How effectively is mathematics taught in your school? There is evidence that the answer to the question above is, "Not very". The National Assessment of Educational Progress reports that mathematics learned in this country is dominated by computation, teacher explanations, computational exercises, and a focus on lower-order thinking skills.

Additional evidence of poor mathematics teaching exists from international comparisons. Japanese students who perform at the fiftieth percentile on standardized tests in Japan score better than the top five percent of students in the United States in mathematics (McKnight, 1987). Other evidence (Kearns, 1987; National Alliance of Business, 1987) indicates that the U.S. ranks at or near the bottom of the industrialized nations in mathematics achievement.

How do you know if your mathematics program is part of the problem? How can you evaluate your mathematics program? There are two areas on which to focus when evaluating your program: the teachers and the curriculum. To begin, let's consider observations of two teachers. This is the first day they are introducing division of mixed numbers by a fraction to their classes.

As you watch Teacher A you see the following:

[Teacher A is at the front of the room, demonstrating how to work a division problem involving fractions.]

• Teacher A: "I've just shown you two examples of dividing a mixed number by a fraction. Now, let's work an example together. For this problem \((1 \frac{1}{2} \div \frac{1}{4})\), what needs to be done first? Susan?"

Susan: "Well, \(1 \frac{1}{2}\) needs to be changed to the improper fraction \(3/2\). Then you have to find the reciprocal of \(1/4\), which is \(4/1\). Finally you have to multiply."

• Teacher A: "Excellent, Susan! What's the answer? John?"

John: "First you multiply the numerators \((3 \times 4)\) which is \(12\), then multiply the
denominators \((2\times 1)\) which is 2. The answer is \(12/2\) or 6 in lowest terms."

- **Teacher A:** "Good. Does anyone have any questions? *(Class is silent.)* Okay, let's do problems 1–20 for tomorrow."

If students are attentive, most observers might consider the above teacher to have done an acceptable job of teaching about division of fractions. If the observer were to look at the written work of the students the next day, it would likely seem that **Teacher A** was successful. There are several difficulties, however, with the approach used by **Teacher A**.

Research suggests that the example above is typical for many American classrooms (Dossey, 1988) and only low-level thinking is being encouraged. There was no application of the knowledge and no evidence that any "mathematical connection" with prior knowledge existed for the students. There was no suggestion that the students had any understanding of when such problems occur. **Teacher A** did not even encourage estimation of the answer to help students see if their solution made sense. The work assigned required no reasoning skills, no development of what the numbers represent, and no communication about the mathematics involved. That type of work would only be of use to students in a math class — never to someone encountering mathematics in a "real-life" setting. Research suggests that students taught in the manner of **Teacher A** will not be able to apply what they've learned to life outside a classroom.

As a contrast, let's observe **Teacher B** to see how the same content might have been approached more meaningfully:

> [As you enter the classroom, you see students working together with measuring cups and buckets of sand. They are not sitting silently in straight rows, but are working on some task.]

- **Teacher B:** "Please measure \(1^{1/2}\) cups of sand in your cups *(students fill one of their containers to that mark).* Now, tell me how many groups of \(1/4\) of a cup you could pour into a second measuring cup without wasting sand or using it a second time. *Jill?*

Jill: "Well, *(after counting marks on her cup)* it looks like I could fill it about 6 times, so there must be 6 groups."

- **Teacher B:** "Very good. Let's see if Jill's estimate is correct. Bill, you have \(1^{1/2}\) cups of sand. Pour out \(1/4\) of a cup into another container *(Bill carefully pours out \(1/4\), then repeats it five more times – the class watches).*"
Teacher B: "Good observation! You were correct, Jill — we were able to pour 6 groups of 1/4 of a cup. Do you know what kind of problem we've just done? (Some discussion by class members but no one responds.) Okay, let me ask a related question — if I asked you to show me how many groups of 2 chips there are if you have 6 chips, what would you do?

Student: "I'd take 6 chips and put them in piles of 2."

Teacher B: "Good. Let's do the same thing with this problem. We have 1 1/2 cups, and we want to know how many groups of 1/4 of a cup there are. Is there at least one group of 1/4 of a cup if we have 1 1/2 cups? Mark?"

Mark: "Yeah, there are six groups of 1/4. Oh, yeah, it's kind of like division!"

You observe Teacher B discuss and provide examples of the fact that division asks, "How many groups of this size are there if we have this much?" Teacher B provides several measurement examples involving cooking and building scale models. The students are then shown the computation, asked to complete a few textbook problems, and are encouraged to give a situation to apply them to.

Teacher B spent considerable time developing students' understanding of the meaning of division with fractions. The measurement model illustrated why the answer was 6, and the additional examples showed when the operation could be applied. Asking students to provide examples helped the teacher check their understanding and also made the mathematics "real". The questions asked by Teacher B required the students to "think about" what the numbers represented. Finally, students had to communicate their reasoning to others.

The skills mentioned above are different than the rote memorization required by Teacher A. At first glance, it would appear Teacher B's students didn't understand as much as Teacher A's. Certainly there were more questions and more mistakes made during responses. This really illustrates that teaching for understanding is more difficult and time consuming than teaching rote memorization. Even so, teaching students to understand helps prevent errors, and promotes retention and application of the concepts.

What do you look for when evaluating mathematics teachers? Following is a list of characteristics which you might not normally identify. These are in addition to other items which are usually assessed and which you expect to see in good teachers.

1. Is the teacher a member of NCTM? Does (s)he read NCTM publications? Is
the teacher an active member of the state and local mathematics associations? Teachers who are aware of new trends in methodology and who make an effort to find new ideas are likely to be more successful than those who aren't. The NCTM and affiliated groups are the best sources of ideas for practicing teachers, and members are likely to be more knowledgeable about what's effective and what isn't.

2. How long does the instructional part of the class last? In the examples above, Teacher A spent very little time teaching. Most of the period was spent "doing problems". Teacher B, conversely, spent much more time asking questions and demonstrating the concepts involved. Students usually can learn new arithmetic procedures using only half the number of textbook problems included in most textbook series. Students learn the procedures they practice, and too much practice using an incorrect algorithm will make it difficult to unlearn that erroneous procedure. Additional problems serve only to "keep the students busy" and do little to reinforce what's been taught. Effective teachers spend about 30 minutes on class instruction (including about eight minutes at the beginning of the period for review) compared to about 15 minutes of time for seat work (Good and Grouws, 1979).

3. Is instruction designed to develop student understanding of concepts rather than rote memorization? Understanding does not just mean the ability to do computation. It requires the knowledge of why that computation works and when that type of problem is appropriate. Using manipulatives and structuring activities to develop conceptual meaning are the best ways to build that understanding.

4. Are the students actively engaged in the learning process? Have students been involved in gathering data to answer a problem? Were decisions made by the students regarding how to interpret or organize information they've used? Were problems solved from a textbook or were they related to some task students did in class? Each of these questions may provide insight into the effectiveness of instruction your students are experiencing.

The second area to evaluate is the mathematics curriculum. Most districts have a curriculum guide identifying what to teach in a given grade. How current is your curriculum? In 1989, the NCTM published curriculum guidelines to address the shortcomings of current U.S. curricular trends. Many recommendations might surprise you. For example, the lesson presented by Teacher A would not likely occur in a curriculum following the new NCTM recommendations. Topics recommended to receive less emphasis include long division, computation with fractions,
multiplication of 3-digit by 3-digit numbers, and drill and practice on manipulation of symbols (NCTM, 1989).

Several topics recommended to receive more emphasis include probability and statistics, estimation, and applications of mathematics. Skills involving the use of calculators, technology, and communication about mathematics are recognized as increasingly important. The emphasis is clearly on critical thinking and problem solving skills rather than rote mechanical procedures. Instruction should involve exploring, analyzing, and applying mathematics to real-world problems.

Your district curriculum may not yet reflect these changes, but placing priority on developing understanding doesn’t require new curricular content. Many changes are occurring in the curriculum and in the way mathematics is taught. Standardized tests and new textbook materials already reflect some of these changes, but change is a slow process. Regardless, mathematics education must focus on making mathematics meaningful and useful to students, irrespective of the topics covered. Careful evaluation can help identify and direct the focus of instruction in your school toward more successful practices.

References


