Department of Mechanical and Aerospace Engineering

Flow Control Optimization using Genetic Algorithms and DMD Reduced Order Model

Steven Murawski, Dr. Datta Gaitonde

INTRODUCTION

The theory and application of active control has been extensively studied for years. Conventional control methods often require use of a transfer functions to represent the physical system and controller [1]. However, this proves troublesome for many complex systems where the transfer functions may not be easily defined.

One such example of a complex system is noise generated by jet. The detailed information required because of the non-linear governing equations makes definition of a transfer function extremely difficult. Genetic algorithms (GA) offer an attractive alternative as it is agnostic to the governing equations and offers many benefits in comparison to traditional optimization algorithms such as brute force and gradient.

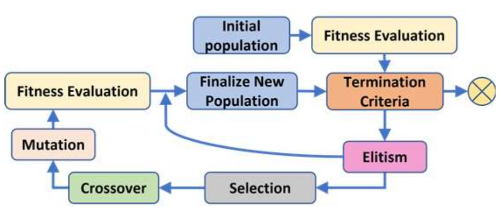
How does a Genetic Algorithm Work?

Genetic algorithms optimize functions in a process that mimics natural selection [2].

A flow chart of the algorithm is shown in the figure below:

1. Initial Population: Create first population with values
2. Fitness Evaluation: Determine the fitness of individuals within the population
3. Termination Criteria: Determine if algorithm has reached optimal criteria
4. Reproduction: Create new population
   - Elitism: Best individuals placed into new generation
   - Selection: Take best individuals from previous population
   - Crossover: Make new individual by combining genes of parents from previous generation
   - Mutation: Randomly mutate certain individuals to ensure full search space is being explored
5. Continue process until you reach optimal value

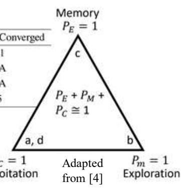
Genetic Algorithm Flow Chart

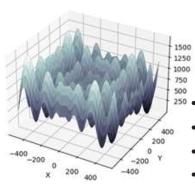


Verification with Schwefel Test Function

A two-dimensional, algebraic test function (Schwefel [3] function) was used to validate and test the genetic algorithm with a variety of reproduction parameters. Exploitation parameter set found to be best and be used in DMD-ROM configuration.

Figure	Testing Category	P_S	P_C	P_M	P_E	Generation Converged
(a)	exploitation	10	0.85	0.1	0.1	121
(b)	exploration	10	0.1	0.85	0.1	N/A
(c)	memory	10	0.1	0.1	0.8	N/A
(d)	large population	40	0.85	0.1	0.1	15

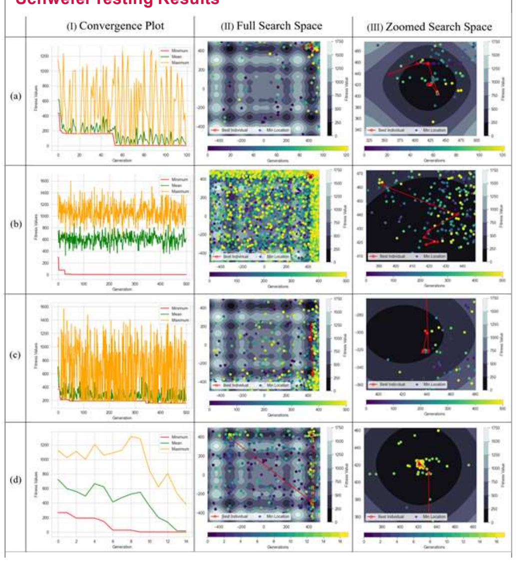




Schwefel Function Definition:

- $f(x) = 418.9892d - \sum_{i=1}^{d} x_i \sin(\sqrt{|x_i|})$
- $x_i = -500 \; to \; 500, d = 2, \;\; \Delta x_i = 0.001$
- $x_{min} = 420.9687$
- 10^{12} total data points

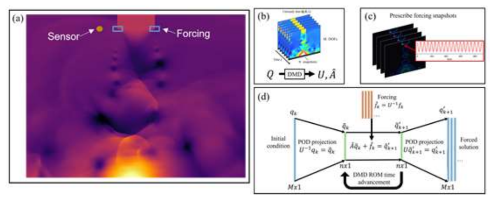
Schwefel Testing Results



Controlling Flow with DMD-ROM

Following validation, the GA was then applied to a supersonic planar impinging jet [5,6] to optimize a controller using the σ function shown below. Running a full computational fluid dynamics (CFD) simulation each time would be too complex, therefore a reduced order model (ROM) based on dynamic mode decomposition (DMD) was used to reduce computational costs.

Planar Impinging Jet Database and DMD-ROM Flowchart

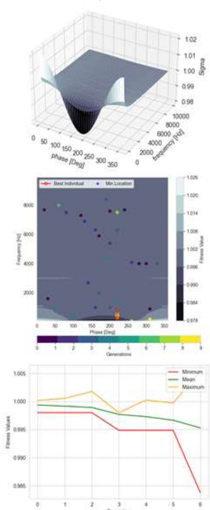


Objective Function: $\sigma = \frac{||\widetilde{Q_f}||}{||\widetilde{Q}||}$

$\widetilde{Q_f}$: Forced Solution Reduced Snapshots

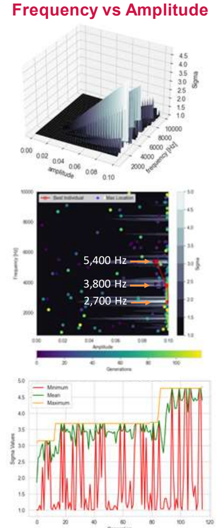
$\widetilde{Q}$: Unforced Solution Reduced Snapshots

Frequency vs Phase



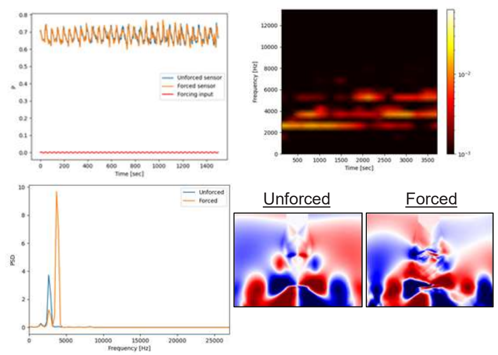
- GA: 70
- Brute Force: 3,267

Frequency vs Amplitude



- GA: 1,170
- Brute Force: 9,500

Forcing at 3,800 Hz DMD-ROM Full Space Results



CONCLUSIONS

The Schwefel test function demonstrated the ability of the GA to find the optimum much faster and efficiently than a brute force search. The GA was then applied to a DMD-ROM of a supersonic planar impinging jet, where it increasing convergence speed by 88% when compared to brute force. When forcing identified by the GA was applied to the system, it was able to significantly alter the dynamics of the system. The coupling of the GA with the DMD-ROM also showed how powerful a genetic algorithm can be when coupled with a lower cost objective function that reduces total computational time. With a more targeted fitness function, this GA could be used to determine the optimal control settings to reduce noise out of the planar impinging jet or for a variety of other flow control applications.

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