Digital Imaging in the Mathematics Classroom

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The National Council of Teachers of Mathematics (NCTM, 2000) emphasized the need for teachers to provide students with opportunities to learn about mathematics by working in contexts outside of mathematics. One way in which teachers can connect mathematics to such contexts is through the task of digital imaging: the process of taking a digital picture and then superimposing some type of mathematical function on top of that picture.

For example, students can take a picture of a rooftop and insert it into The Geometer's Sketchpad (GSP). The simplest method for doing this is to open the image in a program such as MS Paint, crop it, copy the image to the clipboard, and then paste it into GSP. Students then work at creating a line that has the same slope as the rooftop (see Figure 1). Teachers can pose questions to students regarding the mathematical function involved. Lessons may also be developed so that students are asked to re-size the picture either horizontally or vertically and determine how the slope of the roof changes.

Figure 1 shows a rooftop image with the linear function superimposed.

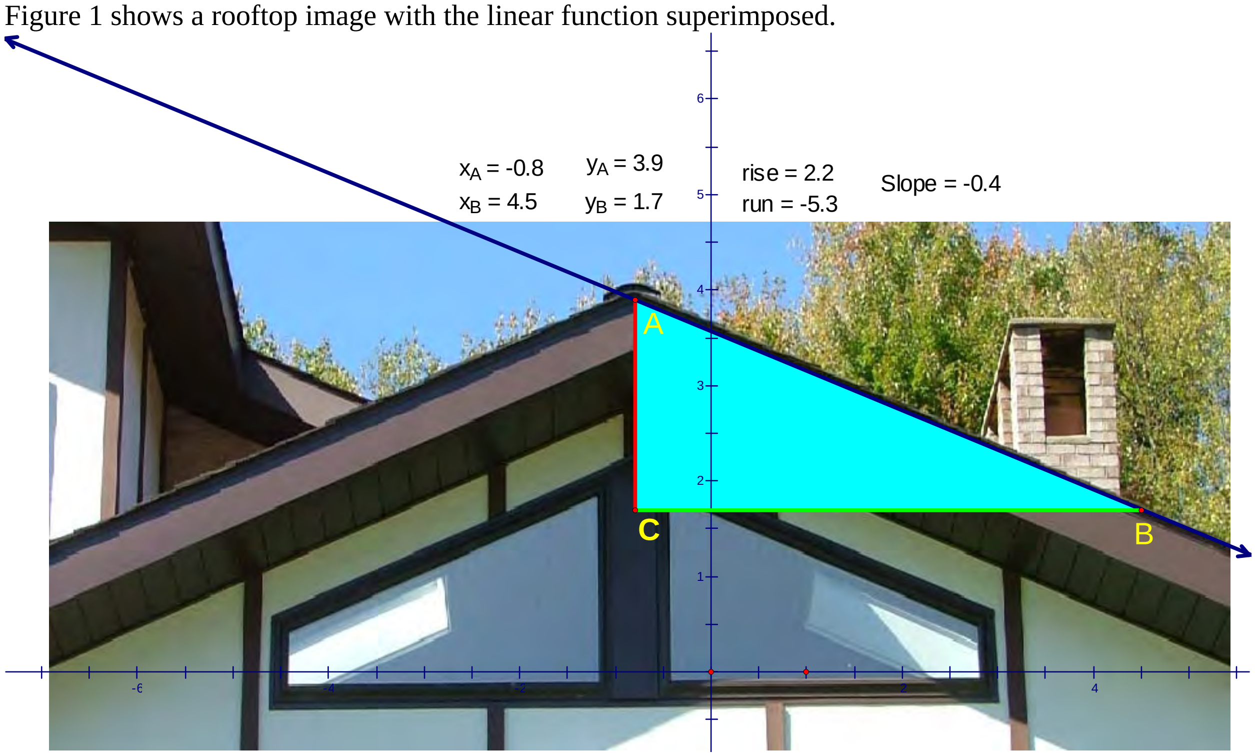

Figure 1: Rooftop of House

This image was used with a handout that posed two questions:

1. The Smiths are extremely upset because they do not like how their new addition looks from the outside. They insist that the contractors did not properly align the windows with the roof; they think that the slope of the window just beneath the roof is different than the slope of the roof. Using GSP, find out if their claims are valid. Make sure you check each side of the roof.

2. "Re-size" the roof so that it has a slope of -1 (on the right) and +1 (on the left). Use the GSP tools to determine the resulting ratio of the rooftop height to the width of the roof shown (using the image provided). What is the relationship? How did the re-sizing affect the ratio?

Students may also be encouraged to take their own pictures and write questions related to those pictures as well as superimpose a mathematical function over the image. Figure 2, the side view of a windshield, was taken by one of my students.

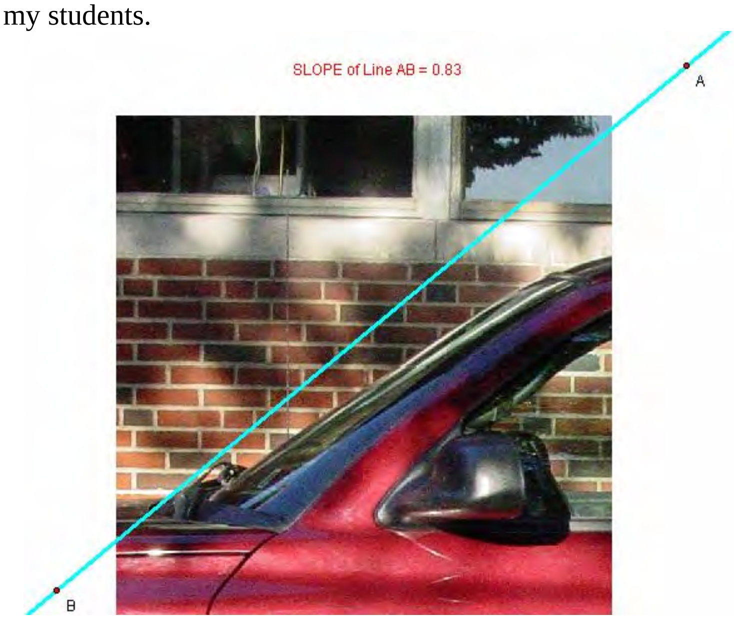

Figure 2: Vehicle Windshield

He posed the following scenario and questions for the picture:

The slope of a windshield varies from vehicle to vehicle. Some large trucks have windshields that are nearly vertical, while sports cars have windshields which are closer to being horizontal. Using the figure above, drag point A and move it slowly so that the line is parallel with the windshield.

a. What is the slope of this line?
b. If the average pick-up truck has a windshield with slope of 0.5 to 0.6, how does your slope for this vehicle's windshield compare to the average pick-up truck?
c. Is this vehicle's windshield steeper or less steep than most pick-up trucks?
d. Do you think this vehicle is a pick-up truck? Why or Why not?
e. How could you measure the slope of your vehicle's windshield at home?

In the process of taking pictures, many of the students wanted to use a chain suspended from both ends to represent a parabola (see Figure 3). However, when they tried to find an equation of the function that matched their picture on GSP, they had difficulty. I tell my students not to feel bad, Galileo also thought this curve was a parabola. (However, I am not sure that Galileo's similar mistake provides them much comfort.)

Figure 3: Chain Fence

In fact, the chain suspended from its endpoints is not a parabola, but rather a catenary. This misconception provided an opportunity for me to discuss the characteristics of parabolas and to introduce the catenary, a curve that most of them had never heard of before. Furthermore, we were able to discuss the relationship between the parabola and the catenary.

The American Heritage dictionary refers to a *catenary* as "the curve formed by a perfectly flexible, uniformly dense, and inextensible cable suspended from its endpoints". It is identical to the graph of hyperbolic cosine. Figure 4 is a sample graph of a catenary on the coordinate plane:

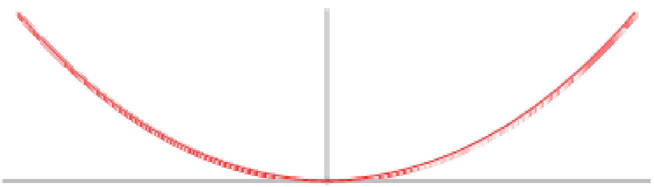

Figure 4: Graph of Catenary

The equation for a catenary is given by: $y = (^{a}/_{2})(e^{x/a} + e^{-x/a})$ or $y = a \cosh(x/a)$

As an extension of the catenary versus parabola discussion, I ask students to use GSP to determine whether the Gateway Arch in St. Louis, Missouri is a parabola or a catenary (Figure 5). The curve looks parabolic, but is, in fact, a catenary. Furthermore, it is a great challenge, to have students find an object in the real-world that does simulate a parabola.

Figure 5: Gateway Arch

The discussions which ensue within the process of using digital imaging are invaluable to a mathematics classroom. They provide a starting point for helping students learn about mathematics in contexts outside of the classroom. Furthermore, digital imaging is a great tool for relating mathematics to contexts with which students are familiar.

Digital imaging in mathematics gives students an opportunity to show they know and understand mathematics while at the same time integrating their world into the classroom in a meaningful manner. Do you find yourself looking at objects in our world and thinking about superimposing a mathematical function on them? This is exactly what my students tell me happens to them. Many of them bring in pictures or ideas about how mathematics relates to something in their world. Exploring and evaluating objects that are found in the real world is, for some, the true fun and beauty of mathematics!

References:

The American Heritage® Dictionary of the English Language: Fourth Edition, 2000.

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics,* NCTM: Reston, VA.

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