

Exploring Exponential Functions

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Part A - Playing with Rice

The Double Rule

I. Gathering the data

1. There are many grains of rice in the container labeled zero. Designate one member of the group as the banker and another member of the group as the starter. The starter starts with the zero container and writes the ordered pair $(0, 1)$ on a sticker to represent that there is one grain of rice before the banker makes any distributions.
2. Distribute the empty cups as evenly as possible to all members of the group. Some group members may need to take more than one cup.

Round 1

The banker gives the person with container 1 double the grains of rice that are in container 0. The person with container 1 writes the ordered pair $(1, \text{\# of grains in this container})$ on a sticker to represent that there are this many grains of rice on round 1. Place the sticker on the graph at the appropriate point.

Round 2

The banker gives the person with container 2 double the grains of rice that are in container 1. The person with container 2 writes the ordered pair $(2, \text{\# of grains in this container})$ on a sticker to represent that there are this many grains of rice on round 2. Place the sticker on the graph at the appropriate point.

Rounds 3-6

Repeat in the same pattern.

II. Creating the graph

Draw a smooth curve to connect the points.

III. Analysis Questions (Note: some answers might be fractions)

1. If the rounds continued, what would be the number of grains of rice in container 8? container 9? container 10? If possible, determine the largest number of grains that might be in any container. Justify your answer.
2. Imagine that there is a container numbered -1. How many grains of rice do you think are in container -1? Container -2? Container -3? Will the number of grains of rice ever equal 0? Justify your answer.
3. The next challenge is to create an algebraic representation of the number of grains of rice in each container. In other words, construct a formula for $f(x)$ where x is the number of the container and $f(x)$ represents the number of grains of rice in that container.

The Half Rule

I. Gathering the data

The starter must count the number of grains of rice in the zero container, write the ordered pair $(0, \# \text{ of grains of rice})$ on a sticker, and place the sticker on the graph at the appropriate point. The starter now divides the rice into two equal piles and gives one pile to the person with container 1. The pattern of play continues for 6 rounds.

II-III. Creating the Graph and Analysis Questions

Repeat from the Double Rule tasks.

The Third Rule

I. Gathering the data

The starter must count the number of grains of rice in the zero container, write the ordered pair $(0, \# \text{ of grains of rice})$ on a sticker, and place the sticker on the graph at the appropriate point. The starter now divides the rice into three equal piles and gives one pile to the person with container 1. The pattern of play continues for 4 rounds.

II-III. Creating the Graph and Analysis Questions

Repeat from the Double Rule tasks.

Part B - Discovering and Analyzing Exponential Functions

Finding a Function to Match the Story

Let x represent the day and $f(x)$ represent the number of grains of rice the king give to Pong Lo on day x . In the story there is one grain of rice on day 0, so $f(0) = 1$. The first row of the table is done for you. Complete the table of values for the first 10 days according to the system used in the story.

x	$f(x)$	$f(x) = 2^x$
0	1	2^0

1. What relationship do you see between the first and third column?
2. Create a function to represent this pattern. Write the function and explain why you believe it works.
3. Graph the function represented by the table of values you have constructed.

Changing the Initial Amount

In this section we consider how the function changes if the initial amount of rice changes. For example, assume Pong Lo begins with 7 grains of rice. On the each subsequent day the king will give 14 grains of rice, 28 grains of rice, 56 grains of rice, etc.

Complete a table for the first 10 days for each of the following situations.

Case 1: Assume Pong Lo began with 3 grains of rice.

x	$f(x)$	2^x	Express $f(x)$ using 2^x
0	3	$2^0 = 1$	$3 \cdot 2^0$

- Write a function to represent the pattern when Pong Lo begins with 3 grains of rice: $f(x) =$ _____
- Graph the function represented by the table of values you have constructed.

Case 2: Assume Pong Lo began with 5 grains of rice.

x	$f(x)$	2^x	Express $f(x)$ using 2^x
0	5	$2^0 = 1$	$5 \cdot 2^0$

- Write a function to represent the pattern when Pong Lo begins with 5 grains of rice: $f(x) =$ _____
- Graph the function represented by the table of values you have constructed.
- What part of the function changes when the initial amount of rice is changed? How are these functions related to the function $f(x) = 2^x$?

Changing the Rule

In this section we consider how the function changes if the amount of rice changes by a different rule. For example, assume the king must give Pong Lo 5 times the amount of rice from the day before. Suppose the king begins by giving Pong Lo 1 grain of rice, the next day Pong Lo gets 5 grains, the next day 25 grains, the next day 625 grains, etc. Complete the following two tables.

Tripling each day: Assume Pong Lo starts with one grain of rice. Complete the following table assuming he asks for the number of grains to be tripled each day.

x	$f(x)$	Express $f(x)$ using 3^x
0	1	
1		
2		
3		
4		

- Write a function to represent the pattern when the king triples grains of rice each day: $f(x) =$ _____
- Graph the function represented by the table of values you have constructed.

Halving each day: Assume Pong Lo starts with one grain of rice. Complete the following table assuming he asks for the number of grains to be halved each day.

x	$f(x)$	Express $f(x)$ using $\left(\frac{1}{2}\right)^x$
0	1	
1		
2		
3		
4		

1. Write a function to represent the pattern when the king halves the grains of rice each day: $f(x) =$ _____
2. Graph the function represented by the table of values you have constructed.
3. How do you think the function changes if you change the rule for how the amount of rice changes each day?

Putting it All Together

1. Suppose Pong Lo had asked the king for 7 grains of rice to be tripled each day for 100 days? Think about the tables you just created. Write a function to represent the situation and explain why you believe it works.
2. Suppose Pong Lo had asked the king for 11 grains of rice to be reduced by $\frac{1}{3}$ each day for 100 days? Write a function to represent the situation and explain why you believe it works.
3. Using the variables a , b , and x , write a general expression for an exponential function. Using the information from the activity, define each of the variables. Explain how you arrived at that rule. Use the tables that you created to help you.

Become an Author

Now you be the “wise one.” Use the idea of the folk tale, *A Grain of Rice* to write your own story that contains a situation that can be represented by an exponential function. Describe the function and construct a graph to represent it. Be prepared to share your story with the class. Be creative and enjoy!

The Power of Visualizations!

“Professor Leo Standing, a Canadian psychologist, [asked psychology students to view 100 pictures for 5 seconds each]. He brought them back in a week, and showed them the pictures again, mixed with 100 new pictures, ... The students correctly recognized more than 90% of the pictures, having seen them only once, for just five seconds. [The same 90% results were returned for 1,000 pictures, and then for 10,000.]”

Lynch, G. & Granger, R. (2008). *Big brain: The origins and nature of human intelligence*, 158. Palgrave MacMillan. New York, NY.