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CHARTS FOR LONG-DISTANCE FLYING

O. M. Miller

Wright Air Development Center
Contract No. AF33(616)-5566

For those readers familiar with the research program and publications of the Mapping and Charting Research Laboratory of The Ohio State University, we would like to call attention to the new organizational heading for this and subsequent publications.

As a result of a recent reorganization, the sponsored research formerly conducted under the Mapping and Charting Research Laboratory will be continued as a part of the program of the Institute of Geodesy, Photogrammetry and Cartography.

The new address of the Institute is 1314 Kinnear Road, Columbus 12, Ohio.

W. A. Heiskanen, Director

INSTITUTE OF GEODESY, PHOTOGRAMMETRY AND CARTOGRAPHY
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by

O. M. Miller
Cartographic Research Specialist

Report to

Wright Air Development Center
U. S. Air Research and Development Command
Wright-Patterson Air Force Base, Ohio
WCLGN

Under

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Project No. 8(610-6190)
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The Ohio State University
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FOREWORD

This report was prepared by Mr. O. M. Miller, Cartographic Research Specialist of the Institute of Geodesy, Photogrammetry and Cartography of The Ohio State University, under Air Force Contract No. AF 33(616)-5566, OSURF Project No. 816. The contract is administered under the direction of the Weapons Guidance Laboratory, WCLGN, Wright Air Development Center, Wright-Patterson Air Force Base, Ohio, with Mr. A. L. Sidnell as Contracting Officer. Project No. 816 of The Ohio State University Research Foundation covers work performed in the Institute of Geodesy, Photogrammetry and Cartography under the supervision of Dr. Weikko A. Heiskanen, Director of the Institute.

This technical paper constitutes a portion of the Heading Compatibility Study under Contract No. AF 33(616)-5566. The work is applicable to Task No. 50785 of Project No. 8(610-6190) with Lt. William Moothart, WCLGN-1, as the WADC Project Engineer.

ABSTRACT

An attempt is made to give a coordinated background to a group of papers issued by the Institute of Geodesy, Photogrammetry and Cartography on the subject of "Heading Compatibility" on long-range high-speed flights. A brief review of the present transitional state of aerial navigation is given, which concludes, that in spite of rapid development of automation in navigation, charts will still be required in flight for some time to come.

A scale of 1:10,000,000 is suggested as suitable for charts used in the intermediate stages of long-range high-speed flights, and several map projection systems on this scale are discussed from the standpoints of continuity, variation of scale and curvature of plotted great-circle routes. The possible advantages of flying routes indicated by straight lines on aeronautical charts when grid navigation is employed are reconsidered in the light of possible future technological developments.

TABLE OF CONTENTS

I. HEADING COMPATIBILITY IN THE USE OF CHARTS

This paper is one of a group concerned with a study of "Heading Compatibility" on long-distance high-speed flights. The writer does not know who coined the expression and finds difficulty in making a direct definition. On the other hand "Heading Incompatibility" would seem to refer to the complications that arise when different reference directions such as magnetic north, true north, "grid north" and different map projection systems are used in one and the same navigational operation. The expert navigator is not as a rule confused with heading incompatibilities; it is after all a simple matter to transform a heading from one system to another, especially if the situations are visualized by graphical constructions. However, on long-distance high-speed flights there is very little time available to practice the conventional methods of navigation. Thus there would seem to be a need for simplifications in the terms of reference such that various navigational and monitoring systems can be applied with equal ease all over the world.

Most of what is written in this paper may not be exactly relevant to the subject of heading compatibility and may be obvious or commonplace knowledge to many pilots and navigators and to cartographers, instrument designers, and mathematicians interested in navigational problems. The attempt will be made nevertheless to give in relatively simple terms an outside-looking-in perspective concerning the use of charts on high-speed long-distance flights.

II. AERIAL NAVIGATION IN A STATE OF TRANSITION

Anyone who has given the subject any consideration knows that aeronautical charts were initially designed to aid in the planning and the execution of flights on the assumption that land would be visible while flying [0]. Moreover contact flying when this is possible either by eye or radar is still considered by many to be the best way of getting from one place to another. As the range of aircraft increased and long flights out of sight of land were undertaken utilizing dead reckoning techniques, magnetic compass bearings and astrofixes, aeronautical charts were still required for planning and plotting purposes because much of the work of keeping track of position and direction could be most easily undertaken by graphical constructions on the charts. If magnetic compasses are used, it is still essential to delineate on the charts lines of equal magnetic variation in order to determine the requisite changes in magnetic compass heading to be made during flight.

The introduction of gyroscopically controlled true-north-indicating compasses simplified the navigator's chart work but introduced new computational and mechanical problems. Notwithstanding, it was only a short step to the introduction of automatic pilots controlled by automatically precessed gyroscopes. And later the introduction of inertial and doppler systems of navigation would seem to forecast that eventually charts and human pilots will no longer be required for many types of flight operations.

The calibration, maintenance and operation of the instruments involved in completely automatic navigation cannot at this stage be considered altogether reliable; and monitoring systems—electronic ground aids to navigation on established airways and astrofixes elsewhere—are still deemed necessary. These monitoring systems themselves are subject to heading incompatibilities and to errors which vary with circumstances. Thus any completely automatic navigational system which includes monitoring inputs should have the capability of translating these to a common denominator and weighting them [1]. To laymen, and this probably includes even the expert pilot and navigator and most certainly this writer, this thought is somewhat overwhelming— though no doubt the mathematicians and technicians can solve the problem for certain types of airborne vehicles.

There would seem to be, at any rate in the present state of the art, a reasonable desire amongst a considerable body of experienced navigators to keep control of the intermediate stages of a long-distance flight, so that individual initiative and judgment can be used in making navigational decisions. Under these circumstances perhaps the ideal solution to the problem is a compromise. This would consist in automation up to the point where decisions have to be made. Conveniently the dead reckoning position and the monitoring evidence confirming or refuting it would be displayed to the navigator or pilot in a way that final decisions concerning position and the correct course to be taken at any stage of a flight could be easily reached.

Reduced to simplest terms, most individual checks on dead reckoning whether initiated on the aircraft or on the ground result in a line of position. The intersection of two such position lines, provided the checks are made simultaneously, gives a point of position. If the checks are not made simultaneously——and this is a situation commonly experienced——an envelope of position lines results which if plotted on a chart can be interpreted by the skilled navigator taking into account the relative change in dead reckoning position between checks.

It may be argued that on very long distance flights position checks need not be frequent except at the beginning and end of such flights. Nevertheless the time available to make plotted position-line checks on charts useful is more or less inversely proportional to the speed of the aircraft. This is, of course, the reason why every effort is being made to supply navigators with automatic computers to transform monitoring input into simple output which the pilot can use without chart work, such as distance and heading to destination.

The completely automatic or partially automatic solutions can hardly be available as yet on the majority of manned aircraft. Furthermore, in the event of war reliable maintenance of highly complex apparatus becomes increasingly difficult, and communication over long distances between ground and air stations may be hazardous or altogether impossible. Thus the old and proven techniques of navigating out of sight of land with the use of charts will still be desirable for many types of operations for some time to come and must in any event be kept in reserve in case of emergency and not forgotten.

III. CHARTS ON THE SCALE OF 1:10,000,000

If the foregoing statement is valid and considering the comparatively recent advent of extremely high speed aircraft capable of long-sustained flights without refueling, a need would seem to have arisen for navigating charts without map projection discontinuities covering much longer distances than heretofore. The physical dimensions of a single chart however must not be so large as to make its handling difficult. A great-circle arc 5000 nautical miles in length is represented on the scale of 1:1,000,000 by a line over 30 feet long.

There is therefore a trend towards smaller scales for the charts to be used in long-distance high-speed flights. One has only to examine the most recent issue (current as of July 1959) of the excellent USAF Catalog of Aeronautical Charts, published by the ACIC [2], to realize that this is true. In addition to a variety of sectional charts on scales of 1:1,000,000 and larger, including the International World Aeronautical Navigation Charts, there exist amongst other series the USAF Jet Navigation Charts on the scale of 1:2,000,000, the USAF Global Loran Navigation Charts on the scale of 1:5,000,000 and the USAF Minimal Flight Planning Charts on the scale of 1:10,000,000. Admittedly the purpose of this last series is for "planning long range flights exploiting the advantages of pressure pattern navigation." Nevertheless the time may be rapidly approaching when the scale of 1:10,000,000 may be considered quite suitable for inflight navigation on long flights, provided larger scale charts are used at the beginning and end of such flights. On this scale 1 inch represents about 137 nautical miles. One nautical mile is represented by about 0.007 inch or about twice the width of the finest line that can be plotted or printed on a chart.

With a scale of 1:10,000,000 in mind brief consideration will be given now to the suitability of several map projection systems, including those in current use for larger scales, from the standpoints of continuity, variation of scale and curvature of plotted great-circle routes, and the differences between great-circle routes and the corresponding routes indicated by straight lines on the charts. The possible advantages of flying the latter type of route over long distances when grid navigation techniques are employed will also be briefly discussed.

IV. GREAT CIRCLES AS STRAIGHT LINES

It would obviously be highly desirable were it possible to show all great-circle routes as straight lines on charts. The gnomonic projection and the two-point azimuthal projections, that can be derived from it by perspective transformations, have this property. They are not used for inflight charts because a single projection of this type cannot be extended to cover large regions and because, within the regions that are mappable, scale variation is excessive and is not constant in all directions in the neighborhood of a point.

Taking the case of the gnomonic projection with a geographical pole at the center, the scale varies along a meridian as the secant squared of the colatitude and the scale of the parallels of latitude varies as the secant of the colatitude. Thus, if we define the scale ratio as the scale in the neighborhood of a point divided by the nominal scale of the chart, whereas the scale ratio at the pole would be 1, at latitude 60° it would be 1.333 along the meridian and 1.155 along the parallel. These numerical examples for the gnomonic projection of the disparity of scale along a meridian and normal to it and the consequent distortion of angular relationships emphasize the need for conformal projections, particularly if charts are to be used for plotting position lines.

V. CONFORMALITY AND THE REPRESENTATION OF MERIDIANS AS STRAIGHT LINES

On a conformal map projection all points (with possible exceptions called singular points) are mapped so that angles about them are correctly represented, and it follows that scale in the neighborhood of a single point is constant in all directions. This unfortunately does not imply that scale is constant along plotted great circles or that these circles are represented as straight lines. That angles are correctly represented is of great importance in extracting directions from a chart, as is illustrated in Figure 1 (a, b, c), page 17. Even though in b one great circle is curved and in c both great circles are curved, it is easy to measure the intersecting angles with a protractor by constructing tangents to the curved lines at the points of intersection.

Thus, in spite of the fact that conformal projections as a group cannot in general show great-circle routes as straight lines, the use of such projections for all types of aeronautical charts regardless of scale is now almost universal. There are innumerable conformal map projections; but the further desirability of showing meridians as straight lines or nearly so, thus making the recovery of course headings with respect to true north quite simple, further limits the projections most suitable for inflight navigation to the conic conformal projections together with the limiting stereographic and Mercator, including in the last case strip charts based on transverse and oblique forms.

VI. PROJECTION CONTINUITY

In using the conic conformal types of projection, in order that there should be no great variation in scale along a great-circle route or excessive curvature of the plotted representation of this route, it is customary to divide the surface of the earth into a series of zones of latitude and to base each on a different projection. It is now rapidly becoming standard practice on the World Aeronautical Charts and the corresponding USAF Operational Navigation Charts, both on a scale of 1:1,000,000, to use 4° zones of latitude from the equator to latitude 80° with standard parallels in each zone $2^{\circ}40'$ apart, and to use the stereographic projection for the remaining polar regions. This of course enables scale variation to be kept within extremely low limits, and great-circle courses within each zone of latitude are portrayed with very little curvature. However, because the representation of the convergence of the meridians changes from zone to zone, angular discontinuities must occur in the plotting of almost every great-circle course which passes from one zone to another. Should, for example, a long-distance flight be planned to start in the neighborhood of the equator and to end at latitude 62°, with the end points varying in longitude, 15 projection discontinuities, that is to say, 15 different convergence factors, would be involved. It would seem obvious, therefore, as far as continuity is concerned, that the old system for the WAC charts, which divided the range of latitude into four zones, namely, 0°-24°,

24°-52°, 52°-72° and 72°-90°, is preferable for long-distance flights.

At this point a few numerical examples may be revealing. Referring to Figure 2, page 17, if L_1 and L_2 are the lower and upper limits of latitude for one zone, the longest possible great-circle route within this zone will be one that starts and ends at latitude L_1 and reaches latitude L_2 at its midpoint C, where it will cut a meridian at right angles. By spherical trigonometry, if d is the arc distance of this maximum great-circle route,

$$\cos \frac{d}{2} = \frac{\sin L_1}{\sin L_2} .$$

Table 1 shows the values of this maximum d in nautical miles for all zones of latitude of the new projection system. Table 2 shows the same for the old system. In either case it is interesting to note the rapidity with which d decreases as latitude increases. In practice, of course, the vast majority of flight lines of lengths comparable to those listed in Table 1 or 2 must be indicated on two or more zones of the projection system. In many cases much shorter flight lines must also be so indicated. From the navigator's point of view, particularly inconvenient cases of this kind occur when a flight line crosses over from one zone to another for only a short distance and returns to the first zone. This can be obviated to a certain extent by providing considerable overlap between zones. To do this most effectively, however, it would be necessary to change the spacing of the standard parallels to minimize scale anomalies.

Of special interest in this connection is the Mercator projection. It can be easily shown that if the arc width of a strip chart on a normal, transverse or oblique Mercator projection is equal to 2α, all great circles intersecting the central great circle at an angle less than α will be shown in their entirety on the chart provided the strip is sufficiently extended.

It is for this reason that from time to time the suggestion has been made that a system of oblique Mercator charts be designed and compiled so that long great-circle routes can be depicted without projection discontinuity. The implications of such a system for charts on a scale of 1:10,000,000 from the standpoints of both the cartographer and navigator are discussed later in this paper and also in the preceding paper of this series.

VII. SCALE AND CURVATURE CONSIDERATIONS

Navigation methods which use monitoring systems such as Loran [3] or the British Dectra [4] depend on the delineation on charts of precomputed networks of intersecting position lines. Two types of position lines can, however, be plotted on charts while in flight with reasonable accuracy. One is the locus of a distance from a point, namely, a small circle on the sphere, and the other is a line of constant direction from a point. It is important in plotting these types of position lines that the scale variation of the map projection and the curvature in the representation of great circles be small.

In order to get some idea as to scale variation and curvature Table 3 has been prepared. Referring to Figure 3, page 17, which gives a generalized diagrammatic outline of part of a conical projection, points A and C lie on the selected limiting latitudes of the projection. D is a point on the line of minimum scale. B and B' represent points at the same latitude on the great circle passing through C and thus cutting the meridian at C at right angles. They are assumed in Table 3 to be 150 nautical miles from C, making the total distance between B and B' 300 nautical miles. The table is otherwise self-explanatory. It should be noted however that the figures given are only approximate as the earth has been assumed to be a sphere. Also the total change in the direction of the representation of the great-circle arc BCB' on the chart is an approximation based on the assumption that this representation is the arc of a circle satisfying the formula

$$\tan \frac{\beta}{4} = \frac{\Delta y}{\Delta x}$$

where Δx and Δy are the differences in the map coordinates of B and C and β is the total change in direction.

The maximum curvature (the reciprocal of the radius of curvature) of the representation on a chart of an elementary arc of a great circle is found in the neighborhood of a point where the rate of change of scale is at a maximum in a direction normal to the elementary arc [5]. On the projections considered this would occur at either point A or C. However the curvature depends on the scale of the chart. For the same elementary arc it would be 10 times as great, for example, on the scale of 1:10,000,000 as on the scale of 1:1,000,000. For this reason and on the assumption that the figures given in the last two lines of Table 3, though obtained rather informally, give a realistic and sufficiently accurate picture of maximum change in direction regardless of scale, the actual maximum curvatures were not computed.

It requires little thought to realize that on the vast majority of flight routes, which come within the limits of each projection being discussed, the overall directional changes in their representation on the charts will be very much smaller than the quantities given in Table 3. Moreover long distances scaled from these charts will on the whole have considerably greater accuracy than the scale ratios at A, C and D would seem to indicate.

It may be concluded at this point that there is little to choose between the four projections under discussion provided the limits of latitude suitable for each projection are not greatly exceeded. Moreover the figures given in Table 3 for the normal Mercator are equally applicable to the transverse and oblique forms of this projection by substituting arc width of strip for range of latitude.

VIII. ALTERNATIVE PROJECTION SYSTEMS FOR CHARTS ON THE SCALE OF 1:10,000,000

The conic conformal projection systems so far discussed would be quite suitable for 1:10,000,000 charts for use on long-distance flights were it not for their lack of projection continuity. Several alternative systems are possible.

In the USAF Jet Navigation series of charts on the scale of 1:2,000,000 transverse Mercator projections are used in the polar regions, and each hemisphere is otherwise covered by two conic conformal projections with standard parallels at latitudes 37°-65° and 6°-30°. The sheets in this series are very large but can be conveniently cut into strips for inflight use. Should this projection system be reproduced on the scale of 1:10,000,000 the sheet sizes would of course be more practicable. However though the continuity problem is considerably relaxed in this series, the scale and curvature anomalies are larger than on the old system of projections for the WAC charts.

Another approach is to divide the earth's surface into a series of regions each bounded by four great-circle arcs such that adjacent sheets match at their borders, even though a separate projection is made for each sheet. The aim here is to obtain the largest average area coverage for the sheets within which scale and curvature anomalies introduced by the map projections can be kept within practicable limits. This approach has been investigated by Dr. Arthur J. Brandenberger and fully described by him in another technical paper in this series [6].

A set of strip charts on oblique Mercator projections already mentioned in this paper is a third approach. The objective here is to provide the minimum number of strip charts which will show every conceivable great-circle course on at least one strip. This problem has been solved by Mr. Jesse Schreiter for strips 30° of arc in width. The system of 40 charts that results is described and fully illustrated in his paper [7].

At the risk of repeating some of the thoughts in Mr. Schreiter's paper, it seems desirable to comment on the cartographic practicability of this set of oblique Mercators when reproduced on the scale of 1:10,000,000 as this scale is being given special consideration in this paper. One objection to the system is that it is extremely uneconomical in that there must of necessity be much more overlap than is customary. This situation is not as serious as it might at first appear. In compiling ground features on the overlapping charts such as coast lines, major drainage features and the location of major cities, airports and so on, details must be considerably generalized on the scale of 1:10,000,000. Once they have been compiled on one strip they can be transferred to another overlapping strip with little or no adjustment because the projections are conformal and because the variation in scale and the curvature are so small on all strips. The computation and plotting of the map projection graticules will be the most exacting task in the production of this series. In the Schreiter solution however as many as 8 strip charts may be on projections which are identical save for the fact that the poles of the projections vary in longitude. His solution actually calls for the computation and drafting of only 6 projection graticules.

One distinct cartographic advantage that oblique Mercator charts have over the other projection systems discussed would be the uniform rectangular shapes of all sheets. On the scale of 1:10,000,000 one strip would have dimensions of 13.05 x 154.86 inches, and if each strip is divided into 6 sheets their dimensions would be 13.05 x 25.81 inches.

These dimensions are sufficiently small to make it practicable to reproduce this series of charts as continuous strips on microfilm. Thus if it is possible to use an optical projecting apparatus in flight, then, with suitable mechanism, the projected image of a strip could be moved continuously over a small screen at a speed proportional to the estimated ground speed of the aircraft. In this way this series of strip charts could be incorporated in a simple manner into various display systems and tracking devices.

Extracting directions from oblique Mercator charts would not be quite so convenient as in the case of the normal conic conformal projections, because meridians would not in general be represented exactly as straight lines and each sheet would not have a constant meridional convergence factor. Nevertheless it may be argued that continuity of map projection is sufficiently an asset for high-speed flights over long distances to offset this disadvantage, which is relatively unimportant provided the map projection is conformal (see Figure 1).

Though the three map projection systems that have been discussed here are probably the most promising for small-scale charts, there is a fourth which is not without interest. It would be truly complementary to the Schreiter system and would consist of 80 stereographic projections centered at the poles of the 40 oblique Mercators. Each projection would extend at least 15° of arc from its center. Such a system of projections would completely cover the earth's surface. Every sheet would be exactly the same size and have the same excellent scale and curvature characteristics. However, because adjacent sheets would not match along their edges, lack of continuity would rule this series out for the purpose of long-distance navigation.

IX. GRID TRACKS VERSUS GREAT-CIRCLE ROUTES ON LONG-DISTANCE FLIGHTS

Great-circle routes for flying are considered desirable because they are the shortest routes on the earth's surface. They are not necessarily the routes flown over in the shortest time, owing to the effects of wind, and they have never been easy to navigate precisely owing to the necessity for continually changing the course heading in respect to true north if a true north indicating compass is used or in respect to magnetic north if a magnetic compass is used. Thus when a great-circle route is to be followed the conventional practice is to break it down into a number of short legs which are then flown on the assumption that the average heading of a leg can be followed without introducing significant errors in course position. Only on the normal Mercator projection are lines of constant bearing (rhumb lines) truly straight lines.

If another route could be navigated more easily than a great-circle route then, provided it did not exceed the latter in length by some acceptable amount, there seems little reason why such a route should not be chosen. It would not matter if it deviated considerably in other respects from the great-circle route. At any rate such a statement would seem to be valid if the steering is done manually with reference to a direction indicating instrument.

Grid navigation has been introduced for the practical purpose of reducing the number of changes in course headings during a flight between two places. It is, as is well known, of particular importance for flights in the polar regions [8] where extremely rapid changes in true or magnetic north headings must in general occur. For grid navigation a grid of parallel lines is superimposed on a chart. Obviously if a straight line between any two points is drawn on the chart, this line cuts the grid lines at a constant angle. These straight-line routes on charts are used in grid navigation and have conveniently been called grid tracks.

Grid navigation can only be wholly effective on long-distance flights provided grid track routes are flown and provided a direction indicating instrument (such as a gyroscopic compass) can be controlled so that the horizontal direction indicated by the instrument will correspond to that indicated by a grid line on the chart at the planned position of the aircraft at any stage of the flight [9]. Under these circumstances, so long as the aircraft keeps on schedule and course, the grid heading will be constant.

Should, however, by means of some position fix during flight, the aircraft be found to be behind or ahead of schedule or be off course, such a gyroscopically controlled instrument would have to be corrected. We are not concerned in this paper with the mathematical and mechanical problems involved in doing this effectively, but it is pertinent to point out here that a straight line drawn on the chart from the position of the fix to the point of destination would immediately indicate the new grid track and its "constant" grid bearing.

Now that grid navigation is becoming a generally accepted technique, the alternative of flying long-distance routes represented by straight lines on a chart, rather than great-circle routes broken down into short legs, is receiving considerable attention.

Neglecting the effects of the earth's rotation and assuming the earth to be a sphere, if an aircraft is trimmed to fly straight and level, its natural course, provided that there are no cross winds or other disturbing influences, will be along a great circle. If such abstract conditions were possible the pilot would be able to set the aircraft on a predetermined heading at the start of the flight and the aircraft would fly the indicated great-circle route without further attention. In reality such conditions will exist for only short periods of time if at all. Consequently the aircraft must be either periodically or continuously trimmed and kept on course by a human or automatic pilot. Over the short distances which an aircraft would be allowed to fly free of control, the great-circle representation would be indistinguishable from a straight line on the chart on all map projections suitable as bases for aeronautical charts.

Thus, provided the mechanical problem can be solved of controlling airborne compasses which will indicate grid north (grid direction) as easily or nearly as easily as true north, the advantages from the standpoints of graphics of flying grid tracks rather than great-circle routes become apparent, providing the lengths of the former do not greatly exceed those of the latter.

A few examples indicate that the actual excess of distance is very small for flight paths even as long as 6000 nautical miles. Referring to Figure 3, if we now suppose that the points B and B' are 6000 nautical miles apart and that the midpoint C is 15° from a point of minimum scale D, the difference between the great-circle route and that indicated by a straight line on the chart in the case of the conformal conic with standard parallels at 55° and 65° would be only 15 nautical miles or 0.25%. Under the same conditions, if B and B' are 3000 nautical miles apart, the difference is only 5 nautical miles or 0.17%. In the case of the normal and oblique Mercator projections, under the same conditions, for the 6000 nautical mile route the difference would be 32 nautical miles or 0.53% and for the 3000 nautical mile route 6 nautical miles or 0.2%.

In oblique Mercator strip charts having an arc width of 30° it appears (no formal proof is offered) that the maximum differences between great-circle and grid tracks occur for great-circle routes of the BCB' type. Within this subset of all possible great-circle routes the maximum difference, as shown on the graph in Figure 4, is 0.57%. At this maximum the great-circle route is about 120° or 7200 nautical miles in length. Note that the differences are zero when the length of BCB' is 0° or 180°. In the latter case this is because the grid track is also a great circle.

When charts are used in the navigation process another point in favor of flying grid tracks rather than great-circle routes is that in most cases the scale departures will be less. For example in the BCB' type of route on the oblique Mercator strip charts, whereas the scale change and curvature of the great-circle route are at their worst, the grid track route is not only straight but is of constant scale.

It is frequently pointed out that grid navigation on the conic conformal type of projection is simpler than on the oblique Mercator type because on the latter the meridians are not strictly straight lines and the convergence of the meridians is not a constant. There is nevertheless one property of the oblique Mercators which they share with the normal Mercator but which is lacking in the conformal conic projections. If the grid is oriented so as to be perpendicular to the central great circle of an oblique Mercator projection, all grid lines will represent great circles which on the sphere will meet the poles of the projection. Thus the concept of a mechanically controlled airborne instrument which will continuously point to grid north can be replaced by the simpler concept of an instrument which continually indicates the direction to a single point on the earth's surface.

X. THE USE OF STATIONARY SATELLITES IN NAVIGATION

The above property, unique to the Mercator class of projections, may possibly have significance in the future. If and when technology succeeds in producing a means of transmitting a directional signal which can be picked up at distances of about 90° with sufficient accuracy, and should the sources of these signals be established at the geographical positions corresponding to the 80 poles of the 40 oblique Mercator strips proposed by Schreiter, then indeed would the problem of heading incompatibility be solved, conversions between true, magnetic and grid headings would become unnecessary and grid tracks of constant heading could be flown with ease in any part of the world.

In this technological age it is perhaps not embarking too far into the sea of fantasy to suggest the establishment of stationary satellites at the 80 poles. This proposal is based on the assumption that radio compasses or their equivalent would work effectively and accurately if the points from which the directional signals were received were far above the ground. Quite apart from the uses such stationary satellites would have in simplifying terrestrial navigational problems---and several intriguing possibilities will probably leap to the mind---they may prove, as has been frequently suggested, of great importance in the monitoring of other kinds of activities. It is in this connection that the system of 80 stereographic projections that is complementary to the Schreiter system might be found to be of real value.

In conclusion, the potentials of a system of oblique Mercator strip projections for aeronautical charts on the scale of 1:10,000,000 would seem to be sufficient to recommend their compilation as an auxiliary series. It is not suggested however that they should replace, for example, the conic conformal systems already in use on larger scales.

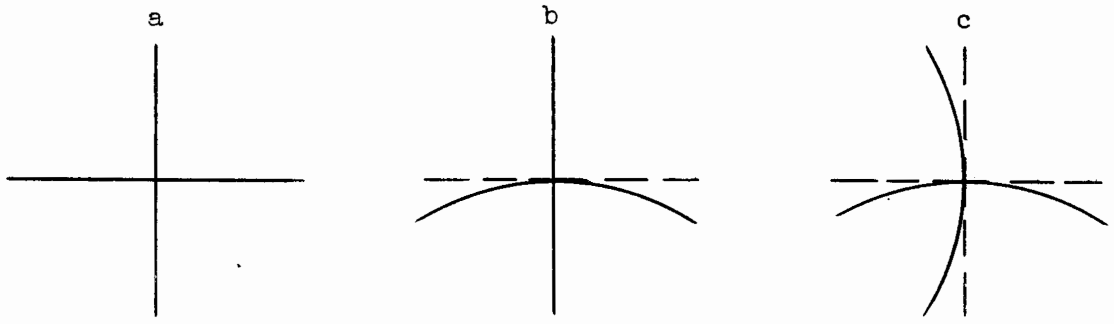

Figure 1.

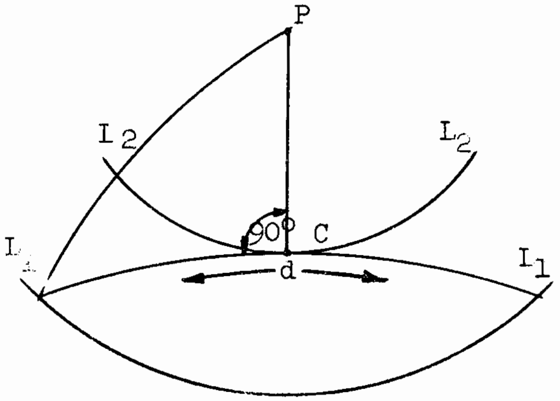

Figure 2.

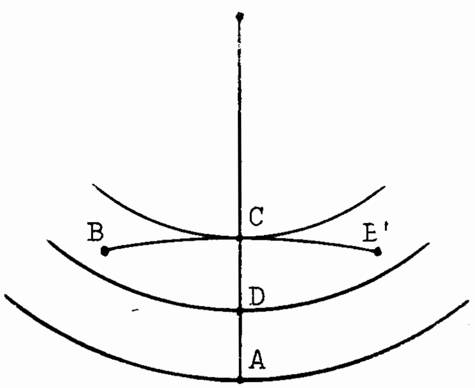

Figure 3.

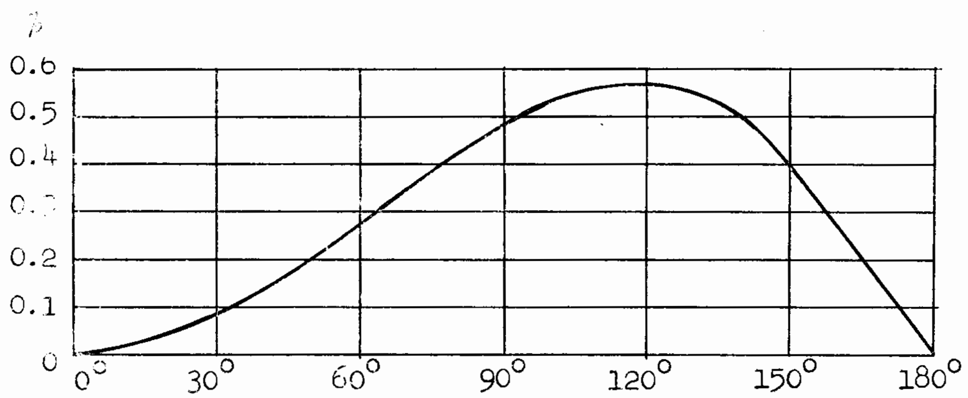

Figure 4. Percentage increase of grid track over great-circle route for B C B' type.

Table 1

L_1	L_2	d (nautical miles)
0°	4°	10800
4	8	7188
8	12	5756
12	16	4924
16	20	4357
20	24	3932
24	28	3597
28	32	3316
32	36	3078
36	40	2866
40	44	2674
44	48	2496
48	52	2332
52	56	2172
56	60	2017
60	64	1864
64	68	1706
68	72	1544
72	76	1370
76	80	1182
80	90	1200

Table 2

L_1	L_2	d (nautical miles)
0°	24°	10800
24	52	7072
52	72	4086
72	90	2160

Table 3

Property	Conic Conformal Projection 1	Conic Conformal Projection 2	Conic Conformal Projection 3	Normal Mercator Projection
Standard parallels	55° - 65°	33° - 45°	7° - 20°	± 10°38'
Selected limits of projection	52° - 72°	24° - 52°	0° - 24°	± 15°
Range of latitude	20°	28°	24°	30°
Scale ratio at A	1.0056	1.0277	1.0214	1.0175
Scale ratio at C	1.0217	1.0222	1.0107	1.0175
Minimum scale ratio at D	0.9962	0.9945	0.9936	0.9828
Directional change on chart of great-circle representation between B and B'	1°22'	1°15'	0°54'	1°21'
Maximum rate of directional change of great-circle representation per 100 nautical miles (approx.)	27' at C (13' at A)	26' at C (24' at A)	(19' at C) 23' at A	27'

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For the Institute of Geodesy, Photogrammetry and Cartography:

Principal Investigator *[signature]* Date Dec 14 1959

Project Supervisor *[signature]* Date Dec. 15 1959

Institute Director W. A. Heiskanen Date Dec. 17 1959

For The Ohio State University Research Foundation:

Executive Director *[signature]* Date Dec 23 1959