

Developing Mathematical Practices: Small Group Discussions

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Abstract

Facilitating small group discussions is essential. Small group discussions determine the quality of whole group discussions and the opportunities for students' mathematical thinking. They also support students' cognitive autonomy, communication, and justification strategies. Realizing these benefits depends on the quality of the small group discussions. In this paper, several practices for facilitating productive small group discussions are discussed. These practices include expecting and assessing students' understanding of strategies used by their peers, and giving students a responsibility to make their strategies accessible to others. Questions to guide and assess small group discussions are included. These practices were observed in several classrooms during a longitudinal study.

Have you ever walked into one mathematics classroom and noticed that students look more engaged in their small group discussions than in other classes? Engagement in small group discussions depends on teaching practices. When small group discussions are not well facilitated, students end up doing their work individually and simply show each other the answers, or some members of the group do the work while others are not cognitively engaged. Several studies (e.g., Baxter & Williams, 2010) reported that facilitating group discussions is a challenge. This paper discusses teaching practices that support productive small group discussions and the benefits of small group discussions. These practices and benefits were observed during a research project with several schools in which students worked on pattern finding activities like in Figure 1.

If one person sits on each side of a hexagon table, how many people would sit around a train of 1, 2, 3, 100, or any number of hexagon tables?

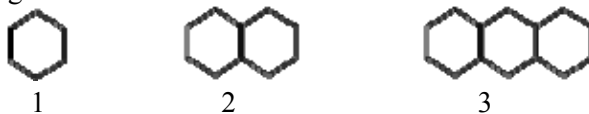


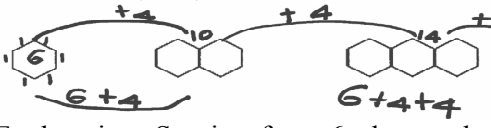
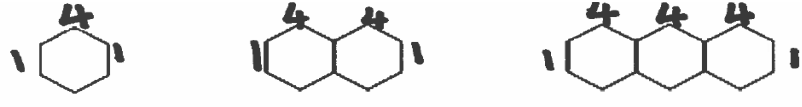
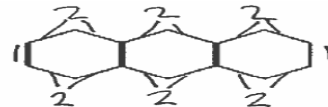
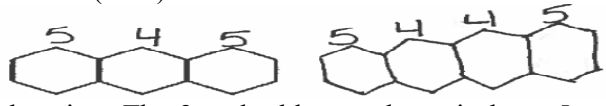

Figure 1: Hexagon tables task

Practices for Facilitating Small Group Discussions

Communicating expectations and reasons for those expectations

It is important for teachers to communicate expectations and reasons for collaboration at the beginning of activities and reinforce them throughout the activities and over time. Teachers should explain to students the reasons for group work. These reasons can focus on how collaboration deepens the mathematical understanding for everyone including those who are already proficient. As one teacher told her students that “we are here (in a mathematics classroom) to broaden our minds, use good thinking, and when we learn from each other we get so much more.” Teachers should use instances from previous mathematics discussions to show how discussions increase understanding. For example, teachers may focus on how sharing strategies from different students as in Table 1 brings a better understanding of the hexagon tables task. Teachers in this study explained that giving reasons for collaboration helps students to see why they should work with others, and consequently motivate students to participate.

Table 1: Strategies for finding number of seats around a train of hexagon tables

Student	Strategy
Jon	<p>Rule: $6 + 4(t - 1)$</p>  <p>Explanation: Starting from 6, the number of seats increases by 4 each time a table is added to the train. Number of increments is one less than the number of tables.</p>
Stacy	<p>Rule: $4t + 2 = s$ where t is the number of tables, and s is the number of seats.</p>  <p>Explanation: There are 2 seats at the ends of each train, and each table on the train contributes 4 seats.</p>
Brenda	<p>Rule: $1 + 2t + 2t + 1 = s$</p>  <p>Explanation: Number of seats at the top of the train, plus number of seats at the bottom plus number of seats on the ends of the train.</p>
Jo	<p>Rule: $5 + 4(t - 2) + 5$</p>  <p>Explanation: The 2 end tables on the train have 5 seats each. The other tables have 4 seats each.</p>
Gwen	<p>Rule: $6t - (2t - 2)$</p>  <p>Explanation: Multiply the number of tables by 6 to find total number of seats before building the train. Then subtract the number of seats lost when building the trains. The number of seats lost is 2 less than twice the number of tables.</p>

Teachers should also make clear the responsibilities for each student during small group work. These expectations include 1) understanding your own thinking or mathematical strategy, 2) helping others to understand your strategy, and 3) understanding other students' strategies. Students can understand their own strategies by explaining to themselves and others why their strategies are mathematically sound. Such arguments should draw from other related mathematical ideas and context of the task. Well-detailed explanations or rules make the reasoning behind the strategies more accessible to both the authors and their peers. Names of variables are an important detail that students need to include with patterning tasks like in Figure 1 (Store, Richardson, & Carter, in press). When student strategies do not include the variable names, for example if Stacy's rule is written as $x + 2$ as it often happens instead of $4t + 2 = s$, many students find it difficult to make sense of it and justify it. Additionally, teachers should encourage students to use pictures or other representations with their explanations to make their thinking more accessible.

It is helpful to give each student the responsibility of understanding their partners' strategies. Asking students to report other students' approaches, state the questions they asked others, and how they explained their ideas to others can reinforce these responsibilities (see Table 2). These discussions should also focus on what is mathematically different or similar between the different strategies.

Grouping Students Purposefully

The outcome of small group discussions significantly depends on how well students work with their partners. While random assignment into groups has a chance of students being with partners whom they can productively work with, purposeful grouping is more likely to support productive discussions. One criterion for grouping students is students' consistent behavior towards each other. As known, students who tend to engage in off task behavior should not be in the same group. In general, teachers should assess how well group members work together. This can be assessed over time as teachers develop strategies based on their knowledge of strengths of each child.

Another important criterion is perceived student abilities. When students perceive themselves as low achieving compared to their partners, they tend to be more receptive of their partner's ideas without critique. In such cases, it is helpful to group students who tend to challenge each other in the same group. Teacher's perception of student abilities is another criterion for grouping students. Teachers in our research study grouped together students with seemingly like abilities. As one teacher reasoned, you usually are going to have one (student) that is a little bit quicker to be able to understand what is going on, and then you will have one that is kind of copying or you know, just saying, okay. So I group them so they are the same levels.

It is important to consider the speed that students tend to complete their tasks, otherwise some students may consistently figure out the math problems before their partners' cognitive engagement thereby robbing them of chances to construct their own understanding. Purposeful grouping creates thinking space for each group member. Like-ability grouping worked very well in the classes we studied in that students showed continued engagement in constructing viable arguments and finding different mathematical strategies. However, readers are reminded that mixed ability grouping is also reported by other researchers to support mathematical understanding. Since mixed ability and like ability grouping may both be argued to have pros and cons, this paper encourages teachers to vary their grouping strategies so that students may benefit from both types of grouping without experiencing disadvantages of tracking (Willis, 2010).

Including different learning styles

It is important for students to work in collaborative environments. However, the uniqueness of some students makes group discussions less productive. In such cases, accommodations should be made so that all students have the benefits of collaborative learning contexts. If exceptions exist that make group work impossible or unproductive for some students, such students can work on the mathematical tasks on their own and then explain their ideas to other students. These students should also have the responsibility of helping others understand their thinking and understanding what other students have done. As such, they should go to other students or groups to ask questions about the different strategies and discuss how those strategies compare with theirs.

Acknowledging Shared Meanings

Shared meanings in classrooms are strategies and mathematical ideas that result from participation in groups (Lave and Wenger, 1991). Acknowledging that participation in small group discussions contributes to mathematical understanding validates the worth of each partner, helps students see the importance of small group discussions and consequently ensures continuity of productive small group activities. Acknowledgement can be through the use of "we" instead of "I" in explanations that others have contributed to. Teachers can make sure students are acknowledging contribution of others during

whole class discussion by encouraging use of “we” when reporting ideas from small group discussions, asking students to “report what they have discussed with their partners,” and referring to the ideas as group ideas. Table 2 contains some useful prompts to help students acknowledge shared meanings.

Assessing if Students Find Collaboration Important

For continuity of productive small group discussions, students should find those discussions helpful for them. Both teachers and students should continually assess if students find small group discussions helpful. Teachers can facilitate such assessments by asking students to be reflective of their experiences and talk about the specific ways the discussions contributed to their mathematical understanding. Questions in Table 2 can help students to be reflective on how group discussions affect their mathematical practices. Such reflections are opportunities to discuss the different correct ways for approaching a mathematical task, the importance of verbalizing ideas to make arguments stronger, and connecting mathematical ideas. Teachers can also check if students found collaboration important by inquiring about students’ feelings when others listened to them as they were “trying to share their best thinking.”

Table 2: Questions for monitoring small group discussions

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| <ul style="list-style-type: none">• What strategies did your group use?• What strategies did you think of?• What strategies did others think of?• How did you convince others that your strategies will always work?• How are your strategies similar to or different from others?• Which of the strategies did you prefer and why?• What questions did you ask?• How did working with others help you today?• How did you feel about working in groups today? |
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Benefits of Small Group Discussions

There are several benefits for facilitating small group discussions well. These benefits include opportunities for students to assess their own understanding, developing communication and justification strategies, and making mathematical connections.

Assessing Mathematical Understanding

Asking students to explain their thinking to others is a tool for students to assess their own understanding of mathematical concepts. With the discussed practices, students are charged with the responsibility to make detailed explanations for others to understand. Through such explanations, students may realize errors in their thinking and the need to evaluate their strategies so that they can make sense. This process of revisiting strategies to evaluate one’s ideas is essential for growth of mathematical understanding (Pirie & Kieren, 1994). Additionally, students develop cognitive autonomy when they assess solutions to problems and explain them to their peers before the teacher validates their thinking.

Using Reasoning by Others as a Tool for One’s Own Reasoning

Naturally, there are times when students find it difficult to reason about some tasks. Students may make some progress in their reasoning that is not enough for them to complete their tasks. Small group discussions present opportunities to incorporate peers’ reasoning into their own for better mathematical understanding (Mueller, 2009). Moreover, students whose strategies do not have more explanation power may use other students’ strategies to clarify their own thinking. With the task in Figure 1, we observed that students who focused on number of seats being lost (Jo, Jon, and Gwen in Table 1) were initially having difficulties justifying their rules. These students made their

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explanations more clear when they were able to understand the strategies that only focused on seats around the train.

Developing Mathematical Communication and Justification Strategies

Working in small groups positions students to communicate to each other. During such communication, students are challenged to think of ways to present their mathematical ideas to others and to convince them that their ideas are valid. Similarly, they are positioned to use viable arguments to critique ideas of others. Developing mathematical communication and justification strategies supports mathematical thinking and participation in classroom activities, which leads to development of mathematical reasoning (National Council of Teachers of Mathematics, 2000).

Making Mathematical Connections

When students productively participate in small group discussions, they are able to see different ways a task may be approached. As they discuss the different strategies, they may ask: what is mathematically similar about the different strategies that enable getting the same answers to a particular task? In this case, for example, how are the strategies for Gwen and Brenda similar and different? Such connections and questioning, potentially leads to deeper understanding of mathematical concepts and is one way to make sense of mathematical ideas within their context and abstractly (Common Core State Standards Initiative, 2010).

Creating Equitable Classrooms

One of the practices for creating equitable classroom is by creating opportunities for each student to participate in verbalizing their reasoning (Bell & Pape, 2010). With time constraints, it is a challenge to create such opportunities during whole group discussions. Additionally as teachers select students for whole group discussions, their patterns of selection may privilege some students more than they may privilege others. Small group discussions create opportunities for all students to participate. Furthermore, students may be relatively freer to draw from tools and representations of their culture as they discuss mathematical ideas with their peers.

As noted from these reasons, and as widely reported by research (e.g. Imm & Stylianou, 2012), small group discussions can support mathematical understanding and align classroom activities with standards for mathematics teaching. Additionally, as Lamberg (2012) explained, “the richness of a whole class discussion is dependent on the quality of small group and partner conversations” (p. 6). Therefore, it is essential to create a context for productive small group discussions.



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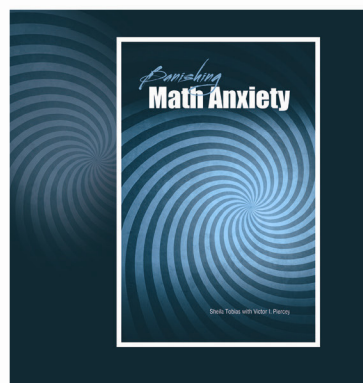
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