DEVELOPMENT OF A SINGLE TOOTH BENDING EXPERIMENTAL METHODOLOGY TO EVALUATE FINE PITCH THIN RIM GEARS

THESIS

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ABSTRACT

In this undergraduate research study, tooth bending fatigue lives of two nominally equivalent gear are evaluated. A methodology to design and develop customized fixtures to enable fatigue experiments of test gears on hydraulic load frames is proposed. The fabricated test fixtures are incorporated in a load frame and two sets of bending fatigue tests with both gear variations are performed. Various statistical analyses of the collected fatigue data are performed to determine whether any statistically significant difference in the fatigue lives of two gears could be detected.

DEDICATION

To my family, friends, and anyone who has helped me get to where I am today.

ACKNOWLEDGEMENTS

I would like to thank Dr. Ahmet Kahraman for giving me the opportunity to complete this thesis and giving me continuous advice and support. This opportunity has been mentally challenging and has helped me further my education greatly. I would also like to thank Dr. David Talbot for serving on my defense committee and Dr. Isaac Hong for the vast amount of guidance he has provided me throughout the thesis process. To other students and co-workers in GearLab, thank you for the support and creating an academic environment filled with challenges and learning. Finally, I would like to thank my family and friends for the continuous support you all have provided and pushing me to be my best every day.

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TABLE OF CONTENTS

Abstract…………………………………………………………………………. i

Dedication……………………………………………………………………… ii

Acknowledgements……………………………………………………………... iii

Vita……………………………………………………………………………… iv

Table of Contents……………………………………………………………….. v

List of Tables……………………………………………………………………. vii

List of Figures…………………………………………………………………... viii

Nomenclature…………………………………………………………………… x

Chapter 1: Introduction…………………………………………………………. 1

1.1 Background & Motivation……………………………………………... 1

1.2 Literature Review……………………………………………………… 3

1.2.1 Gear Single Tooth Bending Experiments……………………. 3

1.2.2 Statistical Analysis of Fatigue Data………………………….. 4

1.3 Scope & Objectives……………………………………………………. 7

1.4 Thesis Outline…………………………………………………………. 8

Chapter 2: Experimental Methodology…………………………………………. 9

2.1 Introduction……………………………………………………………. 9

2.2 Experimental Setup……………………………………………………. 9

2.3 Definition of Anvil Contact Points for STB Fixture……………………. 16

Chapter 3: Experimental Results and Discussion……………………………….. 28

3.1 Introduction………………………………………………….………… 28

3.2 Validation of Test Fixture and Contact Points…………………………. 28

3.3 Test Matrix and Tooth Bending Fatigue Life Results………………….. 29

3.4 Weibull Distribution Analysis of Fatigue Data………………………… 36

3.5 Normal Distribution Analysis of Fatigue Data………………….……... 40

Chapter 4: Conclusion…………………………………………………………... 45

4.1 Summary……………………………………………………………. 45

4.2 Major Conclusions…………………………………………………….. 46

4.3 Recommendations for Future Work…………………………………… 47

References………………………………………………………………………. 48

LIST OF TABLES

Table 2.1: Gear parameters for the test gear. All dimensions are in mm…………… 17

Table 3.1: Test matrix and test parameters……………………………………...…. 33

Table 3.2: Fatigue test results in tabular form…………………………………….... 34

LIST OF FIGURES

Figure 2.1:	The STB test machines used in this study…………………………..	10
Figure 2.2:	Control diagram of a STB test system……………………………....	12
Figure 2.3:	(a) Schematic of a single tooth bending machine according to SAE J1619 and (b) with a floating upper anvil. …………………………	14
Figure 2.4:	Range of test tooth contact points and the chosen contact point for the test gear, and (b) Chosen reaction tooth and contact point for lower anvil….………………………………………………........	22
Figure 2.5:	Chosen reaction tooth and contact point for lower anvil……………	24
Figure 2.6:	Test gear fixture on the STB test machine...………………………..	26
Figure 2.7:	A test gear with removed teeth to make clearance for lower and upper anvil of the STB machine...………………………………..	27
Figure 3.1:	Potential misalignments between the test tooth and upper anvil and expected resultant pressure distributions…………………………..	30
Figure 3.2:	Proper contact between upper anvil and test tooth displayed on pressure sensitive paper…………………………………………….	31
Figure 3.3:	Fatigue test results………………………………………………….	35
Figure 3.4:	Fatigue test results with outliers removed.......................................	37
Figure 3.5:	Weibull mean survival plots and median ranks for (a) Gear A and (b) Gear B…………………………………………………………….	39

Figure 3.6: Weibull survival plots computed for the single tooth bending data computed at confidence intervals of (a) 90% and (b) 95%................. 41
Figure 3.7: Normal probability plot of failure for (a) Gear A and (b) Gear B....... 42

NOMENCLATURE

Symbol	**Definition**
a	Weibull scale parameter
b	Weibull shape parameter
d	diameter
F	probability of failure
H	hypothesis
k	degrees of freedom
l	position of involute
m	module
n	number of samples
N	loading cycles
r	radius
R	load ratio
R^2	coefficient of determination
s^2	standard deviation of the $\log_{10}$ of N
S	probability of survival
t	test statistic
$\bar{x}$	mean of the $\log_{10}$ of N
X	gear profile x-coordinate
y	median ranks of the fatigue data
$\bar{y}$	mean of median ranks of the fatigue data
$\hat{y}$	probability from Weibull survival fit
Y	gear profile y-coordinate
θ	Pitch of gear
σ	Tooth bending stress

Subscripts

a	alternative
A	gear A
b	base circle
B	gear B

cl	lower contact point (reaction tooth)
cu	upper contact point (test tooth)
f	form
max	maximum
min	minimum
n	rotated involute with respect to tooth number
o	null
t	tip

CHAPTER 1

INTRODUCTION

1.1 Background & Motivation

Gears are the primary machine elements used to transmit power in many transmission or drivetrain systems. As any tooth of a rotating gear travels through the loaded gear mesh zone, it comes to contact with a tooth of the mating gear, in the process, transmitting torque momentarily to the mating gear before moving out of the meshing zone to return its unloaded state. These cyclic tooth forces cause different forms of material fatigue. One form of fatigue occurs along the contact surfaces where failures such as pitting and micro-pitting are expected. Considering a tooth is a cantilevered structure, the same tooth forces cause it to bend, creating high stresses along the root fillet away from the contact zone. This results in the second main type of fatigue failures in the form of tooth breakage.

The contact fatigue evaluations must be done within rotating gear test machines such that desired tribological conditions are achieved by gear mesh contacts. Meanwhile, gear tooth fatigue evaluations can be done on non-rotating test benches where a single tooth of a gear can be loaded by an actuator in a manner similar to the loading during the rotating conditions. This is known as single-tooth bending (STB) method. There are numerous published studies (e.g. references [1-3]), which showed that this method is indeed an effective and accurate method for evaluating bending fatigue lives of gears without need for a rotating machine to operate a gear pair under desired load and speed condition. Furthermore, the same test gear is capable of producing multiple fatigue data points. As only two teeth are loaded in any given test, reorientation of the gear between the loaded contacts allows the other teeth to be subjected to the pulsating forces.

The relative simplicity of the STB experimental design as compared to a rotating gear test makes it ideal for rapid development and quick evaluation of bending fatigue lives of a spur gear. A generalized, best practice, design guideline is needed to extend the STB test methodology to any spur gear in production. With this said, there might be complications in applying this concept to any production gear, especially those with relatively fine pitch (low module) and thin rim (low backup ratio). The most desirable STB setup is when the upper and lower anvil contact surfaces are parallel to each other (typically both are on the horizontal plane [1]) and they support the test and reaction teeth along the line of action of the gear. This applies a pair of collinear forces to the gear such that no additional moment is generated to cause any reaction forces on the gear support bearing. However for fine-pitch gears, small tooth height and contact zone might make it difficult to find a gear position in which a tooth can be contacted by a horizontal anvil and loaded against a corresponding parallel contact on a reaction tooth. Any deviation from parallelism moves the contact away from what otherwise would be loading along the line of action if the test gear were mated in a rotating scenario. In addition, it creates bearing forces that must be reactioned by a fixture. A thin rim on a gear as in many aerospace gearings increases the compliance of the gear, potentially introducing undesirable dynamic effects when loading the gear at high speed. Therefore, it is necessary to develop a methodology, which can establish best practice guidelines for loading such a gear for single tooth bending testing. This is the main motivation of this undergraduate research study.

1.2 Literature Review

1.2.1 Gear Single Tooth Bending Experiments

Various methods and fixtures have been used in STB tests to evaluate the bending fatigue lives of gear teeth. Due to the complexity of a gear's geometry, certain methods and fixtures have proven to be more successful than others for a given gear geometry. Larger module ($m > 3$ mm) gears with rigid rims have mostly been used in STB research.

Singh [1] performed STB experiments on spur gears of module 4.5 mm to develop a crack initiation detection technique based on acoustic emissions. He used this technique to separate crack initiation and propagation stages of the total life of a spur gear tooth. Sanders et al. [2, 3] performed STB experiments according to the SAE J1619 standard [4] to evaluate bending fatigues lives of gears with asymmetric and elliptical root shapes. The SAE standard gear has module 4.23 mm and a thick rim with back up ratio of 1.4631. Wheitner and Houser [5] also used STB methods when testing the high module and thick rimmed SAE standard gear to investigate the effects of manufacturing variations and materials on fatigue crack detection methods in gear teeth. They used several nondestructive methods such as visible dye penetrant, ultrasonic testing, a stiffness method (measuring the force applied and the resulting acceleration), and an acoustic emission method (AE) to determine the point at which a fatigue crack had initiated.

Benedetti et al. [6] explored the influence of shot peening on tooth bending fatigue limit of case harden gears with a module of 5 mm. They used a single tooth bending fatigue test method where the load was applied in the direction tangent to the base circle of the gear at a frequency of 50 Hz from a servo-hydraulic machine.

Various methods have been used to conduct single tooth bending experiments. Akata et al. [7] explored single tooth bending fatigue tests of relatively high module and high back up ratio spur gears using a three-point bending method. The test gear was loaded on two gear teeth approximately 180^{o} apart at the highest point of single tooth contact (HPSTC). Since his test gear was loaded on two opposite gear teeth, there is an equivalent shaft reaction force in the upwards direction. This requires that the machine output twice the force that is applied to the test teeth. It also assumes that the teeth will deflect equivalently and that load is split evenly between the two teeth. In addition, this methodology would create higher rim deflections, which would be undesirable and not necessarily representative of operating conditions of a thin rimmed gear. Handschuh et al. [8], and later Krantz and Tufts [9] used a gear with module 3.175 mm to evaluate pitting and bending fatigue lives of a new case-carburized gear steel.

1.2.2 Statistical Analysis of Fatigue Data

As the goal of this research is to develop a new single tooth bending test methodology focused on determining the tooth bending fatigue lives of a thin-rimmed, small-module gear, a robust method must be considered for evaluating fatigue data. Several of the studies discussed earlier also include methodologies for comparing fatigue data sets and producing Stress-Life (S-N) curves. These statistics based methods along with others in literature are found to be adequate to adopt to the data sets that will be measured in this study.

In his STB experiments, Singh [1] performed a life regression analysis on the initiation life, propagation life, and total life data. He assumed that the distribution about the L50 curve was independent of stress and the slope and intercept of the L50 curve was calculated by using a least square minimization technique on all data points. Singh reported that the variation about the L50 line fits both the Weibull and the log normal distributions, but the log normal distribution provided a better fit for each of the three S-N curves.

Coy et al. [10] statistically evaluated gear tooth fatigue data by using a Weibull distribution and plotting the results on a log-log Weibull coordinate plot. They also determined the 90% confidence interval limits for each group of test data, and observed a consistent trend of decreasing life with increasing stress, which indicated a good statistical significance within the data. Coy et al. [10] pointed to earlier studies, one assuming that the Weibull slope is independent of the stress level [11] and another stating that the Weibull slope is a function of applied stress [12]. In their own study, Coy et al. [10] ultimately chose to use an average slope for the Weibull function that will serve in most applications where the applied stress is not unusually high or low.

Gasparini et al. [13] analyzed gear bending fatigue data for case carburized helicopter gears by fitting various curves based on the past experience in helicopter application. Gope [14], acknowledging that fatigue testing is time consuming and costly, sought a minimum sample size required to extract the statistical information for Weibull or log-normal distributions. He attempted to derive relationships between sample size, probability, confidence level, and distribution parameters. He gave error factors for determination of associated error from sample data at 50, 90, and 95% probabilities, 90, 95, 97.5, 99, and 99.5% confidence levels for sample sizes of 3 to 25 for both Weibull and log normal distributions.

Lawless [15] considered the problem of estimating warrantable life for a component in question when the underlying life follows a two-parameter Weibull distribution. His method is applicable to both censored and uncensored data, which means all the information within the data can be used. He describes that the logarithm of a Weibull variate presents the results more naturally.

Numerous studies on predicting fatigue life by using empirical relations or theoretical equations were described in Hwang and Han [16]. They describe that the scattering of fatigue life is well represented by either the Weibull distribution or log normal distribution and prediction of fatigue using distribution functions requires an expression at each applied stress level.

Sivapragash et al. [17] used a Weibull distribution to predict the fatigue strength for cast and welded ZE41A magnesium alloy tested under a stress ratio of 0.1 with different maximum stresses ranging from 135-190 MPa. The probability distribution according to which the material will fail was also obtained using Weibull. In line with other researchers they conclude that the Weibull distribution has the capability to model experimental data of very different characters.

In terms of gear tooth strength reliability, Yang [18] stated that the gear endurance strength follows a normal distribution while gear life cycles at a certain stress level higher than the endurance limit can be best described with either a Weibull or log normal distribution. Using either a Weibull or log normal distribution, a cumulative distribution function can be derived for a specified reliability value. He then tested the fatigue life of case hardened steel gears manufactured using the same processes using a single tooth bending experimental set up. He used median rank values to calculate the probability of failure following a three-parameter Weibull distribution and concluded a reasonable fit to the data.

1.3 Scope & Objectives

Review of the current literature shows that STB fatigue testing of thin-rim, fine-pitch gears has not been attempted, and a dedicated methodology to support and load such a gear must be developed. This proposed study seeks to develop an experimental means to evaluate the tooth bending fatigue lives of a production gear with small-module and low backup ratio. The specific objectives of this research are as follows:

- Design and fabricate a new fixture in order to load teeth of the chosen gear in a high-frequency, linear hydraulic load frame. Special attention will be given to the design in order to ensure proper contact between the surface of the hydraulic anvil and the gear teeth and minimizing bearing forces.

- Run-off the new fixture and develop a visual methodology to check the contact between the anvil and gear teeth.

- Perform a statistically significant number of high-cycle gear tooth bending fatigue experiments to compare the lives of two geometrically equivalent gears having undergone different manufacturing processes.

1.4 Thesis Outline

This thesis is organized in to two main chapters. Chapter 2 introduces the design of a new STB fixture specifically purposed for loading the chosen test gear. A detailed analysis of the contact points at which the designed fixture will hold and load the gear will be provided. An estimation of bearing forces and gear tooth root stresses will also be presented. Integration of the fixture with the hydraulic load frame will be described.

Chapter 3 will introduce an experimental design to make a statistically significant comparison between fatigue lives between two gears of same nominal geometry. Results will be presented and a Weibull evaluation of bending fatigue life distributions will be made to help determine if a statistically significant difference in bending fatigue life exists between the two gears.

Finally, Chapter 4 concludes this research by providing a summary and addressing the main conclusions. Recommendations for future work on this topic are also provided in this chapter.

CHAPTER 2

EXPERIMENTAL METHODOLOGY

2.1 Introduction

This chapter describes the methodology developed to perform tooth bending fatigue life measurements towards achieving the research objectives. The test machine setup, including its key features and an overview of current single tooth bending methodologies is introduced first. Gear design specifications and a formulation in order to solve for proper contact and loading points for the lower and upper anvil for the new STB fixture will be shown.

2.2 Experimental Setup

The test machine employed in this study is a high frequency, linear hydraulic load frame as shown in Figure 2.1. Other hydraulic powered linear pulsating test machines were used in earlier single tooth bending experiments to evaluate the tooth bending fatigue lives

Figure 2.1: The STB test machines used in this study.

of various spur gear geometries [1-10, 13]. This machine uses a custom hydraulic power unit (HPU) to power a Parker FAST series linear hydraulic piston to cyclically load teeth of a gear held statically in a fixture. The cylinder is actuated via a closed loop control system incorporating a Direct Operated Proportional DC valve to actuate the flow of hydraulic fluid. An Interface 1020FPJ-25K-1B load cell mounted on the cylinder provides the feedback control variable and measures the load applied to the gear tooth undergoing the fatigue test. PID control is implemented via a Delta Computer Systems RMC 200 controller. A control diagram of the test machines is shown in Figure 2.2.

The machine is interfaced through a Symbrium Powertrain Group HMI software, which enables a test operator to input the necessary test loading parameters defined as maximum load, loading frequency, and load ratio *R* defined as

$$R = \frac{\sigma_{min}}{\sigma_{max}} \tag{2.1}$$

where σ_{min} is the minimum tooth bending stress achieved in one loading cycle and σ_{max} is the maximum tooth bending stress achieved. A Keyence IL-065 laser sensor measures the displacement of the hydraulic cylinder as a diagnostics tool.

Many of the STB test methodologies reviewed in Chapter 1 incorporated similar test methodologies. One common test is the SAE J1619 standard [4]. This test setup utilizes a linear pulsating load frame such as described earlier with a fixture to hold the test gear through its center with one tooth placed against a fixed anvil. The linear actuator makes contact and produces a cyclic force on another tooth such that the line of force passes through both contact points on both teeth. In the SAE test the upper anvil is connected and

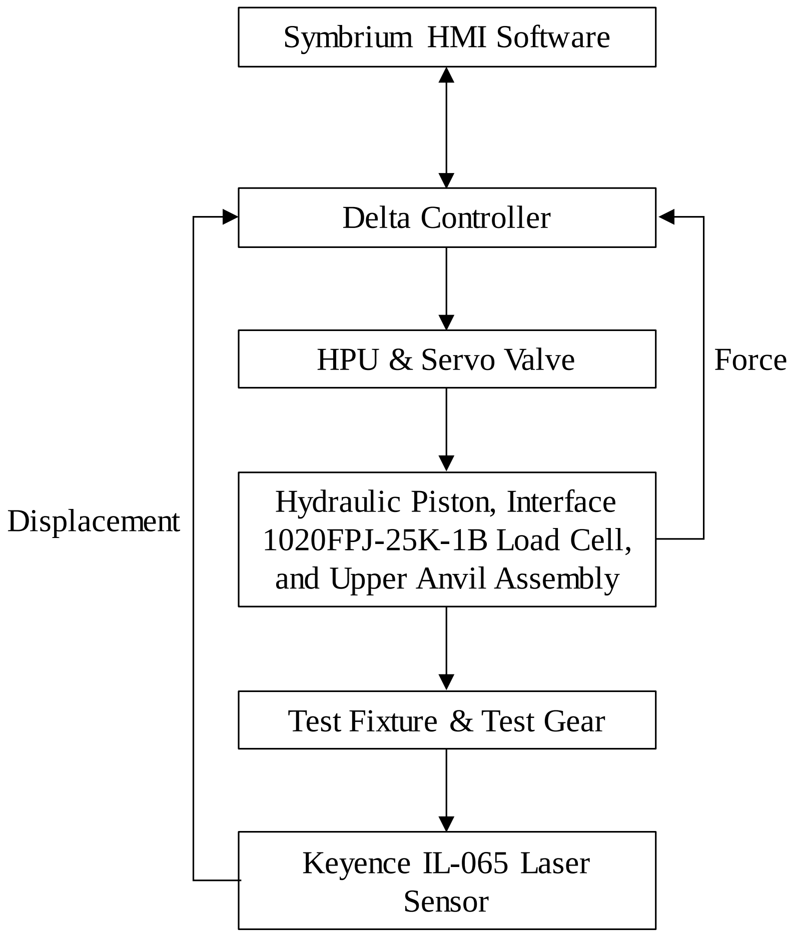


Figure 2.2: Control diagram of a STB test system.

aligned to the fixture through the same shaft supporting the test gear. Other test methodologies employ a floating upper anvil attached directly to the linear actuator. Figure 2.3(a) shows a schematic of the SAE test methodology as compared to one with a floating upper anvil shown in Figure 2.3(b).

Typically, the tooth supported by the lower anvil is designed as the reaction tooth and is intended to have a lower bending stress so that it will not fail before the test tooth. The other tooth loaded by the linear actuator is positioned such that the pulsating forces create a larger bending moment about the root in turn producing higher stresses in the tooth root fillet. This tooth is designed to fail first in the test and is designated as the test tooth. As only two teeth are used for one test, multiple tests can be achieved with only one test specimen by rotating the gear to a new position such that only teeth that have not experienced stress cycles are used.

As stated in Chapter 1, it is ideal to load the teeth such that the forces acting on both teeth are collinear with the axis of the hydraulic cylinder. Forces that are not along the loading axis create bearing forces on the gear bore if constrained, and higher frictional forces on the tooth, which presents multiple load paths through the gear making stress analysis difficult. In addition, any lateral loading can create large bending stresses on the hydraulic cylinder due to the moment arm presented by the upper anvil, load cell, and piston shaft assembly. It should also be noted that edge loading can occur between the upper anvil and test tooth contact if the tooth is loaded too close to the tip and the contact patch spreads to the tooth tip. This is undesirable as large changes and scatter in the resultant root stress can occur when the gear tooth is loaded in such a manner. These issues

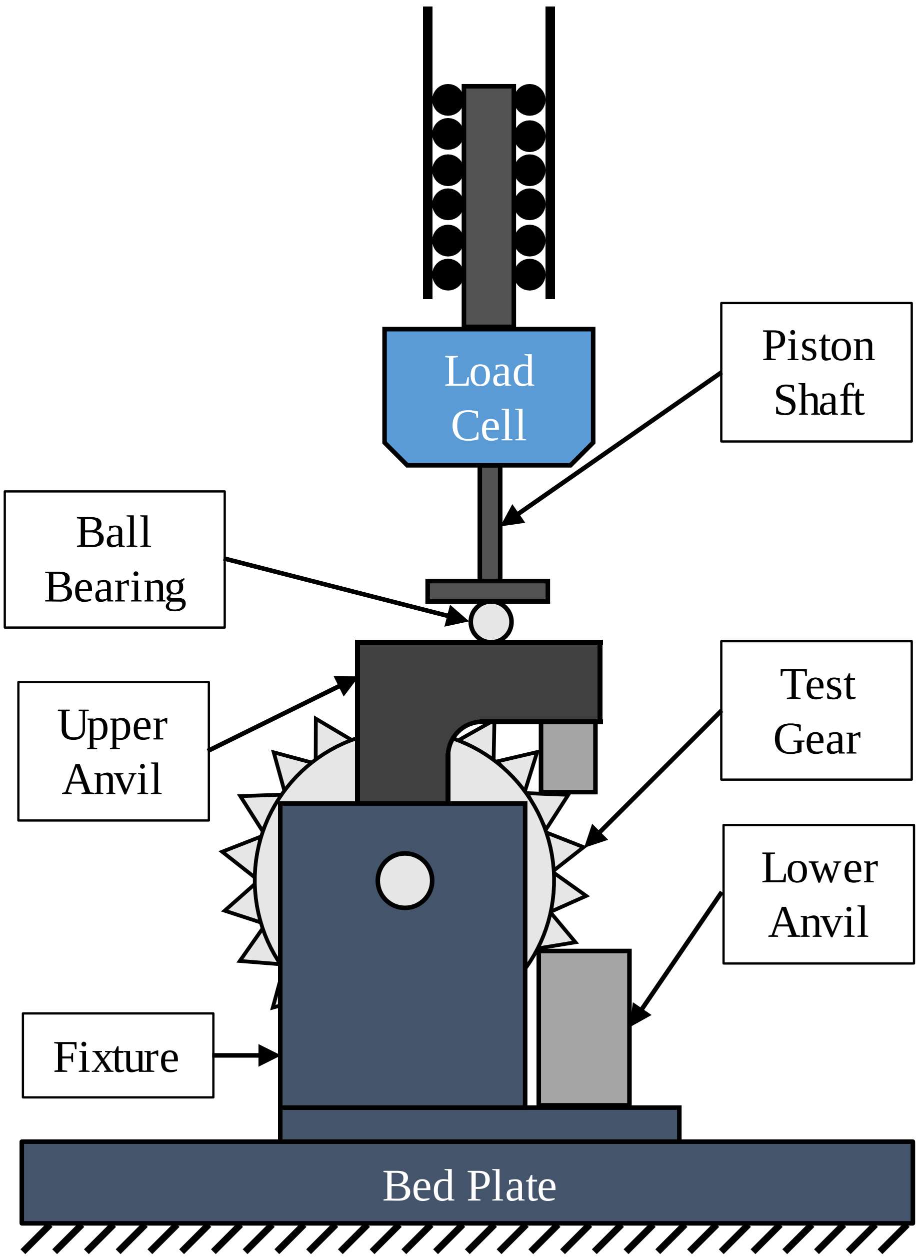
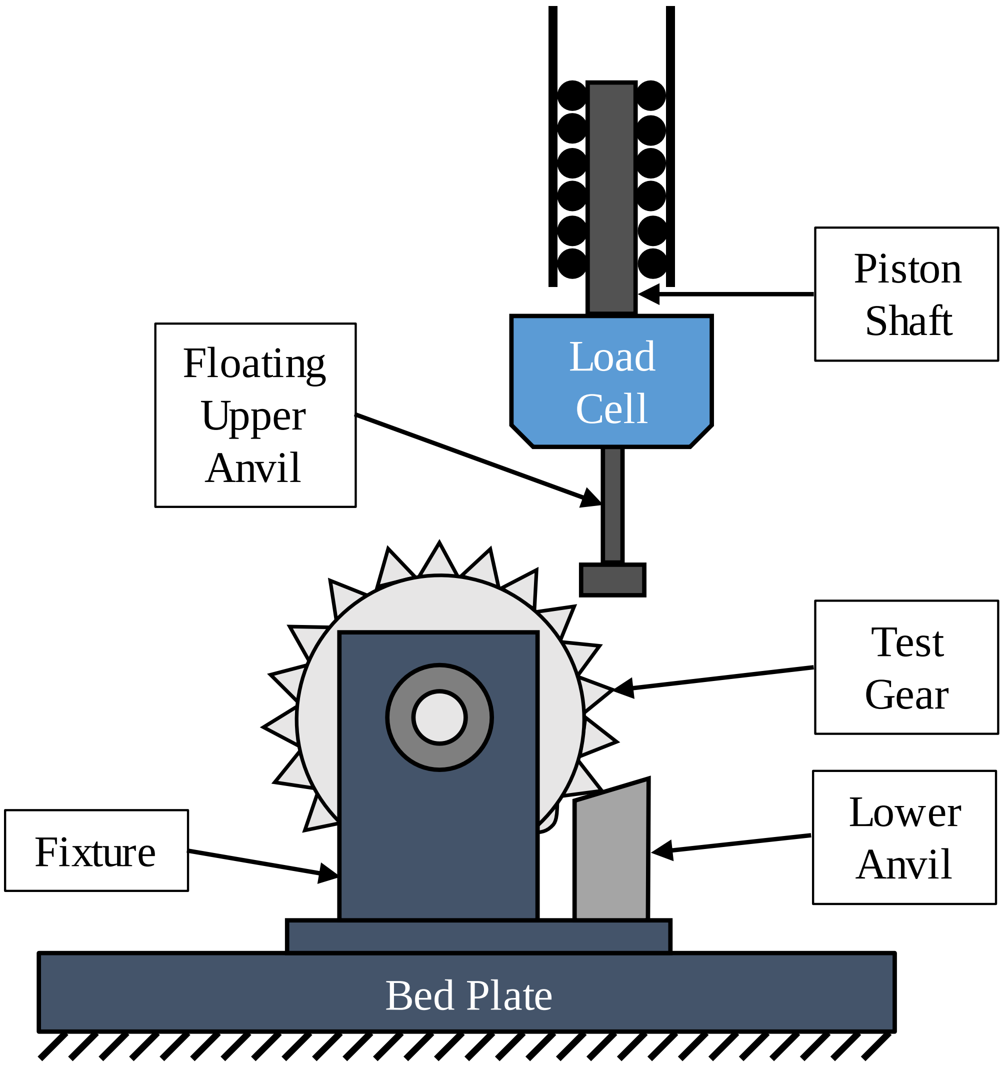


Figure 2.3: (a) Schematic of a single tooth bending machine according to SAE J1619 and (b) with a floating upper anvil.

Figure 2.3 Continued



must be addressed in designing a new fixture to perform fatigue tests on the chosen test specimen.

2.3 Definition of Anvil Contact Points for STB Fixture

The test gear employed in this development is a thin rim, fine-pitch 88-tooth spur gear, geometrically unlike those of previous STB fatigue experiments [1-10, 13]. Notably, the gear consists of a small module teeth and a relatively thin rim and web making the entire gear body compliant as compared to test specimens reviewed in previous testing. Table 2.1 lists basic geometric parameters of the test gear, relevant to the development of contact points for the lower and upper anvil in the single tooth bending machines described in Section 2.2.

A fixture was first designed for the selected test gear by first determining where the lower and upper anvil of the STB test machines will contact the test gear tooth surfaces. In line with previous STB experiments, the goal is to search for a position where two teeth are loaded such that their contact points are concentric with the direction of force. One tooth (test tooth) will be loaded closer to the tip with respect to the loading on the other (reaction tooth) such that it always fails first. It is critical that the force applied to the test tooth is well known. The upper anvil assembly of the STB machine contains the load transducer meaning that its measurement is equivalent to the load applied to the upper tooth. Therefore, it was determined to make the tooth that contacts the upper anvil the test tooth and the tooth that contacts the lower anvil the reaction tooth. The purpose of the reaction tooth is to hold the gear in position by contacting it against a lower anvil so that the gear does not rotate about its center while the upper anvil cyclically loads the test tooth.

Table 2.1: Gear parameters for the test gear. All dimensions are in mm.

Number of Teeth	88
Module	2.540
Backup Ratio	0.88
Tip Diameter	228.30
Form Diameter	218.08
Face Width	14.00
Transverse Tooth Thickness	3.66
Pitch Diameter	223.52
Base Circle Diameter	206.51
Circular Tooth Thickness	3.66

A gear tooth surface can be described by two regions; the dedendum which forms the involute profile from the form diameter to the pitch diameter and the addendum which form the involute profile from the pitch diameter to the tip. Load applied in the addendum of the tooth produce a greater bending moment about the root fillet such that, for a given load, the root stresses will be higher if the load is contacting the gear tooth in the addendum region compared to the equivalent load contacting the tooth surface in the dedendum region. This fact can be used to constrain where on the tooth surface the lower and upper anvil contact the reaction and test tooth. Therefore, it was determined that the upper anvil shall contact the test tooth in its addendum region, while the lower anvil shall contact the reaction tooth in its dedendum region. These constraints were used respectively when searching for a contact point for the test and reaction tooth of the test gear.

The STB machines used in this development have a long upper anvil assembly that consists of an anvil that contacts the test tooth, a load cell, and a piston shaft. Since the upper anvil assembly is very long in length, any horizontal force that this assembly is subject to creates a very high bending moment in the piston shaft. It is therefore necessary that out of plane force between the test tooth and the upper anvil is minimized. This dictates that the upper anvil must contact the test tooth on a horizontal plane to minimize any cyclic bending stresses applied to the linear actuating cylinder. Constraining the upper anvil to contact that test tooth on a horizontal plane also minimizes horizontal deflection of the upper anvil and decreases the chances of the test fixture being misaligned during operation of the STB test machine.

The first step to determine appropriate contact points for the test gear is to mathematically describe the geometry of the gear. A spur gear's tooth surface is formed by an involute with a base circle radius r_b that spans from the form diameter d_f of the gear to the tip diameter d_t. A single involute profile representing the contact flank of the test tooth is defined in a vertical plane using a Cartesian system whose origin aligns with the gears center at $(X,Y)=(0,0)$ such that

$$X = r_b(\cos l + l \sin l), \tag{2.1a}$$

$$Y = r_b(\sin l - l \cos l) \tag{2.1b}$$

where (X,Y) are the Cartesian coordinates. Next, a range of $l \in [l_f, l_t]$ is determined such that the resultant section of the involute profile represented the test gear tooth geometry, spanning from the form diameter to the tip diameter where l_f represents the position of the involute corresponding to the form diameter and l_t is the tip diameter. The same tooth flank for consecutive teeth around the gear are created using a rotation transformation as.

$$\begin{bmatrix} X_n \\ Y_n \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} \tag{2.2}$$

where X_n and Y_n are the rotated Cartesian coordinates of the involute profile. In equation (2.2), the determined coordinates for a single involute of one side of a tooth flank in the *X*-*Y* plane are rotated counterclockwise through an angle θ corresponding to the pitch of the gear about the origin of the Cartesian system. The subscript *n* indexes the rotated involute with respect to tooth number.

With all the involute profiles of one side of the tooth flank created, a single involute for the opposite tooth flank (reaction flank) is created within the same $l \in [l_f, l_t]$ and flipping the direction of the involute according the following equations:

$$X = r_b(\cos l + l \sin l)\,, \tag{2.4a}$$

$$Y = -r_b(\sin l - l \cos l)\,. \tag{2.4b}$$

This yields the coordinates for a single involute of the tooth flank opposite that previously formed. Using the Cartesian coordinates defined by Eq. (2.4), the involute profile was spaced relative to the opposite involute for its respective tooth by spacing it by the circular tooth thickness defined at the pitch diameter of the test gear. The remaining involutes were then created using the rotation transformation defined by Eq. (2.2) while spacing each involute profile evenly around the base circle of the test gear. With all involute profiles for both flanks of the test gear defined, the contact points for the test and reaction tooth can be searched for next.

The contact point between the upper anvil and test tooth was first searched in the addendum region of the test gear. The lower and upper limit for this contact point was found by selecting a single involute and rotating it in space about the test gears center. While the selected involute was being rotated in space about the test gears center, the involute Cartesian coordinates were searched for a point of zero slope tangent to the involutes profile in the addendum region of the test gear. Finding the point of zero slope tangent to the involutes profile guarantees the upper anvil will contact the test tooth on a horizontal plane and satisfies the previously described constraint. The lowest contact point of the test tooth at its pitch point was found to be $(X, Y) = (103.249, 42.780)$ mm. The highest contact point at the tip diameter of the test tooth was found to be $(X, Y) = (103.249, 48.500)$ mm. Between these two limits, an appropriate contact point of $(X_{cu}, Y_{cu}) = (103.249, 45.803)$ mm was selected. This selection met the following criteria:

- Contact point for the test tooth should not be at the tip or too close to the tip of the tooth such that, when under maximum load, the pressure distribution between the test tooth and upper anvil does not reach the tip of the tooth to cause edge loading. Such edge loading would ultimately result in the upper anvil assembly being subjected to horizontal forces.

- Contact point for the test tooth must be as close the tip of the tooth as possible to guarantee higher root stresses compared to the reaction tooth. This constraint guarantees that the test tooth fails before the reaction tooth.

The range for the lowest and highest test tooth contact point and the chosen contact point are displayed in Figure 2.4.

Once an appropriate contact point between the test tooth and upper anvil is established, the rotational position of the gear is fixed along with the involute profiles of all other teeth. Next step is to choose an appropriate reaction tooth based on the following criteria:

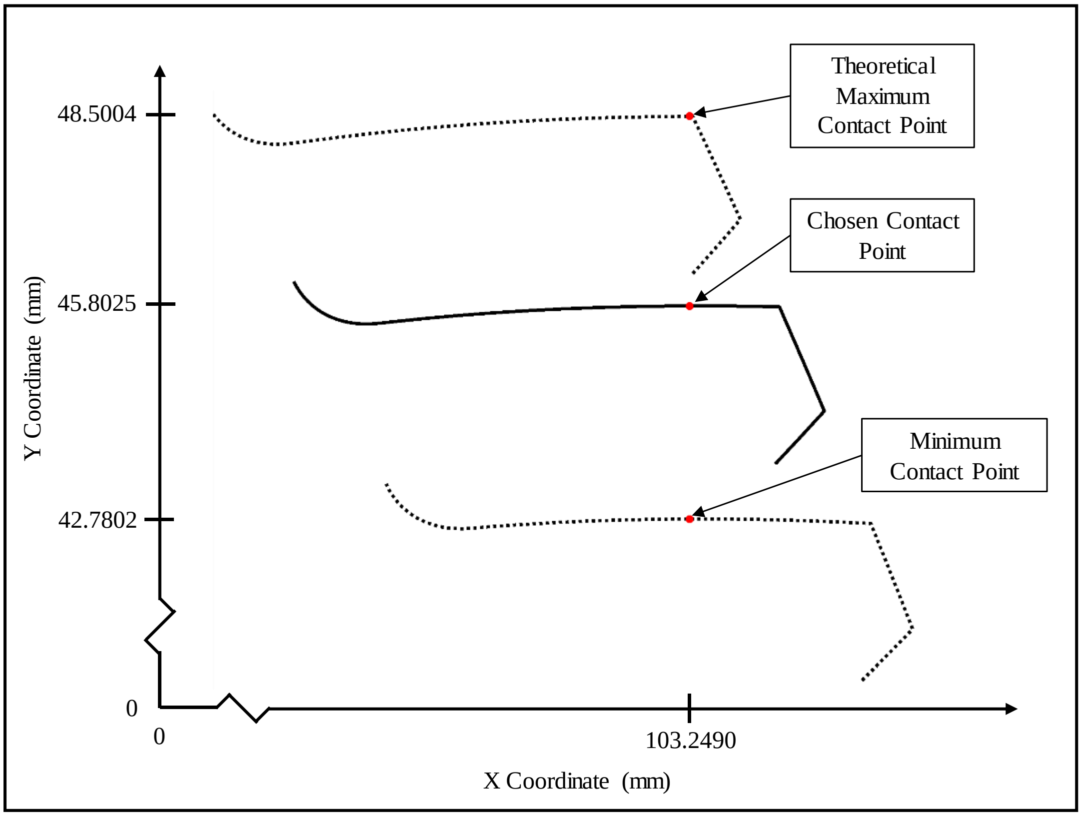


Figure 2.4: Range of test tooth contact points and the chosen contact point for the test gear, and (b) Chosen reaction tooth and contact point for lower anvil

- Contact on the chosen reaction tooth with the lower anvil should be as close as practically allowable to the vertical axis passing through the contact of the upper tooth.

- The reaction tooth and lower anvil should make contact on horizontal plane unless no such point satisfying the other constraints exists. This constraint minimizes the bearing forces within the load path of the test fixture by reducing the horizontal forces produced by the reaction tooth and lower anvil.

- The reaction tooth-lower anvil contact must occur within the dedendum region of the reaction tooth. This constraint ensures that root stresses on the reaction tooth are lower relative to test tooth root stresses. This helps to guarantee failure of the test tooth and not the reaction tooth.

Based on the above criterion, a reaction tooth was selected to be eight teeth from the test tooth. The appropriate lower contact point was computed to occur at $(X_{cl}, Y_{cl}) = (110.010, -12.580)$ mm in the same Cartesian system with the origin at the gear center. The tangent plane at this contact point was at angle of 14.5° from the horizontal as a suitable point resulting in a horizontal contact was not available. It was determined that the lateral component of the forces generated by this contact would be reasonably small, well within what can be carried by the fixture and the bearings. Figure 2.5 displays the chosen reaction tooth and contact point of the test gear.

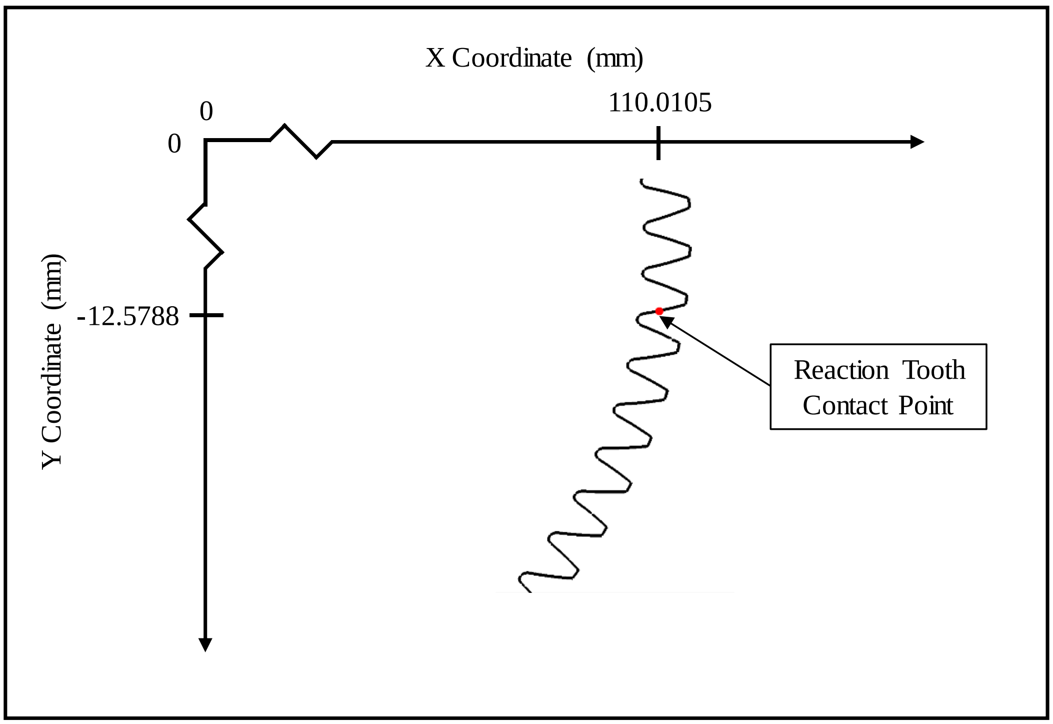


Figure 2.5: Chosen reaction tooth and contact point for lower anvil.

Based on the location of the contact points for the test and reaction tooth developed above, a fixture was designed by a test machine company to hold the test gear in the STB machine such that load would be applied to the gear teeth as shown. This fixture, shown in Figure 2.6 holds the gear at production rolling element bearings. The fixed lower anvil made of hardened carbon steel was aligned to the fixture via dowel pins and holds a flat, ground steel insert at a 14.5° slope such that a tooth on the gear becomes the reaction tooth and contacts it at the point described. The fixture is aligned to the machine cylinder axis via perpendicular keys such that the cylinder axis points at the contact point designated for the test tooth. A floating upper ram containing another hardened ground steel insert is attached to the linear actuator to make the contact with the test tooth.

Figure 2.7 shows a picture of the test gear used in this developmental work. As seen in the picture, there is a pattern of removed full teeth and half teeth around the gear. The purpose of removing full and half teeth is to create clearance for the upper and lower anvil is to provide access to the upper and lower anvils to load the tests and reaction teeth properly. With this pattern, eleven different tests can be performed using the same gear.

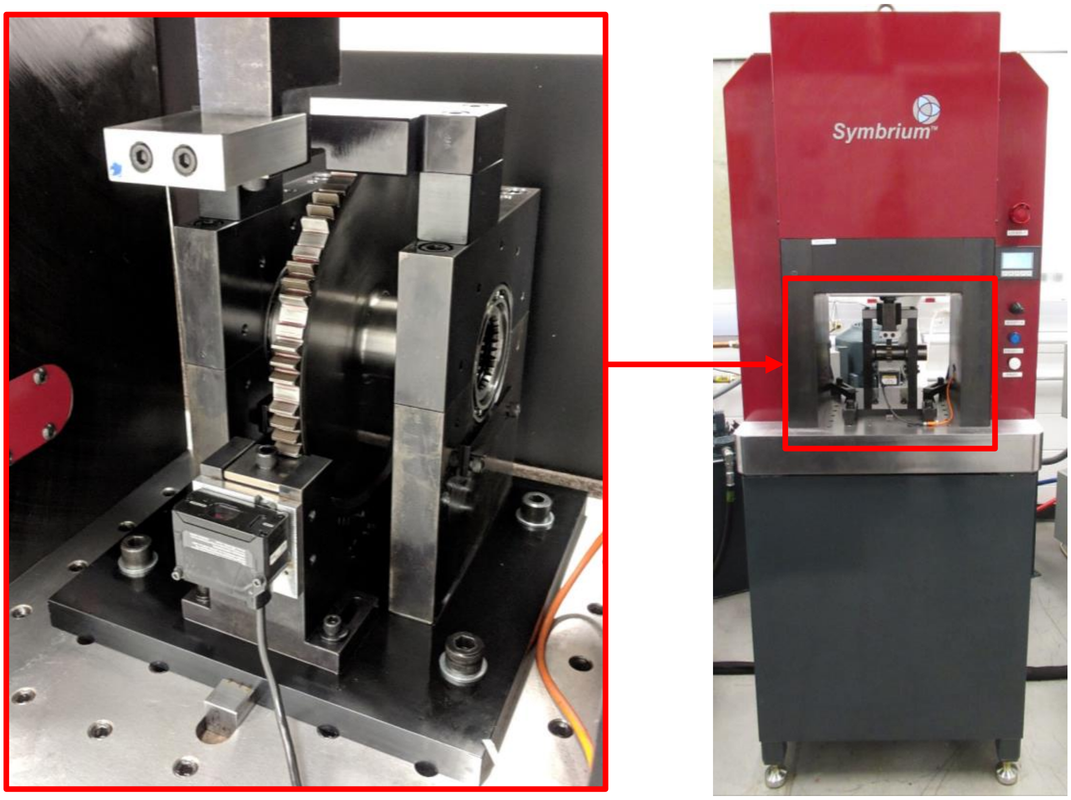


Figure 2.6: Test gear fixture on the STB test machine.

Figure 2.7: A test gear with removed teeth to make clearance for lower and upper anvil of the STB machine.

CHAPTER 3

EXPERIMENTAL RESULTS AND DISCUSSION

3.1 Introduction

This chapter presents the results and analysis of tests performed on two nominally equivalent fine-pitch, thin-rim gears produced using two different manufacturing processes (named A and B here). As such, the STB methodology proposed in the previous section will be employed here to compare the tooth bending fatigue life performance associated with both manufacturing processes.

3.2 Validation of Test Fixture and Contact Points

The first step after installing the test fixture was to check for proper alignment of the fixture base to the upper anvil contact surface connected directly to the linear actuator. Machine tolerances or installation errors can cause the tooth surface to become misaligned with respect to the anvil to cause an asymmetric pressure distribution across the face width of the gear. If each installation were to produce different levels of misalignment, then the applied forces to the tooth would cause variation in resultant root stresses between tests.

As the teeth of test gears had a lead crown modification, a proper pressure distribution is achieved when the highest pressure is centered on the face width. Figure 3.1 shows a schematic of how misalignment of the fixture would produce different load distributions on the gear tooth.

A simple method to check the contact between the test tooth and upper anvil was used here. Proper contact and load distribution were checked before every test using Fujifilm Prescale – High Film (HS) pressure sensitive paper with a pressure range of 7,100-18,500 psi. Figure 3.2 displays a representative acceptable contact pattern measured through the pressure sensitive contact paper. looks like when proper contact is achieved between the upper anvil and test tooth. It is noted that there are no sharp edges and the shape indicates loading with highest load at the center.

3.3 Test Matrix and Tooth Bending Fatigue Life Results

The goal of the fixture design for the gear presented in Chapter 2 was to be able to measure and compare the tooth bending fatigue lives of test gears having undergone different manufacturing processes. The fatigue results of two such test specimens, Gear A and Gear B, are analyzed in this chapter to compare their bending fatigue strength.

All the tests were performed at a single load level that is slightly above the endurance limit of Gear A. For this, a staircase test was performed using Gear A. In the staircase testing, the first test is run at stress levels below an estimated endurance limit. If the tooth survives predefined number of cycles, then it is tested again at a higher stress

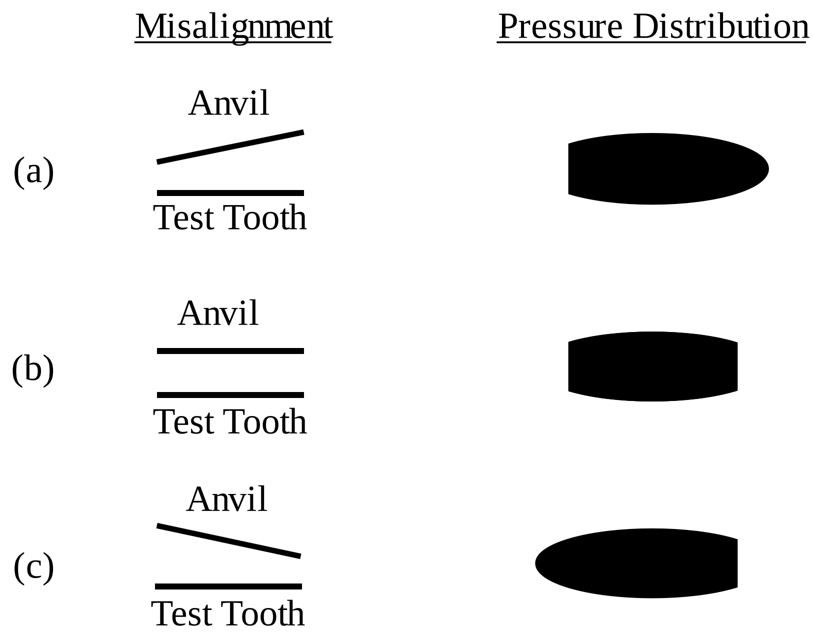


Figure 3.1: Potential misalignments between the test tooth and upper anvil and expected resultant pressure distributions.

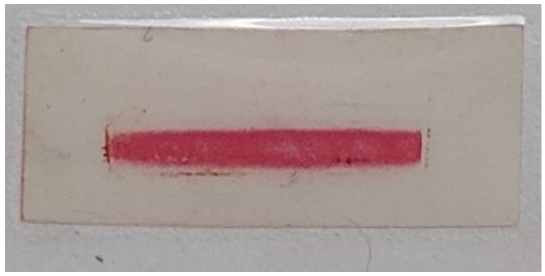


Figure 3.2: Proper contact between upper anvil and test tooth displayed on pressure sensitive paper.

level and so on until failure occurs. This allows for a quick estimation of the endurance limit of the specimen. A maximum applied force of 29,000 N with a load ratio $R = 0.05$ was found to produce bending stresses just above the endurance limit for Gear A. The operating load frequency of the STB test machines for these gears was determined to be 40 Hz. It is desirable to use a high frequency as the time required to perform a test is inversely proportional to the loading frequency. Although the test machines are capable of operation at frequencies greater than 100 Hz, it was clear from initial runoff of the test machine with this new gear and fixture that operating at such high speeds could cause dynamics issues. Table 3.1 provides a test matrix and test parameters for the experiment performed.

The resultant measured fatigue lives from both Gear A and Gear B are displayed in Table 3.2 and Figure 3.3. Eleven tests were performed on Gear A and 10 tests on Gear B. Five tests were suspended without failure at two million cycles on Gear A while three tests suspended on Gear B. Since the load parameters for these tests were determined using a staircase method and purposefully chosen close to the endurance limit, it is likely that the suspended test teeth are exhibiting endurance limit type behavior. As such, the suspended teeth are not expected to fail even if further loading cycles are applied. The endurance limit behavior is not related to the failure rate or fracture mechanism exhibited by the test teeth resulting in failure. As such the suspended tests were assumed to be of a different statistical population and not included in a statistical analysis of the data shown later in Section 3.4. Additionally, Gear A had a tooth fail at 1,804,620 cycles. It has been shown that fatigue fractures on gear teeth at very high cycles typically originate from a sub-surface crack initiation point Hong [19]. This is an entirely different failure mode of the gear tooth with a different population of fatigue lives. Although, a fracture analysis was not able to

Table 3.1: Test matrix and test parameters.

Gear	Maximum Load [N]	Minimum Load [N]	Operating Frequency [Hz]	Suspension Point [Cycles]
A	29,000	1,450	40	2,000,000
B	29,000	1,450	40	2,000,000

Table 3.2: Fatigue test results in tabular form.

Test Number	**Gear A [Cycles]**	**Gear B [Cycles]**
1	2,000,000	1,804,620
2	39,173	59,745
3	57,160	115,063
4	46,971	66,335
5	2,000,000	2,000,000
6	2,000,000	130,000
7	33,296	2,000,000
8	83,604	2,000,000
9	50,984	58,131
10	2,000,000	45,547
11	2,000,000	-

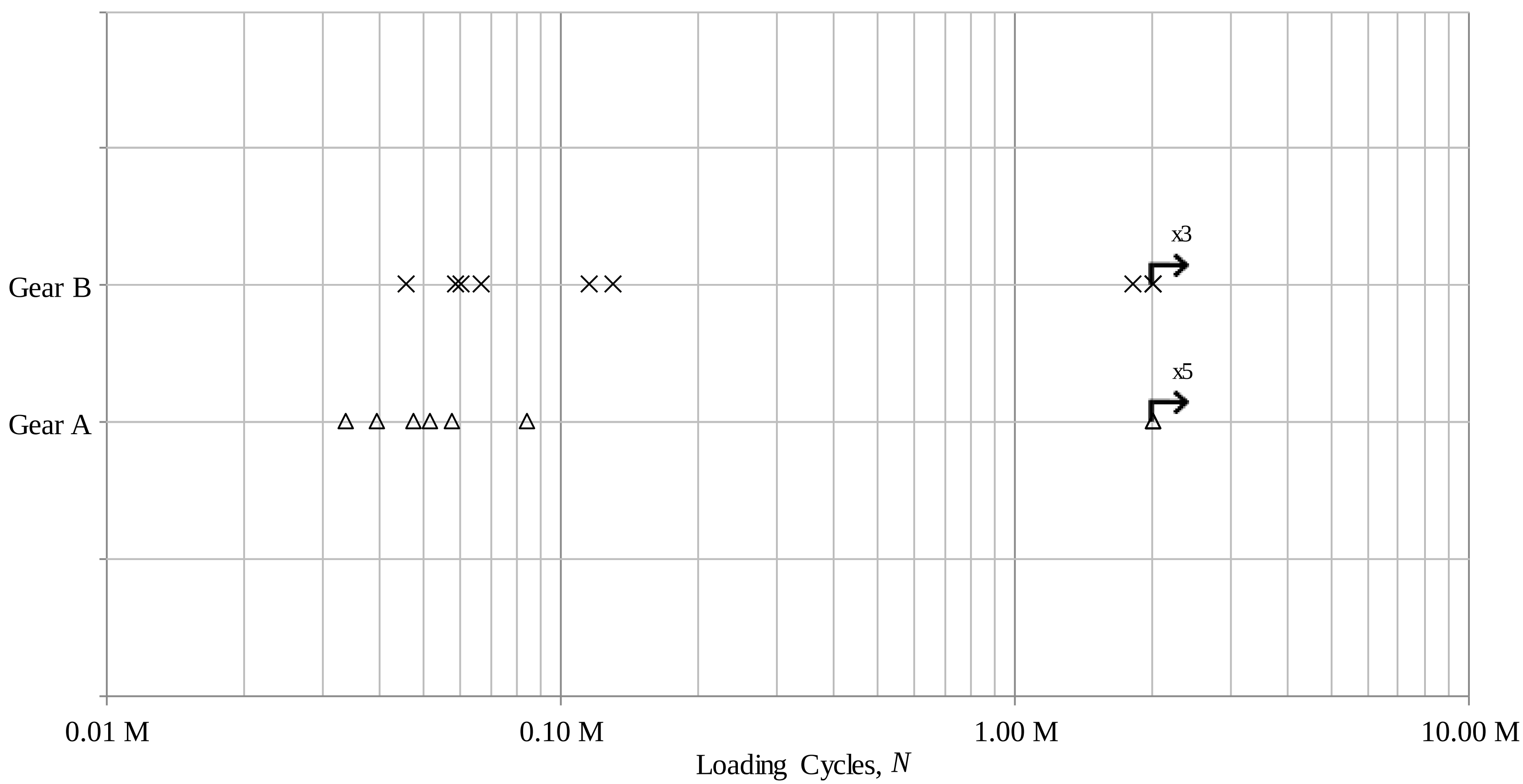


Figure 3.3: Fatigue test results.

be performed, the large difference in the life of this 1.8M cycle failure to the range of 0.04M-0.13M cycle lives suggests that this failure is most likely of a different fracture mechanism and should not be statistically analyzed with the rest of the failures.

The removal of the suspended tests and the one outlier from the statistical analysis leaves six data points for each gear to for a comparison. Figure 3.4 displays the test results with the determined outliers from the initial data set removed. Two statistical techniques found in the literature reviewed in Chapter 1 and commonly used in fatigue data analysis will be adapted to make the comparison of fatigue lives between Gear A and Gear B with these retained data points.

3.4 Weibull Distribution Analysis of Fatigue Data

The first analysis employed is based on the two-parameter Weibull distribution, commonly used in literature with fatigue data such as [10] to describe the distribution of fatigue lives at any single loading condition or stress value. Good practice suggests to establish 90% or 95% confidence intervals on Weibull parameters to observe whether there is any potential overlap on measured Weibull distributions. The two-parameter Weibull distribution is described by its cumulative distribution function (CDF) as

$$F(N \mid a,b) = 1 - e^{-(N/a)^b} \tag{3.1}$$

where N is the random variable corresponding to fatigue life, a is the Weibull scale parameter and b is the shape parameter. When used with fatigue data, the CDF corresponds to the probability of failure at any given number of loading cycles.

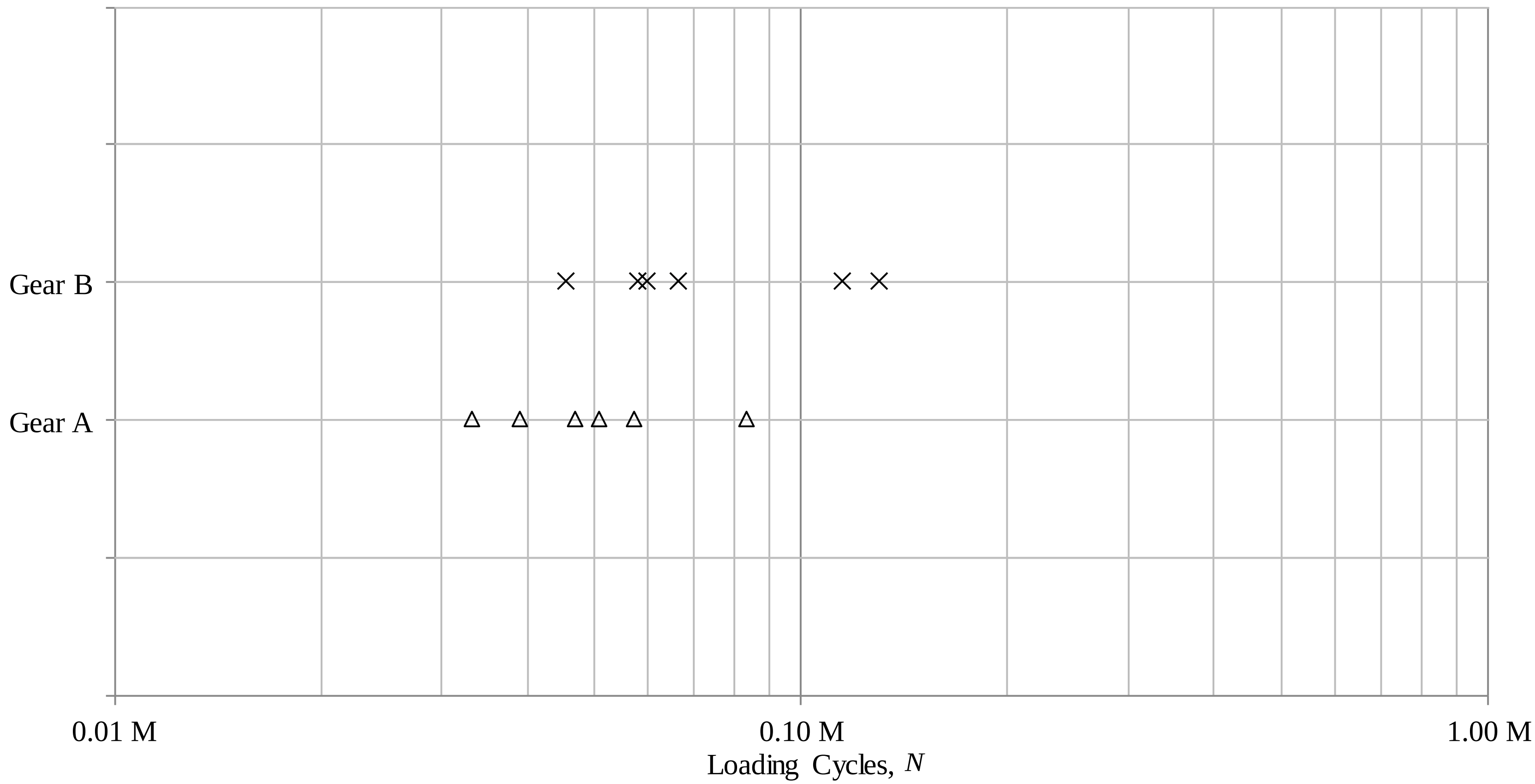


Figure 3.4: Fatigue test results with outliers removed.

When discussing fatigue lives, it is more useful to define probability of survival rather than the probability of failure. Hence, from Eq. (3.1), the survival cumulative distribution function is defined as

$$S(N \mid a,b) = 1 - F(N \mid a,b) = e^{-(N/a)^b} \; . \tag{3.2}$$

Using a commercial statistics software, a Weibull distribution was fit to the test data for both Gear A and Gear B in order to determine respective Weibull Parameters. The resulting Weibull survival curve generated from the computed Weibull parameters and Eq. (3.2) is shown in Figure 3.5(a) and (b) for Gears A and B, respectively. Measured data points are also shown with their associated median ranks to demonstrate the fit of the distribution to the data. This fit can be quantified by computing a coefficient of determination R^2 described as

$$R^2 = 1 - \frac{\sum_{i=1}^{6} (y_{N_i} - \hat{y}_{N_i})^2}{\sum_{i=1}^{6} (y_{N_i} - \bar{y}_{N_i})^2} \tag{3.3}$$

where y_{N_i} is the median ranks of the fatigue data, $\hat{y}_{N_i}$ is the corresponding probability from Weibull survival fit at number of cycles N_i , and $\bar{y}_{N_i}$ is the mean of the median ranks. Data from Gear A fits the mean survival Weibull distribution with a coefficient of determination $R^2 = 0.93$ while Gear B yielded a coefficient of determination of $R^2 = 0.82$. These values indicate that data for both gears suitably fits the Weibull distribution.

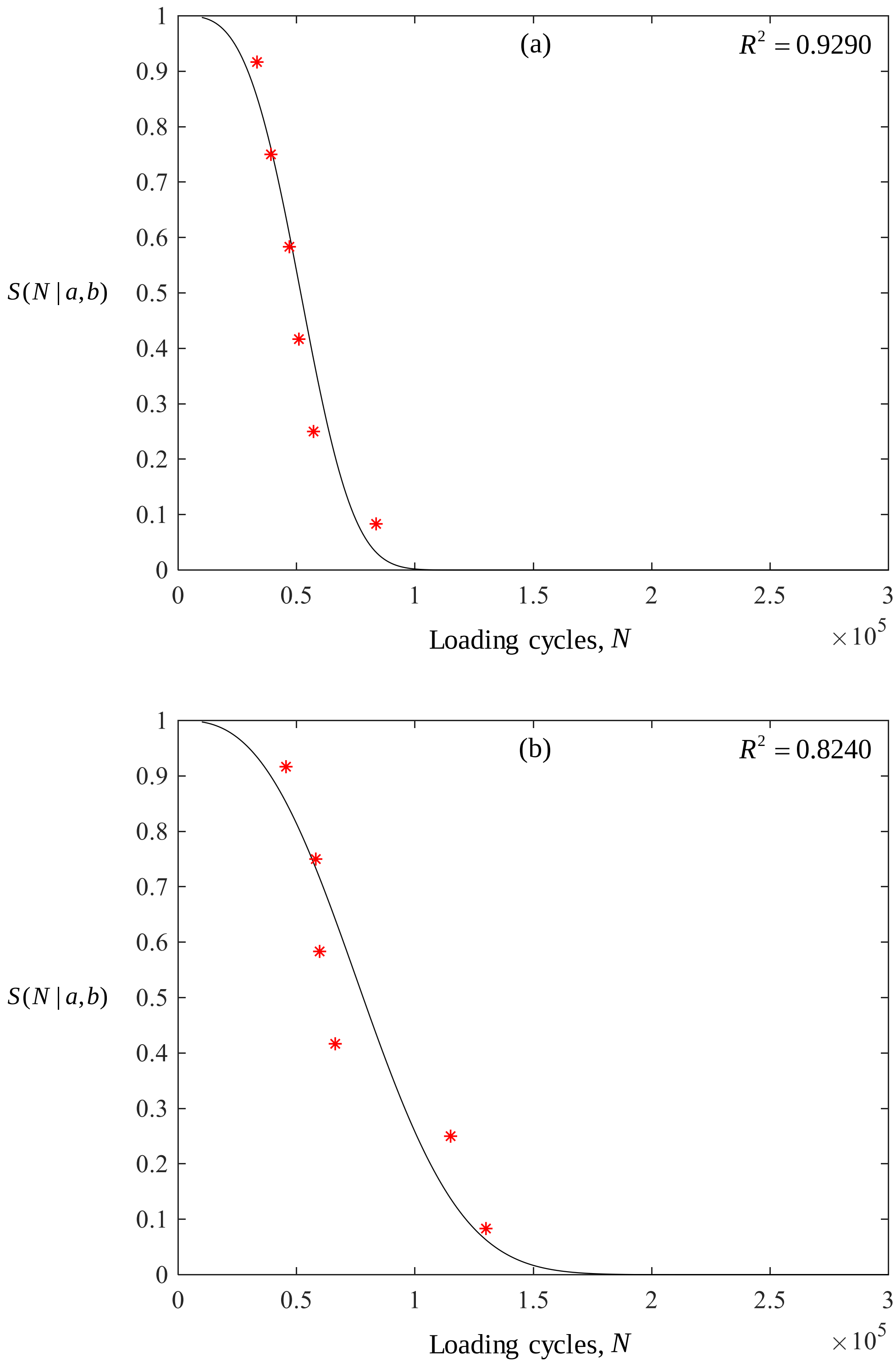


Figure 3.5: Weibull mean survival plots and median ranks for (a) Gear A and (b) Gear B.

The 90% and 95% confidence intervals (CI) on the shape and scale parameters were also found using the same commercial statistics software. The Weibull distributions corresponding to the lower and upper confidence intervals are plotted in terms of survival according to Eq. (3.2) in Figure 3.6(a) for the 90% CI and (b) for the 95% CI. It is observed for both 90% and 95% confidence intervals, that the estimated Weibull distributions for Gear A and Gear B converge when the number of loading cycles is low and survival probability is high. However, as a specimen survives longer and its probability of survival goes down, specimens from Gear B have a longer fatigue life for equal probability of failure even when considering for the 90% or 95% confidence intervals on estimated Weibull parameters.

3.5 Normal Distribution Analysis of Fatigue Data

The Weibull Analysis suggests that there may be some difference in the fatigue lives of gear teeth of Gear A in comparison to those of Gear B. Yet, it does not give a very definitive answer. A common, formalized statistical test of comparison of sample means called the two-sample t-test [20] can be used if the data fits a normal distribution. Although the data has already been shown to follow a Weibull distribution reasonably well, it is also commonly found in fatigue testing of steel that the logarithm of fatigue life follows a normal distribution, which is also known as the log-normal distribution. Figure 3.7 shows the fit of the logarithm of the measured test gear fatigue lives to a normal distribution.

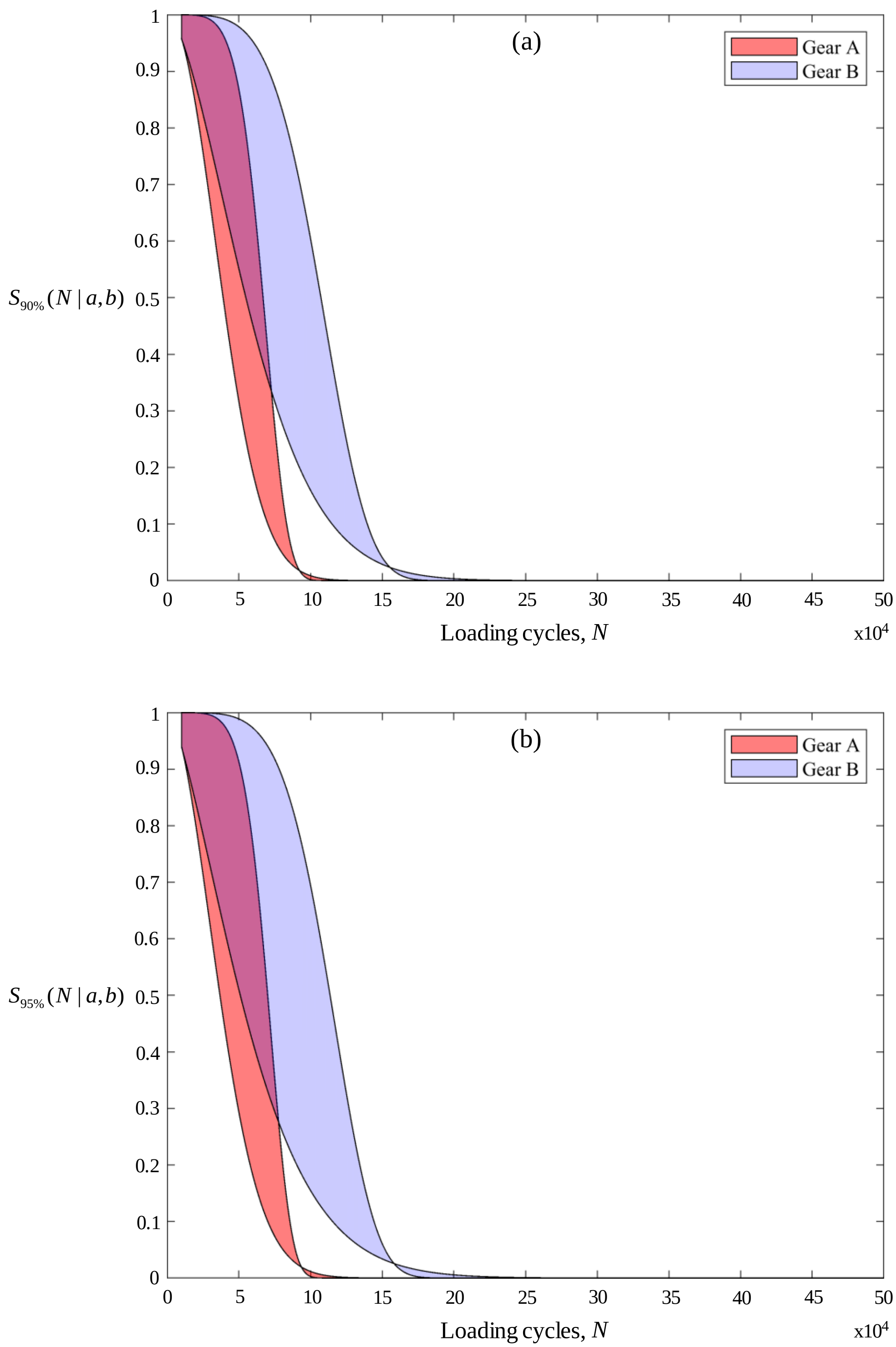


Figure 3.6: Weibull survival plots computed for the single tooth bending data computed at confidence intervals of (a) 90% and (b) 95%.

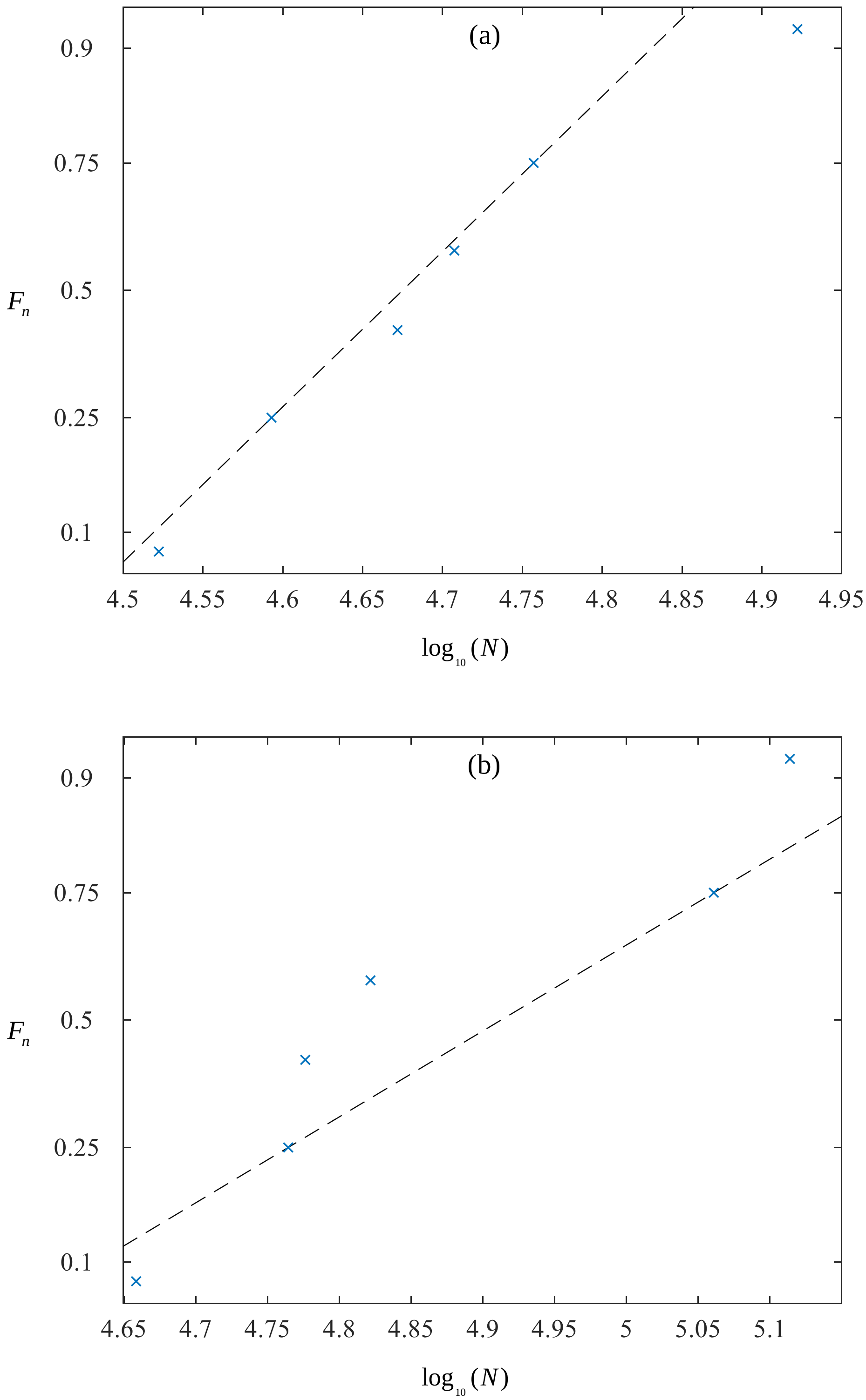


Figure 3.7: Normal probability plot of failure for (a) Gear A and (b) Gear B.

With data shown to fit the log-normal distribution, the t-distribution is also applicable [20]. Several versions of the t-distribution based t-test exist including those for equal and unequal variances. The unequal variance test also known as Welch's t-test is used here as it is the most conservative when stating statistical differences. It is formalized here as

$$t = \frac{\bar{x}_A - \bar{x}_B}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}} \tag{3.4}$$

where $\bar{x}$ is the mean of the $\log_{10}$ of the fatigue lives for the respective subscripted gear, s^2 is the standard deviation of $\log_{10}$ of the fatigue lives of respective subscripted gear, and n is the number of samples of the respective subscripted gear. The t-test is designed to differentiate between a null hypothesis and an alternative hypothesis. For the two-sided t-test the null hypothesis H_o is that $\bar{x}_A = \bar{x}_B$ (i.e. the mean fatigue life of Gear A is equal to the mean fatigue life of Gear B). The alternative hypothesis H_a is that $\bar{x}_A \neq \bar{x}_B$. The t-value computed corresponds to the standardized t-distribution, which has as a two-tailed area under the curve of P representing the probability that the null hypothesis H_o is true. Appling Eq. (3.4) yields a value of the t-statistic of 1.832. In order to compute the P value, the degrees of freedom (k) of the statistic need to be known and are approximated by the following equation

$$k = n_A + n_B - 2\,. \tag{3.5}$$

The degrees of freedom yielded a value of $k = 10$. The statistics software was then used to calculate a P-value of 0.0988. This means that there is a 9.88% probability that Gear A and Gear B have the same fatigue lives.

If the 90% confidence level is established as sufficient, then the test concludes that the null hypothesis is rejected, and the alterative that Gear A and Gear B have different mean fatigue lives is accepted. However, at a 95% confidence level, it cannot be statistically determined that there is a difference in the mean fatigue lives of Gear A and Gear B. Because the P-value is so close to the 95% confidence level cutoff, it would be suggested that more fatigue data to increase statistical power might change the result of the test.

CHAPTER 4

CONCLUSION

4.1 Summary

This work builds on a series of research projects aiming at providing relevant insight into the experimental methodologies and statistical analysis for gear fatigue single tooth bending testing [1-10, 13-18]. This developmental work was performed to investigate the required methodologies in order to accurately evaluate the tooth bending fatigue lives of a fine pitch thin-rimmed gear under nearly fully-released loading conditions on a high speed, linear hydraulic load frame test machine.

Two geometrically equivalent spur gears used interchangeably in the same application were chosen to perform tooth bending fatigue life experiments on a fixture fabricated to load the gears according to the contacts points specified as part of the developmental work. These two gears were fabricated with different manufacturing processes with the intent that their tooth bending fatigue lives would be equivalent. Multiple fatigue tests were done on each gear at a single, equivalent loading condition. A statistical analysis was performed on the resulting fatigue measurements to determine if a statistically significant difference in the fatigue lives could be detected.

4.2 Major Conclusions

Based on the developmental work and analysis presented in Chapter 2 and 3, the following major conclusions can be made:

- Contacting the test and reaction tooth of the test gear on a horizontal plane minimizes the bearing forces seen by the test fixture. This also allows the greatest percentage of the force produced by the test machine to be seen by the test tooth.

- A fixture designed to hold the test gear such that two teeth were loaded in the hydraulic load frame at contact points specified in the development was successfully fabricated and able to produce proper load distributions on the test tooth.

- Fatigue tests were successfully performed with the newly designed fixture on two test gears and was able to precisely load the gear teeth in a cyclic manner to produce fatigue data following conventionally used statistical distributions.

- There is a 9.88% probability that Gear A and Gear B have the same fatigue lives. Thus, at a 95% confidence level, a statistical difference cannot be concluded between the two. However if only a 90% confidence level were required, Gear A and Gear B would be said to have statistically different tooth bending fatigue lives.

4.3 Recommendation for Future Work

As an extensive database on varying experimental methodologies for single tooth bending tests has been established, this work along with other related works act as a stepping-stone to accelerate and streamline the single tooth bending test procedure. The contact point development process and statistical analysis can be employed or adapted in studies investigating the fatigue lives of gears using a single tooth bending test machine. Some potential topics for future work include:

- Compute Hertzian Contact Stress between upper anvil of test machine and gear tooth to better understand the pressure distribution near the tip of the tooth. This will yield a more accurate upper limit for the test tooth contact point.

- Conduct strain gage measurements in the roots of the test gear to compare to root stress calculations from computer load distribution simulations.

- Develop a general MATLAB code to optimize and solve for the reaction and test tooth contact points for any given gear geometry towards streamlining the fixture design process.

- Instrument the gear with accelerometers to measure dynamic behavior of the gear rim and tooth when cyclic load is applied at different and higher frequencies.

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