CUBE BUILDINGS

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This article is a summary of a series of lessons conducted by two prospective elementary school teachers and the mathematics methods instructor in the Teacher Education Center, Youngstown State University.

The unique parts of the series of lessons were: (1) the students were engaged in hands-on activities and found the formulas of volume and surface area of prisms by themselves, (2) the creativeness of the participating students was encouraged as they constructed sugar cube buildings of their own design, (3) the lesson achieved the textbook content through non-textbook activities.

Materials needed to carry out these activities include cubes (commercially produced cubes such as connecting cubes, multilink, or any wooden cubes, will be much better than sugar cubes for the suggested activities), and centimeter grid paper.

<u>Volume of Rectangular Prisms</u>

Group the students so that three or four students become members of a team. Give some sugar cubes to each team. The students will realize that a sugar cube is not really a cube, but a rectangular prism. (The actual dimensions are 1.3 cm by 1.3 cm by 1.1 cm as shown in Figure 1.) Discuss the difference between a cube and a rectangular prism. Inform the students that the shorter side of the sugar cube (1.1 cm side) is placed vertically in building activities in the lesson in order to obtain a solid and smooth appearance.

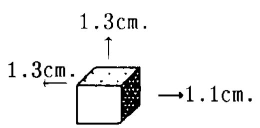


Figure 1

Ask each team to build a rectangular prism with three cubes. Ask each team to build a rectangular prism using 12 cubes. Let the students check other teams' rectangular prisms to determine whether the shape of each building is a rectangular prism and only 12 cubes are used. Introduce vocabulary such as "base" and "lateral faces" of the rectangular prism. Let students count them. In rectangular prisms each pair of opposite faces may be called bases. In other prisms the congruent, parallel faces are bases, with lateral faces being rectangles. More precise definitions aren't needed at this level.

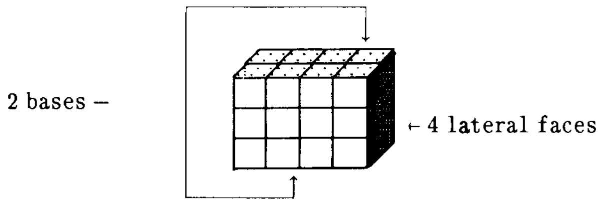


Figure 2

Ask each team to build a rectangular prism using 30 cubes. Each team should check the other teams' building to see if the shape is a rectangular prism and the building contains only 30 sugar cubes.

Now ask one student of each team to explain why his or her team's building is a rectangular prism and why the building contains exactly 30 cubes. At this point many students use their own methods such as (1) count the squares of the front side and multiply that number by the number of layers behind the front face or (2) count the squares of the base and multiply the number of squares by the number of layers between the two bases. Designate the student's or the team's name for the formula, such as "John's Formula", when a student or the team uses the method such as the number of squares of the base times the height of the prism. Since the number of squares of the base is base area, define "John's Formula" as "base area x height". This formula is much more convenient than the traditional "width x length x height" formula because the new formula can be applied in computing volumes of all prisms, such as the hexagonal prism and cylinders, but the traditional formula can be applied only to rectangular prisms. Some of the sample rectangular prisms with 30 sugar cubes are shown in Figure 3.

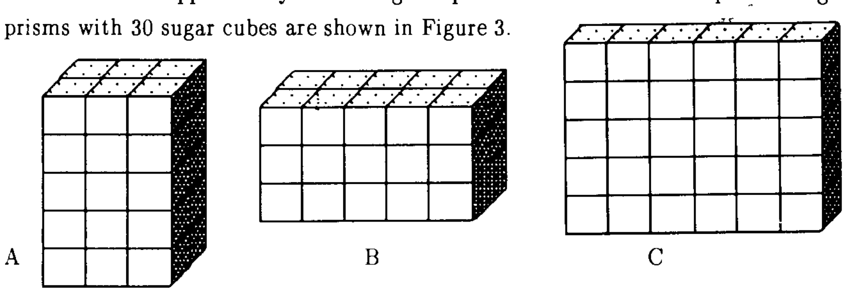


Figure 3

Students should apply the discovered "John's Formula" to find the number of cubes in the rectangular prisms of each team.

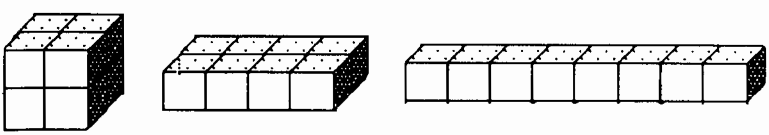
Now, have each team build a rectangular prism with 40 cubes. This time, let them design first by drawing a rectangular prism of their own shape, indicating the "base area" and the "height", then actually constructing the building. Then let them check the other teams' buildings.

By giving a box and a ruler to each team, let them find out how many 1 cubic centimeters, (which is 1 cm by 1 cm by 1 cm), the box can hold. Explain that the formula they discovered to find the number of sugar cubes can be applied in finding volumes of rectangular prisms.

<u>Surface Area of Rectangular Prisms</u>

Give a sugar cube to each team so that students can count the number of faces of the sugar cube. Inform them that total area of the six faces is the surface area of the cube. Have the teams build a rectangular prism with three sugar cubes and count the square units of surface area of the prism. Discuss the difference between the volume and the surface area: the volume is how many unit cubes fill the prism and the surface area is how many square units can cover the prism. Discuss various teams' methods used in finding the surface area of the prism. Usually the following formulas were popular: (base area x 2) + (front side area x 2) + (side area x 2), or the sum of the areas of three adjacent faces x 2. If centimeter cubes are used, each face is one square centimeter.

Let each team build a building using 8 cubes and record the volume and the surface area of their building. Some of the samples are in Figure 4.



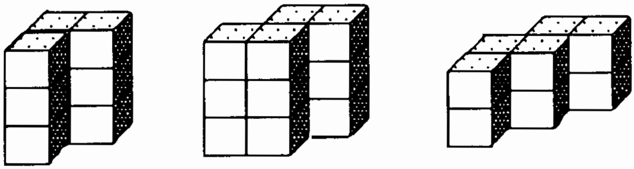
A B C

Figure 4

The students will discover that the rectangular prisms all have 8 cubic units of volume and their surface areas are different (24 square units for A, 28 square units for B, and 34 square units for C). Lead the discussion that the cubical shape of the rectangular prism takes less surface area.

Volume and Surface Area of Prisms other than Rectangular Prisms

Build the prisms in Figure 5 as the models and have the teams build the same prisms:



A B C

Figure 5

Remind the students that any three-dimensional figure that has two bases congruent and parallel and rectangular lateral faces is a prism and the name of the base determines the name of the prism. For example, if the base is a triangle, then the solid is called a triangular prism; if the name of the base is a hexagon, then the solid is called a hexagonal prism. Let them find the solid's name; such as, solid A in Figure 5 is a hexagonal prism; B an octagonal prism; and C a decagonal prism.

Have students find the volume of each figure and check whether the formula that they discovered, base area times the height, works for the prisms. The student will discover that the formula works for any prism, but the formula of "width x length x height" does not work for all other prisms except rectangular prisms.

Let students build their own prism and find the volume and surface area of the prism.

Building Activities with Cubes

Construct a building such as shown in Figure 6, on the teacher's desk. Designate the base, front, right side, left side, and back side of the building. Demonstrate how to draw the different views for base, front, right side, left side, and back side of the building on a piece of grid paper either on the overhead or chalk board as illustrated in Figure 6.

Let each team design and construct a sugar cube building. Give each team a piece of grid paper and have them draw a different view of their building: base, front, left side, right side, and back. One-centimeter grid paper will be helpful to students in drawing the buildings.

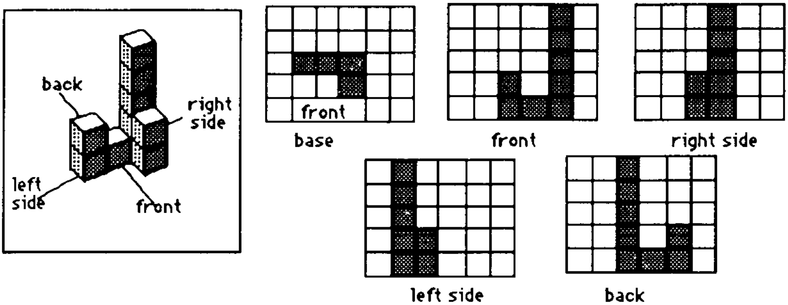


Figure 6

Let students indicate the front side of the building and move to the sides and back when they draw these illustrations.

After completing all five illustrations, students should dismantle their buildings. Then, after exchanging their illustrations with one other team, the students will build their partners' building by studying the five illustrations. Partners should compare the drawings to the finished buildings as a check.

Students may repeat the activities by constructing complicated buildings using many cubes. At the advanced level, the students can construct a building using an unlimited number of cubes. The number of cubes for the building is the volume of the building. The students may be asked to pretend that they are painting the building. In order to paint the building they are asked to count how many squares of the building need to be painted. Only the exposed faces are counted.

Cube buildings teach geometry, enhance spatial visualization, and encourage creativity in the math classroom.

"As every school student knows, a SCALENE triangle is one with UNEQUAL SIDES. SCALENE, aptly, is from SKALLOS, Greek for CROOKED LEGGED."

Word for Word
Cartoon strip by Atchison
Sept. 17, 1991