TRIANGULAR SERIES ON THE SUBTRACTION TABLE: NUMBER PATTERNS

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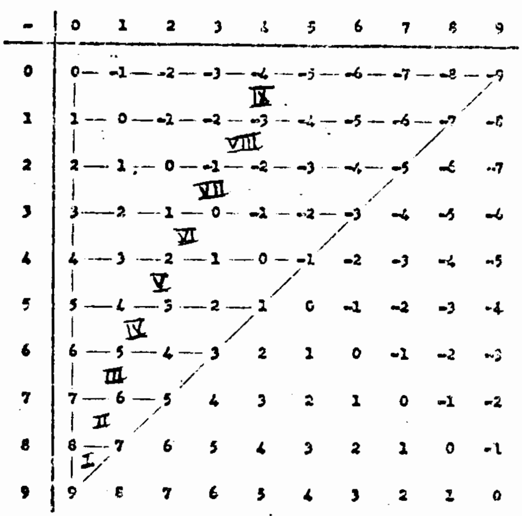
Mathematics teachers are always looking for opportunities to give their students meaningful computational practice. It is an added bonus if number patterns may also be observed.

Suppose that a series of isosceles right triangles are positioned on the subtraction table as shown in Figure I.

Figure I



-	0	1	2	3	4	5	6	7	8	9
0	0	-1	-2	-3	-4	-5	-6	-7	-8	-9
1	1	0	-1	-2	-3	-4	-5	-6	-7	-8
2	2	1	0	-1	-2	-3	-4	-5	-6	-7
3	3	2	1	0	-1	-2	-3	-4	-5	-6
4	4	3	2	1	0	-1	-2	-3	-4	-5
5	5	4	3	2	1	0	-1	-2	-3	-4
6	6	5	4	3	2	1	0	-1	-2	-3
7	7	6	5	4	3	2	1	0	-1	-2
8	8	7	6	5	4	3	2	1	0	-1
9	9	8	7	6	5	4	3	2	1	0

Triangle I passes through the entries 9, 8, and 7; triangle II passes through entries 9, 8, 7, 6, 5, and 7; triangle III passes through 9, 8, 7, 6, 5, 4, 3, 5, and 7; etc. Note that their perpendicular legs lie on the rows and columns of the subtraction table. We shall call this series of triangles the 9-series since the "bottom vertex number" is nine.

Table I represents the sums of the entries that each triangle passes through.

Table I: 9-series Triangles

Triangle	Sum	Factored Sum
I	24	3·8
II	42	3·14
III	54	3·18
IV	60	3·20
V	60	3·20
VI	54	3·18
VII	42	3·14
VIII	24	3·8
IX	0	3·0

Observe that these sums are all multiples of three.

Figure II depicts another series of triangles, each located one-unit to the right of the corresponding triangles of Figure I. These triangles are called 8-series triangles since their "bottom vertex number" is eight. Only the interior of the subtraction table is displayed.

Figure II

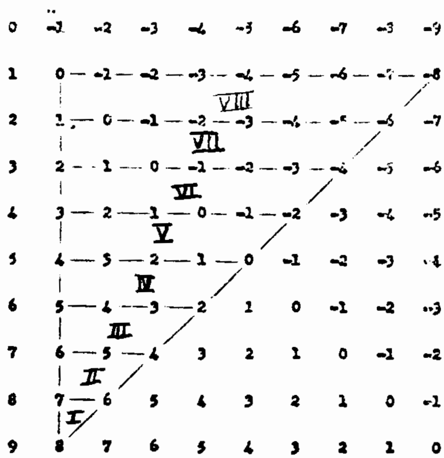


Table II reports the sums of the entries which these triangles pass through.

Table II: 8-series Triangles

Triangle	Sum	Factored Sum
I	21	3·7
II	36	3·12
III	45	3·15
IV	48	3·16
V	45	3·15
VI	36	3·12
VII	21	3·7
VIII	0	3·0

In a similar manner 7-series triangles, 6-series triangles, . . ., 1-series triangles may be drawn. The first triangle in each series is shown in Figure III. In some cases to generate the series of triangles the subtraction table must be extended.

Figure III

0	-1	-2	-3	-4	-5	-6	-7	-8	-9
1	0	-1	-2	-3	-4	-5	-6	-7	-8
2	1	0	-1	-2	-3	-4	-5	-6	-7
3	2	1	0	-1	-2	-3	-4	-5	-6
4	3	2	1	0	-1	-2	-3	-4	-5
5	4	3	2	1	0	-1	-2	-3	-4
6	5	4	3	2	1	0	-1	-2	-3
7	6	5	4	3	2	1	0	-1	-2
8	7	6	5	4	3	2	1	0	-1
9	8	7	6	5	4	3	2	1	0

Table III displays the sum of the entries for all of these series.

Table III

	I	II	III	IV	V	VI	VII	VIII	IX
9-series	3·8	3·14	3·18	3·20	3·20	3·18	3·14	3·8	3·0
8-series	3·7	3·12	3·15	3·16	3·15	3·12	3·7	3·0	3(-9)
7-series	3·6	3·10	3·12	3·12	3·10	3·6	3·0	3(-8)	3(-18)
6-series	3·5	3·8	3·9	3·8	3·5	3·0	3(-7)	3(-16)	3(-27)
5-series	3·4	3·6	3·6	3·4	3·0	3(-6)	3(-14)	3(-24)	3(-36)
4-series	3·3	3·4	3·3	3·0	3(-5)	3(-12)	3(-21)	3(-32)	3(-45)
3-series	3·2	3·2	3·0	3(-4)	3(-10)	3(-18)	3(-28)	3(-40)	3(-54)
2-series	3·1	3·0	3(-3)	3(-8)	3(-15)	3(-24)	3(-35)	3(-48)	3(-63)
1-series	3·0	3(-2)	3(-6)	3(-12)	3(-20)	3(-30)	3(-42)	3(-56)	3(-72)

Note that:

1. All of the sums are multiples of three.
2. In the column of triangle I, the consecutive second factors decrease by 1; in the column of triangle II, the consecutive second factors decrease by 2; etc.

Challenge to the reader: Perform the procedures of this article on the addition table. Do the same (or similar) patterns result?

A tramp, having ambled three-eighths of the way across a mile-long trestle, heard a train approaching at 60 miles per hour behind him. Making a rapid mental calculation, he discovered that he would have just time enough to get off the trestle by running as hard as he could in either direction. How fast could he run ?