DIGITAL CLOCK ARITHMETIC

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Ever since we got our lighted digital clock for Christmas a few years back I've been looking at the time and trying to attach a mathematical significance to each readout. Some simple examples are 12:34, 1:23 and 2:48. We could say that 12:34 represents the set of the first four counting numbers or 1, 2, 3, 4 . Likewise 1:23 could represent 1, 2, 3 as well as the first number, reading from left to right, plus the second number equals the third number or $1 + 2 = 3$. Before reading ahead, try your luck with 2:48, but keep in mind that the order of the digits may not be changed. Very good, that is, if you let 2:48 represent $2 \times 4 = 8$, or the first three terms of a geometric sequence with 2 as the ratio.

But that's enough of this already! My family knows that I'm somewhat possessed with palindromes and naturally 2:32 strikes a familiar chord. What is a palindrome? It is any set of numbers, letters, words, etc. that reads the same from left to right and from right to left. As an example, there was this young girl who stated that her brother was only four and that he could spell his name backwards. An amazed friend inquired about the name and was told that the brother's name was Otto. Some other examples are the name Anna and the words madam and pop. Now try your luck with some more names and words that are palindromes.

Now let's do some digital clock arithmetic involving palindromes! We will neglect the ":" so that 1:11 as well as 11:11 represent palindromes. Also we will consider only a standard 12 hour clock.

1. What is the smallest palindrome?
2. What is the largest palindrome?
3. What is the shortest time between two consecutive palindromes? List them.
4. What is the longest time between two consecutive palindromes? List them.
5. How many palindromes are there on a digital clock? Be careful!
6. What's the next consecutive palindrome after each of these: 2:32, 8:58 and 4:44?
7. Name some jets that are palindromes on our digital clock.
8. Find some palindromes such that the last two digits on the right are a multiple of the first digit or the first two digits on the left.

Well, how did you approach these problems? Did you do a mental exercise, which is good for us on occasion, or did you make a complete list of palindromes first? How you would approach these problems in the classroom would depend upon your own goals. I'm sure that you can add many classroom questions to the above such as the following:

1. Who gets picked up by the bus or leaves for school on a palindrome?
2. Which classes begin or end on a palindrome?
3. What's your favorite palindrome and why? (This could be 2:02 for recess or 3:23 for the end of the school day, etc.)

Let's return to the initial idea of attaching some mathematical significance to each readout. As a special challenge take 6:28 and try to perfect a mathematical statement other than $6 + 2 = 8$. You may have students select readouts at

random or you may investigate some ahead of classtime. As for 11:33 there are numerous combinations for equations and statements such as: $1 \times 1 \times 3 = 3$; $1^1 \times 3 = 3$; $(1 \div 1) \times 3 = 3$; $(1 \div 1) = (3 \div 3)$; $1 - 1 = 3 - 3$; 33 is 3 x 11 or the last two digits are 3 times the first two digits; and the product of the means is equal to the product of the extremes.

I offer the following as additional examples for your consideration:

1. For 12:01, $(12)^0 = 1$
2. For 2:42, $\sqrt[2]{4} = 2$
3. For 11:21, $1^{1/2} = 1$

So far the answers given have assumed that the digits in the readouts are all positive. As a variation you might like to let them be either positive or negative so that 11:33 could also be $1 \times (-1) \times 3 = -3$; $(1)^{-1} \times 3 = 3$; etc.

When you can't sleep, just look at your digital clock and think mathematics!

ANSWERS: 1. 1:01 2. 12:21 3. 2 minutes; 9:59 and 10:01 4. 70 minutes; 10:01 and 11:11; 11:11 and 12:21 5. 57 6. 2:42; 9:09; 4:54 7. 7:27; 7:37; 7:47 8. palindromes 1:01 through 3:03; 3:33; 4:04; 4:24; 4:44; palindromes 5:05 through 6:06; 6:36; 7:07; 8:08; 8:48; 9:09; 11:11 Special problem 6:28. 6 and 28 are the first two perfect numbers.