JOHN NAPIER - MARVELOUS MERCHISTON

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Napier! To the mathematically literate this name is practically synonymous with logarithms. Yet this man would have called himself a philosopher and theologian rather than a mathematician. Laplace once asserted that Napier's invention of logarithms "doubled the life of the astronomer" by shortening the labors involved in computation, and another philosopher has called logarithms "the greatest boon genius could bestow upon a maritime empire"; yet Napier died believing that he would be remembered primarily for his commentary on the Book of Revelation.

John Napier was born at Merchiston Castle, near Edinburgh, in 1550. The recorded history of his ancestors dates back to the early 15th century. According to one legend, the family name was originally Lennox, but one of the clan so distinguished himself in battle that the king declared that he had "na peer" when it came to valor. Accordingly, by royal warrant, this branch of the Lennoxes was renamed. Compounding the identity problem is the fact that no fewer than ten different spellings of the name "Napier" are found in historical records.

The Napiers played prominent roles in public affairs, serving in Parliament, the army, and the Reformed Church of Scotland. Merchiston Castle, probably built by Alexander Napier around 1438, stood on a high spot commanding the southwestern approach to Edinburgh. The old tower was over 60 feet high, with walls averaging six feet in thickness. Today the castle is part of the Napier Technical College of the City of Edinburgh.

John was the eldest child of Archibald and Janet Napier, the latter being a sister of Adam Bothwell, the notorious Bishop of Orkney. No record of Napier's childhood has been preserved, but during the first ten years of his life a fierce struggle between Roman Catholicism and Protestantism raged in Scotland. The country was distracted by civil war, which ultimately resulted in a victory for the Protestant party. Sir Archibald had identified himself with the Protestant cause, and John inherited from his father a fear of the Church of Rome that bordered on paranoia.

In 1563, the year of his mother's death, John entered St. Andrews University, where he studied theology and philosophy. He left there without earning a degree, and may have spent the next several years in study abroad. By 1571 he was back in Scotland, living in another mansion owned by his father at Gartness. The next year he married Elizabeth Stirling, the daughter of a neighboring nobleman. They had a son, Archibald, who would become the first Napier to hold the title "Lord," and a daughter, Joanne. Elizabeth died in 1579, and John subsequently married Agnes Chisholm, who bore ten more children. Napier became the eighth Laird of Merchiston when his father died in 1608.

In 1593 it was discovered that certain Catholic nobles and others were inviting Philip II of Spain to send an army to Scotland to accomplish what the Spanish Armada of 1588 had failed to do. Not only did Napier join with ministers of the Reformed Church in protesting before King James VI but, on his own, he wrote a book entitled, A Plaine Discovery of the Whole Revelation of St. John. This treatise is made up of two parts--the first consisting of 36 propositions and proofs in the style of Euclid, and the second consisting of notes and commentary on each verse of the Book of Revelation. Napier's most astounding theological conclusion was that the Pope is Antichrist. He also predicted that the "day of God's judgement" would fall "betwixt the years of Christ 1688 and 1700."

Napier believed that his life's reputation would rest on this theological work, and it was indeed extremely popular in its day. During the 50-year period following its first publication, there appeared no fewer than five editions in English, three in Dutch, nine in French, and four in German.

Given Napier's concern for the military security of Scotland, it comes as no surprise to learn that he wrote prophetically of several "war engines." These include burning mirrors, reminiscent of Archimedes; a piece of artillery that "can clear a field up to four miles in circumference of all living creatures exceeding a foot in height"; devices for "sayling under water"; and a chariot with a "living mouth of mettle that can scatter destruction on all sides." These latter instruments of war achieved reality in modern times in the form of the machine gun, submarine, and tank.

In many respects, John Napier was a typical 16th-century Scottish landowner. In addition to being active in the Church and in politics, he quarrelled with his half brothers over inheritance and disputed tenants and neighboring landlords over land rights. He paid considerable attention to agriculture, serving as poulterer to the king and also experimenting with the use of salted manure as a fertilizer. He has been credited with the invention of a hydraulic screw for pumping water out of flooded coal pits. Napier died on April 4, 1617--apparently a victim of gout.

Napier's major mathematical achievement--the invention of logarithms--has been characterized by some historians as a "bolt out of the blue," with nothing to foreshadow its discovery. Yet great discoveries in science and mathematics are seldom spontaneous. They are usually the result of a prolonged and agonizing struggle to solve some intractable problem. Napier's logarithms are no exception, and their development is the culmination of efforts to improve the methods of computation.

As Henry Pollack is fond of saying, addition and subtraction are cheap, multiplication is more costly, and division by messy numbers is very expensive. By the mid-sixteenth century, improved methods and instruments in navigation and astronomy made the multiplication and division of numbers with five or more digits quite common. Any device that could reduce the labor involved in such computations would be welcomed by the scientists.

One method already in use was known as prosthaphaeresis. This device involved transforming multiplication into addition by means of trigonometric identities such as: $(\sin a)(\sin b) = (1/2)[\cos(a-b) - \cos(a+b)]$. By the late 1500's the Danish astronomer, Tycho Brahe, and his assistant, Paul Wittich, had developed prosthaphaeresis into a systematic adjunct of astronomy. In 1590 Napier was already wrestling with the problem of how to change multiplication into addition when he heard of Brahe's success. By 1594 Johann Kepler, who had once been an assistant to Tycho Brahe, received sketchy information about a new method due to Napier. Two more decades were to pass, however, before any of it was published.

In 1614 Napier's Mirifici Logarithmorum Canonis Descriptio appeared, consisting of 57 pages of text and 90 pages of tables. Five years later, after Napier's death, a second Latin treatise, Mirifici Logarithmorum Canonis Constructio, was published by John Napier's son, Robert. The latter publication actually consists of material that Napier had written many years earlier, explaining how his tables had been calculated and the reasoning on which they were based. In the Constructio Napier uses the term "artificial numbers" rather than "logarithms," which suggests that the latter word (meaning ratio number) was invented somewhat later. Apparently Napier had withheld publishing the Constructio until he knew whether the public would accept his new concept.

The fundamental idea underlying logarithms is the correspondence between terms of a geometric progression and a related arithmetic progression. Michael Stifel, in 1544, had explicitly stated the four basic rules of exponents in connection with the sequences

... -3, -2, -1, 0, 1, 2, 3, 4, 5, 6 ...

and ... 1/8, 1/4, 1/2, 1, 2, 4, 8, 16, 32, 64 ...

However, today's exponential notation had not yet been invented, and the concept of exponent was still imperfectly understood.

Napier must have been thinking about the trigonometric work of the astronomers, for he actually constructed a table of logarithms of sines of arcs of a circle, whose radius was taken to be 10,000,000. From this radius he subtracted the 10000000th part, obtaining 9999999. From this result he again subtracted its 10000000th part, to get 9999998.0000001. Continuing in this way, he created a hundred proportionals, the hundredth being 9999900.0004950. Napier called this geometric progression his "First Table."

For his "Second Table" Napier again started with a radius of 10,000,000 and subtracted fifty successive 100000th parts until he obtained 9995001.222927. This last number resulted from a computational error (the correct result being 9995001.224804). The "Third Table" consisted of 69 columns, each containing 21 numbers, proceeding in geometric progressions with ratios of 1/2000. Moreover, the first entries in these columns also formed a geometric progression, with ratio 1/100. By interpolating the first and second tables with each column of the third, Napier had, in effect, created a table of 6,900,000 sines in geometric progression, with no two numbers differing by more than unity.

It may help in understanding the above tables to note that, in the First Table, the logarithm of 10000000 is zero; the logarithm of the next number, 9999999, is one; the logarithm of 9999998.0000001 is two; and so on, until the logarithm of 9999900.0004950 is 100. That is, the logarithms (ratio numbers) count the number of ratios that have been used in obtaining a particular sine from the radius 10000000.

In actual practice, Napier used existing tables of sines and chose his logarithms by a rather complex process of interpolation. To assign "artificial numbers" (i.e., logarithms) to each sine, Napier made use of the concept of a moving point that approaches a fixed point so that its velocity is proportional to the distance remaining from the fixed point. According to Napier's own definition, a logarithm is "that number which has increased arithmetically with the same velocity throughout as that with which the radius began to decrease geometrically." For a radius of 10,000,000 this is equivalent (in modern terms) to $dx/dt = -x$, and $dy/dt = 10{,}000{,}000$; whence $dy/dx = -10{,}000{,}000/x$. Consequently, $y = \text{Naplog } x = -10{,}000{,}000 \ln(x/10{,}000{,}000)$.

Napier's logarithms differ from our contemporary logarithms in several respects. As the sines decrease their Napierian logarithms increase. The logarithm of 10,000,000 is zero, while the logarithm of 1,000,000 is 23025851. (Compare this number with 2.3025851, the natural logarithm of ten.) Napier's logarithms actually had no base, but they are related to the base 1/e. Finally, Naplog x + Naplog y equals Naplog(xy/10,000,000), rather than Naplog(xy).

Credit for the development of common logarithms belongs jointly to Napier and an English mathematician, Henry Briggs (1561-1630). Briggs received his mathematical education at Cambridge and, in 1596, became the first professor of geometry at the newly founded Gresham College in London. Although he was a mathematician in his own right, he is best known for his combined efforts with Napier. In the spring of 1615, after learning of Napier's logarithms, Briggs wrote: "Neper, lord of Markinston, hath set my head and hands a work with his new and admirable logarithms. I hope to see him this summer, if it please God, for I never saw book which pleased me better and made me more wonder."

Briggs did visit Merchiston in the summer of 1515, and again in 1516. It is said that, when he and Napier met for the first time, they stood in silence for a full 15 minutes, each gazing in wonder at the other. In spite of his admiration for Napier's invention, Briggs had some improvements to suggest. He had written to Napier, proposing that log 10,000,000,000 = 0 and log 1,000,000,000 = 10,000,000,000. Napier, however, had some new ideas of his own, suggesting that log 1 = 0 and log 10,000,000 = 10,000,000,000. Finally, they agreed to make log 1 = 0 and log 10 = 1, as we know them today.

Since Napier was in very poor health, it was left to Briggs to calculate the new tables. The first 1000 logarithms of the new canon were published after Napier's death, but before December 6, 1617. Briggs' Arithmetica Logarithmica, containing the 30,000 logarithms from 1 to 20,000 and 90,000 to 100,000, appeared posthumously in 1624. The other 70 chiliads were completed by Adrian Vlacq and appeared in 1628.

The first English translation of Napier's Descriptio, undertaken by Edward Wright and published by his son, Samuel Wright, came off the press in 1616. The first publication on the Continent was that of Benjamin Ursinus in 1618. Through this work Kepler became aware of the details of Napier's discovery. He expressed his enthusiasm in a letter dated July 28, 1619, and was very influential in spreading logarithms throughout Europe. Although Napier's canon was not entirely free from error, his calculations were essentially sound and formed the basis for subsequent logarithm tables for nearly a century.

There is one other person who plays a leading role in the invention of logarithms. That is Jobst Bürgi (1552-1632), a native of Liechtenstein who became the court watchmaker to Duke Wilhelm IV in Kassel, Germany, and later to Emperor Rudolf II and his successors in Prague. He was known for his fine astronomical instruments and his careful, precise computation. His manuscripts include a use of the decimal point (sometimes substituting a small arc for the dot), an elaboration of the rule of false position, and tables of sines. While in Prague, he worked for Kepler, computing tables. From about 1584 (nearly 20 years before going to Prague) Bürgi was engaged in the improvement of prosthaphaeresis. There is also evidence that he knew of Michael Stifel's work on exponents.

The idea for logarithms could have occurred to Bürgi by the end of the 1580's, and it is possible that he had computed his tables before coming to Prague in 1603. However, Bürgi's only published work, Arithmetische und Geometrische Progress Tabulen, did not appear until 1620--six years after Napier's publication. After 1620 the scientific and cultural community in Prague disintegrated, with the result that Bürgi's tables went almost unnoticed and had no apparent influence on the development of mathematics. It is generally believed that Napier and Bürgi may each be credited with the independent invention of logarithms.

Bürgi's geometric progression begins with 100,000,000 and has a ratio of 1.0001. His arithmetic progression contains the terms, 0, 10, 20, 30, 40, etc. Bürgi's logarithm of 1,000,000,000 is 230270.022. These logarithms are essentially natural logarithms, but are actually arranged as an arithmetic progression of logarithms printed in red, with their corresponding antilogarithms in black.

Napier's concern for practical methods of computation led him to publish the Rabdologiae in 1617, the year of his death. This work contains several elementary calculating devices, including the celebrated rods known as "Napier's bones." This is really nothing but a mechanical multiplication table, based on the principle of gelosia or lattice multiplication, popular in Napier's day. Another mechanical calculator, which Napier called the "Promptuary," has rods and strips running in both transverse and longitudinal directions. It is more sophisticated than the "bones" and may be considered the first mechanical calculating machine. One further section of Rabdologiae describes a method of multiplication based on binary arithmetic, using a checkerboard with counters.

Fragments of Napier's earlier work in arithmetic and algebra were published in 1839 by a descendant, Mark Napier, under the title De Arte Logistica. These consist mostly of arithmetical algorithms and rules for solving algebraic equations.

Napier also made a useful contribution to spherical trigonometry. If any two of the five parts of a right-angled spherical triangle are known, the remaining parts can be determined. Napier's rules reduce ten formulas to a single relation in which the triangle is replaced by a star pentagon.

Finally, Napier deserves credit for popularizing the use of the decimal point. In the Constructio Napier states: "In numbers distinguished thus by a period in their midst, whatever is written after the period is a fraction, the denominator of which is unity with as many cyphers after it as there are figures after the period. Thus 25,803 is the same as 25 $\frac{803}{1000}$." Napier's authority and the popularity of his logarithms led to this method of decimal fractions eventually being adopted throughout the mathematical community.

It has been said that great discoveries in mathematics are made by great mathematicians. Napier's logarithms have certainly had far-reaching consequences for both science and mathematics--originally as a computational device, and later as a functional concept. For this work John Napier well deserves the title, "Marvelous Merchiston."

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LOSING AT LOTTO

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Many states have instituted a game called Lotto in an attempt to raise funds for various state projects. The common procedure for those games is that the player selects r numbers (usually 6) from a set of n numbers (varying from 30 to 44, depending upon the state). At the end of a given time period (often one week), the state lottery commission randomly selects a winning set of r numbers from the total set of n numbers.

Usually a player wins a cash prize if he/she has selected 4, 5, or 6 of the winning numbers that the lottery commission has identified. In some states a free play is awarded to the player who has matched three of the winning numbers.

A common impression of those who play Lotto is that not only is it very difficult to win anything, but it is very difficult to