OBSERVING AND TREATING MATH ANXIETY

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OBSERVATION

Today, many teachers are trying to help students who are anxious about mathematics class. Our concern for students can be guided by looking at some of the more common psychological indicators of anxiety. Some of the cues which may indicate math anxiety are listed below. Observing several of these indicators can guide our development of corrective teaching strategies.

Dependence — Children who are constantly at the math teacher's desk, asking, "Is this right?" may be very anxious about their math work. They show an extremely strong need for approval; taken with other signs, this indicates math anxiety.

Excessively Cautious — Fearful of taking a risk, these math students spend a great deal of time making sure one problem or exercise is done perfectly. They may be very neat and perfectionistic, a combination that seems to freeze their creativity and spontaneity. Taking an inordinate amount of time to complete a mathematical task, when most of the class members are already done, is another signal of anxiety.

Reduced Responsiveness to the Environment — Anxious people are preoccupied so they may not be paying attention in math class. This is not because they are trying to be rude. Instead, their feelings of inferiority may be blocking any consistent attention span.

Deterioration of Complex Problem Solving Processes — Anxiety affects complex, higher order thinking more than rote, mnemonic skills like spelling or basic fact recall. Some students may recall basic facts very well but be unable to exhibit mathematical reasoning skills necessary for problem solving. Suspension of judgment, reflection, and generation of alternatives can be harmed by high levels of anxiety.

Rejection by Family — Rejection is sometimes more a perception than a reality. Children from single-parent homes may, in reality, be rejected by the non-custodial parent. However, children may perceive rejection in a divorce situation even when both parents try to be supportive of the child.

What will be the period of mod 32? Students will likely guess 48, which is 2 times the period for the preceding power of 2.

**Powers of 3**

<table>
<thead>
<tr>
<th>Mod</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>mod 3</td>
<td>8</td>
</tr>
<tr>
<td>mod 9</td>
<td>24</td>
</tr>
<tr>
<td>mod 27</td>
<td>72</td>
</tr>
</tbody>
</table>

What will be the period of mod 81? The period is 216, which is 3 times 72. Look at the powers of mod 5.

**Powers of 5**

<table>
<thead>
<tr>
<th>Mod</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>mod 5</td>
<td>20</td>
</tr>
<tr>
<td>mod 25</td>
<td>100</td>
</tr>
<tr>
<td>mod 125</td>
<td>500</td>
</tr>
</tbody>
</table>

What will be the period of mod 625? The period is 2500, which is 5 times 500.

Have the students formulate conjectures about the periods of Fibonacci numbers (mod m) where $m = p^k$ for a prime number $p$. A good hypothesis might be that the period equals $p$ times the period for $F_n$ modulo the preceding power of $p$. Also note that, equivalently this is $p^{k-1}$ times the period for $F_n \pmod{p}$. Have them examine the Fibonacci numbers modulo 7 and 11 to test their hypothesis. Remind the students that testing cases may support a conjecture, but only a proof covers all cases.

In summary, students have prepared a table of values by hand, written a computer program to check and extend that table, learned useful algebraic notation, and conjectured a generalization to describe their findings. There are many problem solving skills to be learned by students as they Fibonacci Around the Clock!

"Cauchy, there are limits to your knowledge of real numbers," Dedekind said cuttingly.

*Mathematical Maxims and Minims*
Compiled by Nicholas J. Rose
Rome Press, Inc., 1988, p. 135
60 FB3=FB2+FB1
70 FM3=FB3 MOD M
80 PRINT "",FM3;
90 IF FM3=0 AND FB2=1 THEN GOTO 130
100 FB1=FB2
110 FB2=FM3
120 GOTO 60
130 NEXT M
140 END

To have the computer print the period (length of each cycle) insert the following lines.

35 P=2
55 P=P+1
90 IF FM3=0 AND FB2=1 THEN GOTO 125
125 PRINT " Period = ",P

For those using Applesoft Basic, the following changes need to be made to the program.

65 GOSUB 1000
(Delete line 70.)
1000 IF FB3 < M THEN GOTO 1040
1010 FB3=FB3-M
1020 GOTO 1000
1040 FM3=FB3
1050 RETURN

With the help of these computer programs, students can quickly generate various cases, where they will see patterns appear and be able to predict the period for a specific modulus. For example, if the modulus is a power of a prime, we get the following results:

<table>
<thead>
<tr>
<th>Powers of 2</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>mod 2</td>
<td>3</td>
</tr>
<tr>
<td>mod 4</td>
<td>6</td>
</tr>
<tr>
<td>mod 8</td>
<td>12</td>
</tr>
<tr>
<td>mod 16</td>
<td>24</td>
</tr>
</tbody>
</table>

In the case of the two-parent family, rejection may be very subtle. A parent might say at an open house, "Johnny is a very poor math student", or "Sue really is not close to her brother's ability".

Psychologists suggest that a four-to-one ratio of praise to criticism be provided for children. A home environment offering a derogatory comparison of children, emotional aloofness, and criticism can contribute to math anxiety.

**Hostility** — Irritability, impatience, and anger are likely to increase when individuals are anxious. For example, a student may flare up in anger at his/her inability to solve a problem.

**Expectation May Exceed Ability** — Excessively high goals can be related to anxiety. True enough, positive expectations are important but not everyone is going to attend Harvard. Some students, for a variety of reasons, may feel pressure to be perfect. Even a 90% success rate is considered low by them. Keep an eye on students who never seem to be satisfied with themselves.

**Physiological Symptoms** — In a mathematics class, the classic sweaty palms symptom may be present during a mathematics test. Some children may pull their hair, have stomachaches, headaches, a need to go to the bathroom, among other symptoms, when performing mathematical tasks.

**Compulsive Behaviors** — Compulsive behaviors are non-task related actions that drive a person and inhibit the performance of a required task. For example, some students spend an inordinate amount of time arranging their desk for mathematical work. They may arrange their pencils in order, put paper in a certain pile, clean their desk top, sharpen pencils, etc. Performance of these activities are substitutes for mathematical work. Instead of thinking about mathematics, these students are controlling their environment to gain a feeling of mastery.

**Avoidance Behaviors** — Individuals sometimes avoid situations which they perceive as anxiety creating. Students may miss assignments, avoid class, and attempt to escape being called upon in class.

**Low Self-esteem** — At the heart of the math anxiety dilemma is poor self-esteem. Students sometimes say that they were "never any good at math". They may attribute success to luck and failure to a lack of ability. Their self-talk of negativity affects their math learning. Instead of focusing on the lesson, these students are rehearsing old scripts which inhibit their performance.
TREATMENT

Now let us see what we can do to help these children. Awareness of a problem can be a major step in its solution.

Raise Self-Esteem  Children with high dependency needs and poor self-esteem must be assured that they are adequate as people. Frequent praise, sometimes in the form of motivational awards such as certificates saying "Most Improved Problem Solver", "Estimator of the Week", etc. can be helpful. Praise and awards need not be called out at a semester assembly; more frequent weekly awards can be given. These children need to say out loud, "I am good at math." True enough, their performance may not quite approach excellence, but the saying of a phrase such as this can substitute and break the cycle of self-talk which drones on with phrases such as, "I never could do math well." Verbalizing an affirmation can sometimes image the successful completion of a skill.

Alleviate Tension  — Physical symptoms may be altered by simple deep-breathing exercises. Slow, deep inhaling followed by slow, complete exhaling, done three or four times can produce calmness. Results with adult hypertension patients have shown these techniques to have calming effects (Cousins, 1989). Why not try these with a small group of math students?

Use Humor  — Humor can help reduce anxiety, lower unrealistically high goals, and increase responsiveness to the environment. Is there a Fourth of July in England? How much dirt is in a hole three feet by two feet and five feet deep? These are just a couple of the many riddles and puzzles that can help break the ice during a math class.

Teach Problem Solving Strategies  — Problem solving can be aided by the now widespread approach to teach strategies such as making a table, creating a diagram, or working a simpler problem. This gives the anxious student a structure for solving problems. The student at least can be praised for trying something even if the attempt is unsuccessful.

Introduce Creativity  — Students who are compulsive and extremely cautious need to experience creative approaches to instruction instead of rote mechanical procedures. Problems with more than one answer or more than one method of solution such as, "how many ways can you use $25 to plan a class party" are helpful. Drawing a picture and creating a story problem can be another creative

To count to 7 on this circle, starting at zero, you would go around the circle two times and 1 more tic mark. So we say that 7 is congruent to 1 modulo 3, written \( 7 \equiv 1 \pmod{3} \). After the students are familiar with modular arithmetic, ask them to generate the Fibonacci sequence using different moduli and look for new patterns. By making a list, they can see the patterns more easily.

Examples:

\[
\begin{align*}
F_n \pmod{2} & : 1, 1, 0, 1, 1, 0, \ldots \\
F_n \pmod{3} & : 1, 1, 2, 0, 2, 2, 1, 0, 1, 1, 2, 0, 2, 1, 0, \ldots \\
F_n \pmod{4} & : 1, 1, 2, 3, 1, 0, 1, 1, 2, 3, 1, 0, \ldots \\
F_n \pmod{5} & : 1, 1, 2, 3, 0, 3, 3, 1, 4, 0, 4, 4, 3, 2, 0, 2, 2, 4, 1, 0, \ldots
\end{align*}
\]

The students should notice that the sequence starts to repeat whenever the terms 1, 0 occur in succession. After 3 terms, the mod 2 sequence repeats, after 8 terms mod 3 repeats, 6 terms are needed for mod 4, and after 20 terms for mod 5. The sequence of residues modulo \( n \) between repetitions is called the cycle. The number of terms it takes to complete a cycle is called the period. For example, the period for the Fibonacci numbers modulo 5 is 20. What else might be noticed? How many times in each cycle does zero appear? In each cycle of \( F_n \pmod{2} \) zero occurs once, mod 3 twice, in mod 4 once, and mod 5 four times. After generating the sequence for more moduli, students might hypothesize that, for any modulus, the cycle will contain 1, 2, or 4 zeros only. This exercise helps students develop skills of looking for patterns, making lists, and formulating hypotheses.

To further investigate this topic, students can use a computer. Modern technology provides terrific teaching tools for mathematics classrooms. First challenge the students to develop a computer algorithm to generate the Fibonacci sequence, then to modify their algorithm to compute the sequence modulo \( m \) for a user-specified value of \( m \). The following program stops once the sequence has come to the end of its cycle.

Sample GW – BASIC PROGRAM

```basic
10 REM FIBONACCI NUMBERS MOD M
20 FOR M=2 TO 25
30 FB1=1
40 PRINT "FIBONACCI NUMBERS MOD" M
50 PRINT FB1
60 FOR I=1 TO M
70 FB2=
80 PRINT FB2
90 NEXT I
100 PRINT
110 NEXT M
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"One o'clock, one o'clock, two o'clock rock. Three o'clock, five o'clock, eight o'clock rock. We're gonna Fibonac around the clock!"

Problem solving is a struggle for most students. By making problems more interesting and by showing students how to use problem solving strategies, teachers can turn confusion into confidence. Fibonacci numbers can help do just that. Activities with the Fibonacci numbers illustrate many problem solving strategies, such as using a formula, looking for patterns, considering various cases, making a list, and developing an algorithm.

When introduced to Fibonacci numbers for the first time, students can apply the problem solving strategy of looking for a pattern. Show the students the first few numbers of the Fibonacci sequence (1, 1, 2, 3, 5, 8, ...), and let them tell you in words how these numbers are formed. A student should say something like "The sequence starts with two ones and each number that follows is the sum of the two previous numbers." Next, ask the students to express the same idea by means of a formula. Here is an opportunity to introduce subscript notation. Let \( F_n \) be the number we want to find, where \( n \) is the number of the term in the sequence. Then, \( F_{n-1} \) and \( F_{n-2} \) are the two previous numbers and \( F_n = F_{n-1} + F_{n-2} \). Have the students use this formula to generate the next several numbers in the sequence (... 13, 21, 34, 55, 89, ...).

In order to investigate some additional patterns involving the Fibonacci numbers, we introduce modular arithmetic, better known to some as "clock arithmetic". Modular arithmetic involves dividing numbers by an integer, called the modulus, and using the remainders, called residues. For example, we will examine the number 7 (modulo 3). Three divides 7 two times with a remainder of 1. This can be demonstrated on a 3-hour clock or circle. Divide the circle into thirds, and label the divisions 0, 1, and 2.

venue to try with these students.

**Encourage Small Group Activities** — Some students, especially avoidant types, probably need small group or individual activities. Large group games where the child must stand up in front of the class should be avoided for the time being.

**De-emphasize Timed Activities** — Speaking of time, timed activities should be used discretely. Children with extremely high goals often put undue pressure on themselves. Whenever possible, try to structure instruction so that timed activities are proportional to the student's mental and emotional state. As adults, we surely have experienced examples where we have felt overcome by the multitude of tasks confronting us. Running errands, taking care of children or elderly parents, and job demands sometimes stress us greatly. Actually, some anthropologists feel that modern man has less free time than Neanderthal man! A moderate number of activities, done carefully and in depth, can provide a productive yet calm classroom environment.

**Encourage Cooperative Projects** — Our society is an extremely competitive one. Some children with exceptionally high goals may feel overwhelmed with constant competitive activities. Cooperative learning approaches (Johnson & Johnson, 1987) offer ways to reduce competition and develop creative projects. Anxious students often feel isolated, but small group activities where all members of the group contribute can help reduce the effects of isolation. A metric system scavenger hunt where all members find objects weighing a certain number of grams or having a selected length is a nice cooperative project. Small group projects in problem solving, especially problems where each child can make a contribution in gathering tabular information, also are helpful. For example, "How many ways can a pro football team score 17 points?" One member of the group can record results, and each child can contribute at least one or two responses.

Finally, reducing math anxiety requires awareness, sensitivity, and solid mathematics instruction. Emphasizing process, explaining concepts with concrete manipulative materials, and allowing children to generalize and discover math concepts are the foundation of effective math instruction. These techniques are especially helpful with math anxious students. We hope to teach our students the beauty of mathematics and, as Henry David Thoreau once said, "You cannot perceive beauty without a serene mind."
References


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**MATH SCRAMBLER**

Unscramble these four mixed-up math terms, one letter to each blank:

REM E T  

S T R I F  

RE U S A Q  

T I N N E Y  

Now, rearrange the letters in the boxes to form the answer to the riddle below:

AFTER A DIFFICULT ARITHMETIC EXAM, THE CLASS EXCLAIMED, "THAT WAS _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ 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