The NCTM Standards call for assessment to be an integral part of teaching. We should evaluate what we teach, including a broad range of mathematical aspects such as problem solving, reasoning, and communication. We also should use a variety of techniques that demand different types of thinking and that correspond to the type of information sought. One approach is to use the seminar as an assessment tool.

I recently gave my first-semester calculus class an open-ended two-week project on finding the area of a "Blob" (similar to the one shown in figure 1) drawn on graph paper and on finding the area of the region bounded by \( y = x^2 \), \( x = 0 \), \( x = 3 \), and \( y = 0 \) (London, 1989). The students were given this problem before we discussed integration. I explained that there was not a specific "right answer"; there might be several good ways to approach the problem. They could use any math texts they liked, use the computer if they chose, discuss their ideas with each other or with me. I wanted the students to solve a problem new to them and to attack it with their own creativity and reasoning capabilities.

At the end of two weeks, they were to participate in a seminar. To help them prepare, I described my expectations at the beginning of the project. The students were to discuss with each other and with me what they had done: what worked, what hadn't worked, how they knew some technique didn't work, their process of deciding what to do, and their results. Each seminar would include at most eight people.
I held the seminars in a college classroom by having eight students at a time come to class for 30 minutes. (Small-group seminars could be conducted in a high school class by working with eight students in one corner of the room while the other students worked on other projects.) It was important that the groups be small so that many students had a chance to discuss their original ideas and methods. In our seminars, there was repetition of several approaches in different groups. By being in separate groups, the students with similar ideas had a chance to shine and to explain their own contribution. In a large classroom setting, once one student presents an idea for solving a problem, other students with the same idea lose the chance to participate, thus losing both the pride of authorship and opportunity for recognition.

During the seminar, each student explained the way the problem had been attacked, including any methods that led to dead ends, and explained the final results. The participants listened carefully to each other and questioned the things they didn’t understand. Thus, each student had to clarify his or her thinking enough to explain what had been done, both in the initial presentation and in responding to questions. In listening, each student had to follow another's logical arguments and judge the validity of those arguments.

Throughout the semester, I had discussed with the students my philosophy of teaching mathematics. My goals were to engender not just learning of skills but also problem solving, oral and written communication, and reasoning, and to instill the self-confidence needed to attempt the "dreaded calculus".

Assessment should be aligned with one's goals and with the mathematical content being learned. Observing the students' participation in the seminar was an appropriate mechanism for evaluating their projects on area. (Any similarly open-ended project would lend itself to assessment by seminar.) I was not looking for the right answer but instead was observing the students' problem solving processes, mathematical reasoning, communication, and disposition. Another advantage to having small seminar groups was that I could record observations about each student. I assigned grades rather generously, acting on my conviction that the students should be rewarded for attempting something new rather than be penalized for not reaching the exact answer. All students who actively participated received an A, A-, B+, or B. The grade depended upon the creativity the student showed, the reasoning used to support what was done, and the details included in
The result of this approach was that, instead of dreading an assessment of their work on a test, the students were quite enthusiastic about their seminar experience. They enjoyed showing their colleagues what they had accomplished and also seeing how others had solved the same problem. Furthermore, I gave them much-appreciated positive feedback about their progress and mathematical thinking by praising them in front of their peers. Negative comments written on a test are largely ignored by students. During the seminars, however, "constructive criticism" could be stated in a non-threatening way (sandwiched between positive comments) and with a much higher probability of being heard. The students also appreciated the fact that only eight people participated in each seminar, because they found it much less intimidating to talk mathematically in front of a few people instead of the whole class. Finally, as one student later wrote, "The fact that we didn't have to get anything right made it a little easier to attempt some rather outlandish solutions."

I feel that use of this seminar as an assessment tool met several of the goals of the NCTM evaluation standards, by focusing on the student's progress in learning to solve problems, to reason and to communicate mathematically.

References


Riddle: I think; therefore I yam!