AN EDIBLE METHOD OF EVALUATION

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As the school year stretches into June, both the students and the teacher wish to be outside or at least doing something different. Faced with a general mathematics class containing 28 students who score below the 25th percentile on the Iowa Test of Basic Skills mathematics composite, a real effort must be made to keep these students functioning. These students have a "test phobia" and an effort must be made to make testing a low stress endeavor.

The last quarter of the year covers consumer mathematics, loan application forms, and probability. The latter includes simple probability (coin flipping, for example), expressing probability as odds, and mathematical expectation. Keeping in mind the contents of this last quarter and the need for retention of previous material, the challenge then becomes to create a non-threatening method of evaluation for these students.

In keeping with the NCTM standards, using manipulatives provides a learning experience with less of a threat to the individual than straight multiple choice or customary paper tests of problem solving. A familiar manipulative makes the testing even less stressful than usual and also makes the class more interesting. Manipulatives that can be eaten after the test is finished encourage the students to get on with things and not waste time. With these thoughts in mind, the lead author developed the following test for use in her general mathematics class as a final examination for the second semester. Each student contributed $.50 to the teacher who then purchased packages of M&Ms for each student. These packages were given to each student at the beginning of the exam period.

**Math I Exam**

All work must be shown on this paper

1. Count the number of candies and report to the teacher.
   
   _______ candies.

2. Sort by color and record the number of each color.
   
   _______ red

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**FIBONACCI NOTE**

A note from Catherine Mulligan, Fenwick High School and Miami University—Middletown, who was one of the referees for "Magic Squares a la Fibonacci":

I got "hooked" on this. I did some crank and grind stuff with polynomial arithmetic to verify the property of the Fibonacci square (and to show that regular magic squares also have the property). What I did isn't elegant or profound, however.

### Fibonacci

<table>
<thead>
<tr>
<th>8x + 13y</th>
<th>x</th>
<th>3x + 5y</th>
</tr>
</thead>
<tbody>
<tr>
<td>x + y</td>
<td>2x + 3y</td>
<td>5x + 8y</td>
</tr>
<tr>
<td>x + 2y</td>
<td>13x + 21y</td>
<td>y</td>
</tr>
</tbody>
</table>

Sum of product of rows =

Sum of product of columns =

\[ 34x^3 + 133x^2y + 167xy^2 + 66y^3 \]

### Regular

<table>
<thead>
<tr>
<th>x + 7n</th>
<th>x</th>
<th>x + 5n</th>
</tr>
</thead>
<tbody>
<tr>
<td>x + 2n</td>
<td>x + 4n</td>
<td>x + 6n</td>
</tr>
<tr>
<td>x + 3n</td>
<td>x + 8n</td>
<td>x + n</td>
</tr>
</tbody>
</table>

Sum of product of rows =

Sum of product of columns =

\[ 3x^3 + 36x^2n + 114xn^2 + 72n^3 \]

Also, look at a "multiplicative" magic square:

<table>
<thead>
<tr>
<th>ax^7</th>
<th>a</th>
<th>ax^5</th>
</tr>
</thead>
<tbody>
<tr>
<td>ax^2</td>
<td>ax^4</td>
<td>ax^6</td>
</tr>
<tr>
<td>ax^3</td>
<td>ax^8</td>
<td>ax</td>
</tr>
</tbody>
</table>

Sum of product of rows =

Sum of product of columns =

\[ 3a^3x^{12} \]

I wanted to look at 4 x 4's, etc., but ran out of time.
MAGIC SQUARES A LA FIBONACCI

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For years people have been fascinated with magic squares as well as the Fibonacci sequence of numbers. Put the two together, and you have an especially intriguing mathematical pastime.

In a "regular" magic square, the sum of each row, column, and diagonal is the same number. Consider a regular, 3 by 3 magic square that uses the counting numbers 1-9:

<table>
<thead>
<tr>
<th>8</th>
<th>1</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>2</td>
</tr>
</tbody>
</table>

The Fibonacci sequence is 1, 1, 2, 3, 5, 8, 13, 21 ... where each term is the sum of the preceding two terms. Take any sequence of nine Fibonaccis and pair them up with the magic square counting numbers 1-9. The square below uses the Fibonacci terms 2, 3, 5, 8, 13, 21, 34, 55, 89:

<table>
<thead>
<tr>
<th>55</th>
<th>2</th>
<th>21</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>13</td>
<td>34</td>
</tr>
<tr>
<td>8</td>
<td>89</td>
<td>3</td>
</tr>
</tbody>
</table>

A new magic square is formed, but not with the usual properties. In a Fibonacci magic square, the sum of the products of each row is equal to the sum of the products of each column. (Isn't that pretty??)

Try the same procedure with a different sequence of nine Fibonaccis. In fact, why not try it with your own sequence, created a la Fibonacci (example: 3, 3, 5, 9, 15, 24 ...)?

3. Create a bar graph in the space to show how many M&Ms of each color were in your sample. (5 pts)

4. Using class data from the board, find the: (4 pts)
   a. mean
   b. median
   c. mode
   d. range

5. What are the following probabilities assuming you replace before drawing again? (5 pts.)
   a. \( p(\text{orange}) \)
   b. \( p(\text{green}) \)
   c. \( p(\text{purple}) \)
   d. \( p(\text{orange, red}) \)
   e. \( p(\text{brown, tan}) \)

6. What are the following ratios (in lowest terms)? (4 pts.)
   a. \( \frac{\text{red}}{\text{total}} \)
   b. \( \frac{\text{green}}{\text{orange}} \)
   c. \( \frac{\text{brown}}{\text{tan}} \)
   d. \( \frac{\text{yellow}}{\text{orange}} \)

The letter grade scores on this test were, on average, one letter grade higher.
than the multiple choice examination given at the end of the first semester. The students thought this a less demanding, more relaxed exam than the other exams they took as part of their final exam series. They were deceived. The examination still addressed five of the twelve performance objectives covered during the year. The objectives were chosen from all four quarters. Only one day was allowed for review for this test as the students were still learning new material until the final examination period.

The five objectives covered and the questions related to them were:

1. Without the aid of electronic equipment, the student will perform the four basic operations using whole numbers with 75% accuracy. (Q2&4)

2. Without the aid of electronic equipment, the student will perform the four basic operations using fractions with 75% accuracy. (Q5&6)

3. Given a set of data, the student will organize, interpret, and analyze the data with 75% accuracy utilizing graphing as well as other forms of organization. (Q3&4)

4. Given the direction, the student will solve and correctly use ratios, proportion, and rate problems with 75% accuracy. (Q6)

5. Given the direction, the student will use elementary notions of probability with 75% accuracy. (Q5)

Questions 5 and 6 require the teacher to do calculations for each individual student. This takes more time in grading than is required by using a multiple choice format or by using one package of M&Ms and doing the counting for the class. But the additional time required is a small price to pay for the additional information provided about each student. It is obvious from students' answers if they understand the statistical terms. If the answer is incorrect for each of the terms, it is easy to note whether the answer is caused by confusion between terms, ignorance of the definition of the terms, or lack of concern on the part of the student. Since all calculations must be shown on the paper, the same also is true of questions 5 and 6. Questions 5d and 5e receive partial credit if the students indicate the multiplication but do not perform the multiplication. Partial credit is given also in question 6 if the numbers chosen are correct but are not reduced to lowest terms.

fun of discovery is the highly valuable exercise of writing a precise description of the recognized language. As an interesting example, determine the language accepted by the machine in Figure 7.

![Figure 7](image)

Reference


Once you have the necessary items, plug the RS–232 connector on the back side of the Mini Modem 2400 into the RS–232C connector on your computer, printer or terminal, then screw up. — *Installation instructions for a modem made in Taiwan.*

Don't worry, we will!

From *The New Yorker*
October 28, 1991, p. 84
state is called "the language recognized by M". (Incidentally, we shall assume that all sequences are nonempty.) In the next example and succeeding ones, we follow the convention of simply numbering the states 0, 1, 2, ... , understanding that 0 is the starting state and double-circled states are the accepting ones. Figure 6 is the state diagram of a machine M having three states, two of which are accepting. M reads sequences of x's and y's and recognizes a certain language that contains the sequence yxxyy, among others.

![State Diagram](image)

Figure 6

To check that claim about yxxyy, start in state 0 and read the sequence from left to right, using it as a set of directions to guide you among the states of M in Figure 6. The initial y leaves M in state 0, the first x sends M to state 1, and second x leaves M in state 1 and the next y sends M to state 2. The final y leaves M in 2, which is an accepting state, and so the sequence yxxyy is accepted. As one can verify, the language recognized by M consists of all sequences that end with a y.

To illustrate a way of recording the progress of M as it reads a sequence, consider xyyyx as the input sequence. The idea is simply to record below each symbol, and slightly to the left of it, what state M is in when it encounters that symbol. Since the initial state is 0, put "0" at the beginning of the state list. While in state 0, the machine reads the initial symbol x and switches to state 1. Keeping track of the computation so far, we have this:

<table>
<thead>
<tr>
<th>Sequence</th>
<th>States</th>
</tr>
</thead>
<tbody>
<tr>
<td>x y y y x</td>
<td>0 1</td>
</tr>
</tbody>
</table>

The rest of the computation creates this record:

<table>
<thead>
<tr>
<th>Sequence</th>
<th>States</th>
</tr>
</thead>
<tbody>
<tr>
<td>x y y y x</td>
<td>0 1 2 2 2 1 (final state not accepting)</td>
</tr>
</tbody>
</table>

Students enjoy the challenge of puzzling out the language recognized by a finite-state machine, and they like to create examples of their own. Alongside the

Question 1 is graded on the basis of the answers given in question 2. The total number of M&Ms in question 2 must equal the number given in question 1. Even if the answer to question 1 is incorrect, it is still used as the basis for grading questions 5 and 6. Students will probably refer to their answer to question 1 throughout the test and it is unfair to penalize them for this.

The students enjoyed this examination and appeared interested in doing the work asked of them. The authors feel that using a common manipulative helps relieve the stress many students feel during evaluations. Eating part of the evaluation also may relax the students. The students involved are not the "academic elite". Many of them are counting minutes until school ends. This evaluation instrument holds the students' interest and allows students to succeed. Nothing is more important than that in educating students.

AN ANNOUNCEMENT FROM:

RCDPM (Research Council for Diagnostic and Prescriptive Mathematics)

With the cooperation of Educational Testing Service (ETS), the New Jersey Department of Higher Education and Trenton State College, the Research Council for Diagnostic and Prescriptive Mathematics (RCDPM) will hold its Nineteenth Annual Conference in the Princeton, New Jersey area 14-16 February. Program Chair Richard Lesh (ETS) has announced two themes: Assessment at All Levels and Collegiate Mathematics Education. The conference will include about 50 research reports and thematic presentations as well as plenary addresses by Mathematics Educators Jan de Lange and Thomas Romberg and Science Educator William Aldridge. Presenters are being encouraged to provide time for interchange with participants. RCDPM represents a broad range of interests in Mathematics Education research, but has traditionally placed strong emphasis on individual and clinical work with students at all levels. For registration and additional information, contact David E. Boliver, General Conference Chair, Department of Mathematics and Statistics, Trenton State College, Trenton, New Jersey 08650-4700; (609)771-3042. The deadline for reduced rate conference motel rooms is Dec. 1.