrete situation to a more abstract setting. They communicate with each other as they use problem solving skills to make conjectures. This part of the lesson correlates well with the NCTM *Principles and Standards for School Mathematics* which suggest mathematics classrooms should be places where interesting problems are regularly explored using important mathematical ideas. Figure 2 provides actual samples from student work.

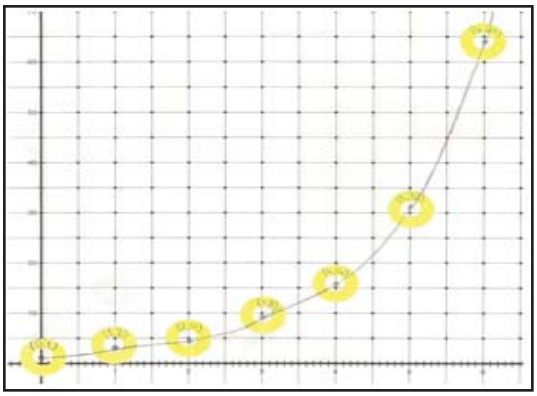

Fig 1 Student example of the doubling graph

Repeating the Process

Students continue their exploration by repeating the process two more times, once with the half rule and once with the third rule. The students have a hands on experience to "see" the functions $\left(\frac{1}{2}\right)^x$) and $\left(\frac{1}{3}\right)^x$). The students explore the various exponential functions, observing similarities and differences. This supports the suggestion in the *Standards* that recommends students "identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationship" (NCTM, 2000, p. 296).

Story Time

Story time now begins! As the teacher begins to read the fairy tale, the students listen with rapt attention. The simple tale is delightful. Despite the pleas from the class to hear the entire story, the teacher stops at the point where the future of the kingdom is at risk. She promises to complete the reading when the homework for the lesson is complete. Students make sure the promise is kept.

The exploration of the exponential function continues as students consider two new questions. What happens if Pong Lo begins with 3 grains of rice? What happens if Pong Lo begins with 5 grains of rice? Students complete tables of values and are asked to represent these situations with a function that includes the expression 2^x. Students seem to readily determine that the new functions are $3 \cdot 2^x$ and $5 \cdot 2^x$. In the analysis of the two new functions students must explain what part of the

Question 1: *If the rounds continued, what would be the number of grains of rice in container 8? Container 9? Container 10?*

Sample:

If the rounds continued the number of grains of rice in container 8 would be 256; in 9 there would be 512 and in 10 there would be 1024.

x	y
8	256
9	512
10	1024

Question 2: *If possible, determine the largest number of grains that might be in any container. Justify your answer.*

Sample 1: There couldn't be a largest possible number because if you keep doubling the value it could go on forever.

Sample 2: The largest number of grains that might be in any container is any positive real number. This can be expressed as (0,∞).

Question 3: *Will the number of grains of rice ever equal 0? Justify your answer.*

Sample : The grains double as the containers increase, so if the containers drop below 0, then the gains will be cut in half. The grains will never equal 0 because you always have to double the denominator so it would be 1/□.

Note: Students actually begin to intuitively understand the notion of a limit. The number of grains of rice gets close to zero, but never equals zero.

Fig 2 Analysis-style questions and sample student responses

function changes when the initial value changes. Next, students assume Pong Lo starts with one grain of rice and explore the functions that would represent the situation assuming the number of grains of rice is tripled each day and then halved each day. The students discuss the ideas and find that the new functions can be represented by 3^x and $\left(\frac{1}{2}\right)^x$. Proceeding to a more abstract analysis, students must create a general rule for an exponential function using *a, b*, and *x*. Students use the information from the story to define the meaning of each of the variables. This work supports the suggestion from the NCTM *Standard for Algebra* that states "students should be able to "represent and analyze mathematical situations and structures using algebraic symbols" (p. 296). See Figure 3 for a typical response; the students get it!

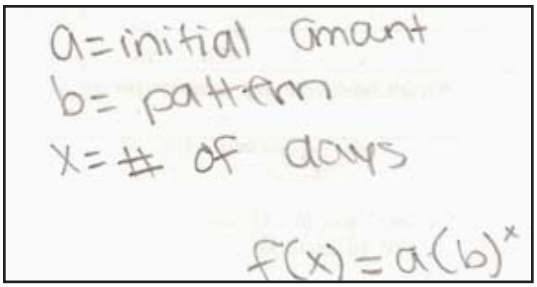

Fig 3 Sample student response

Final Steps

The final part of the lesson provides an opportunity for students to create a story containing a situation that can be represented by an exponential function, actually write the function, and create a sketch of its graph. Many students, particularly those who take pleasure in creative writing, immerse themselves

Students assume Pong Lo starts with one grain of rice and explore the functions that would represent the situation

in this part of the lesson, relishing the concept of outwitting a character of their own making and taking the opportunity to be the "wise one," like Pong Lo had been in *A Grain of Rice.* This part of the lesson correlates well with the communication standard in NCTM *Principles and Standards for School Mathematics* which states "writing in mathematics can also help students consolidate their thinking because it requires them to reflect on their work and clarify their thoughts" (p. 60). This project helps students develop skills in mathematical communication that will serve them well both inside and outside the classroom. Using these skills will in turn help students develop deeper understandings of the mathematical ideas about which they speak, hear, read, and write.

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Assessing with Stories

Students are excited to share their stories with their classmates and showcase their creative talents. The stories provide a wonderful assessment that clearly illustrates student understanding of increasing and decreasing exponential functions. One of my favorite stories begins "Once upon a time there was a young boy named Randy. He was a normal boy who didn't make himself standout. He got average grades and lived in an average sized house. His parents had normal jobs and he was happy with his normal life. One day he was walking home from school and a genie promised to grant him one wish. The only exception was he couldn't wish for unlimited wishes. Randy thought for a minute . . .." The story goes on to explain that Randy's first wish was that on each subsequent day he would have triple the number of wishes he was allowed to make the previous day. Another story tells the tale of "a horrible chemistry teacher who gave 324 minutes of homework EVERY night!" An accident in the lab results in the students saving the teacher's life. In gratitude, the teacher asked if she could give them something in return. The clever students "asked for their homework to be reduced by 1/3 every night . . . They gave her an exponential function, $f(x) = 324\left(\frac{1}{3}\right)^x$. Since she was a chemistry teacher, and not a math teacher, she agreed out of gratitude." The story goes on to describe how within a very short time, "the students had virtually no homework."

In Conclusion

The enthusiasm generated by this unusual, standards based introduction to the exponential function is a delight to behold. In addition, the initial hands-on experience of playing with rice provides a concrete representation of both increasing and decreasing exponential functions. The subsequent investigation of the mathematics provides students with the opportunity to analyze and understand how the exponential function can be used to represent real world situations. The classroom comes alive as students work together in their pursuit of mathematical understanding. Ω

References

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

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What is thought?

"There are perhaps 100 billion neurons in your brain, each of which may, at any moment, send a signal via a brief, tiny pulse of electricity to other neurons, via roughly 100 trillion synapses. Each neuron can, in turn, re-route the message to still other target neurons. The spreading activity, coursing through the pathways of the brain, constitutes the message. That activity is the substance of thought."

Lynch, G. & Granger, R. (2008). *Big brain: The origins and nature of human intelligence, 107.* Palgrave MacMillan. New York, NY.

Where does math fit?

"Whereas innate tasks are almost universally acquired by all people, non-innate tasks must recruit brain areas that were not specifically evolved to perform them, and thus will fail in some percentage of people."

Lynch, G. & Granger, R. (2008). *Big brain: The origins and nature of human intelligence, 198.* Palgrave MacMillan. New York, NY.