SOME IMPLICATIONS OF PIAGET'S RESEARCH FOR MATHEMATICS TEACHERS

Robert J. Sovchik and L.J. Meconi
The University of Akron
Akron, Ohio

The purpose of this paper is to explore the diagnostic and remedial mathematics implications of Piaget's research. Since one can devote a lifetime of study to the theory of Piaget, this paper is designed to be of a "practical" nature. The content developed in this paper is not viewed as a thorough treatment of Piaget's theory.

Essentially, Piaget has studied children's thinking through interview techniques. He proposes four basic stages in the development of mental structures. These stages are the following: 1) the sensori-motor stage (from birth to one and one-half years); 2) the preoperational stage (beginning at one and one-half or two years of age until approximately seven years of age); 3) the concrete operations stage (from approximately seven to eleven years of age); 4) the formal operations stage (emerging at approximately eleven or twelve years of age).

Because of the nature of most teachers' work, our concern will be with the preoperational stage and the concrete operations stage. Certainly, there is a variance among children as they move through these stages. That is, all children do not move through these stages at the same rate. Also, there is criticism of Piaget's research techniques and findings.[1] Some of the criticisms are that young children are highly unreliable interview sources; that using manipulative material like candy may evoke imaginative responses rather than logical responses; and that true experimental controls are lacking. Our interest will be to consider Piaget as a means of broadening the base of diagnostic-remedial teaching options so that we can have alternative procedures at our disposal.

Considering the preoperational and operational stages, Almy[2] suggests that a key feature of the preoperational child is that the child is unable to deal with the operation of reversibility. For example, children at the preoperational stage think a ball of clay weighs more than the same clay stretched into a sausage shape. They are unable to reverse the process, in other words, to squeeze the sausage-shaped clay to its original ball-shaped form and thus determine that the weights are the same. Children who are able to go beyond the perceptual stage by using the operation of reversibility are said to be conserving weight. Thus, children able to do this are classed at the concrete operation stage for the weight concept. In summary, two key words are related here. Conservation means the child is able to use the operation of reversibility to go beyond perceptual change and thus determine that the concepts are the same. We will look at conservation tasks later on in this paper.

It should be noted that for some mathematical concepts, children do not leave the preoperational stage until nine or ten years of age. For example, conservation of number appears at approximately age seven, while conservation of volume does not appear until around ten or eleven years of age. We will describe these terms in the context of the activities that follow.

There are many activities with which Piaget experimented. We will look at logical classification, class inclusion relations, and the various conservation tasks. After each description, you may want to answer the accompanying questions. Bear in mind two thoughts. First, Piaget's work is not without its critics as we have previously said. Secondly, the "passage" from the preoperational stage to the concrete operations stage is relative, varying from child to child and from concept to concept.

I. Logical Classification Questions

One of the key concepts in Piaget's approach to studying logical classification is the relation of class inclusion (roughly speaking, part-to-whole relationships). Piaget asks this question (he resided in Geneva, Switzerland): "Are you Swiss?" The child answers: "No, I'm Genevan." Up to age nine, according to Piaget, 75% of the children deny being both Swiss and Genevan.

Question 1. Can you write several other class inclusion relation questions with a more appropriate vocabulary for Sometown, Ohio?

Question 2. Form G of the Metropolitan Mathematics Achievement Test contains at least two items which assume a class inclusion relation competency.

Item 6 in Test 3: Mathematics Problem Solving. "In the kindergarten class, there are 13 boys and 17 girls. Five more children are coming. How many children will there be then?"

Item 3 of the same section of the Metropolitan contains a similar class inclusion relation. "Our class had a pet show. There were 4 dogs, 3 cats, and 7 birds. How many animals were in the show?"

Find other items which exhibit a class inclusion relation.

II. The Relations "All" and "Some"

According to Piaget, it is not until nine to ten years of age that a child is able to use these words correctly in a logical sense for classification. Here are two items from the Metropolitan that seem to involve an understanding of the term "all."

Item 7 of Test 2: Mathematics Concepts of the Metropolitan, "If I eat one-half of a pie, how much of the pie is left?" The B response is "almost all of it."

Item 11 of the Mathematics Concepts section has this question. "If 7 children each paint 2 pictures, how many pictures will there be in all?" A procedure that Piaget uses to determine whether children understand this concept is presented below.

Materials Needed: Three red squares, two blue squares, and three blue circles.

Procedure: 1) First ask the child to identify the colors and shapes. "What color is this?" "What shape is that?"

2) "Are all the circles blue?"

3) "Are all the blue ones circles?" "Why?"

Question 1. See if you can develop other questions which require an understanding of the relations "all" and "some."

Question 2. List as many troublesome mathematics vocabulary terms as you can which involve this concept.

Question 3. What suggestions can you make for teaching these vocabulary terms?

III. Addition of Classes

Piaget has an activity which shows how the class inclusion relation affects the operation of addition. The activity is presented below:

Materials: 1) Nine wooden beads--seven brown and two white (other colors may be substituted).

2) A piece of string or wire to use as a necklace.

Procedure: 1) Ask the child of what the beads are made (wood).

2) Then, what color is this (pointing at brown) and this (pointing at white)?

3) Are there more brown or more wooden beads?

4) Ask, "If I made a necklace of wooden beads and a necklace of the brown beads, which would be longer? Why?"

According to Piaget, until age seven or eight, children will respond that there are more brown beads than wooden beads. Psychologically speaking, the child can see or perceive either the whole set or its parts but not both <u>simultaneously</u>. The child does not have reversibility of thought which allows him to recombine the parts into the whole and consider the whole and its parts at the same time.

IV. Right-Left Relation

Sometimes right-left relations are seen as absolutes rather than as relations to each other. To determine this, take two objects. We will call them A and B and write A   B. The child identifies A as left of B, but if a third object C is added, A   B   C, most ten years of age or less will not admit that B can be left of C.

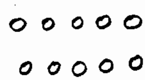
Question 1. See if you can find a number line situation where confusion can arise if right and left are seen as absolutes.

The conservation tasks are presented in order of development. Approximate age for emergence is presented in parenthesis.

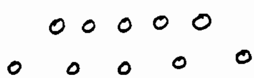
V. Conservation of Number

1) Conservation of Number (Age 7)

A youngster observes two sets of beads.

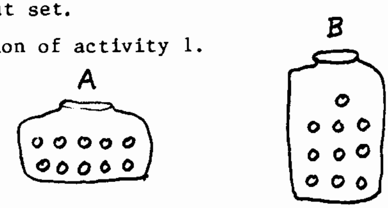


He agrees they contain the same number. Then one set is spread out.



Are they still the same? According to Piaget, most children under age seven say there are more in the spread out set.

2) A variation of activity 1.



Here you have two jars, one taller than the other. A child is given a pile of beads. The child takes the beads with both hands and places one bead in the jar on the right with his right hand and one bead in the jar on the left with his left hand. For each bead placed in jar A, a corresponding bead is placed in jar B. Are there the same number in both jars? Children who have not conserved number may say that jar B has more because it is taller.

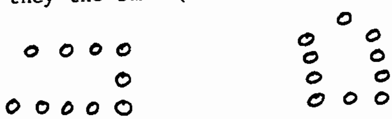
3) Some more variations of activity 1.

Other kinds of figures are used, but the procedures are the same as question one.

a)

Are they the same (in number)?

b)



Are they the same (in number)?

VI. Conservation of Quantity (Age 7-10)

"If I give you some milk in one cup, will it be the same if I pour it into two cups?"

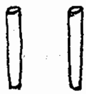
Now if Mother gave you these two cups each containing the same amount of milk and we pour the milk from one cup into that tall glass, which would hold more, the cup or the glass?

Question 1. What language ambiguities are present in this activity?

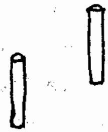
Question 2. List the questions you would ask when developing this activity.

VII. Conservation of Length

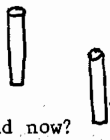
Take two sticks of equal length. Place them in front of the child in the following ways:

1) 

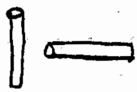
Which is longer or are they the same?

2) 

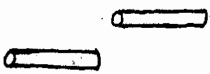
Now is one longer or are they still the same?

3) 

And now?

4) 

And now?

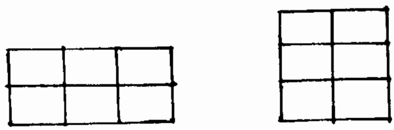
5) 

And now?

VIII. Conservation of Area

To investigate the concept of conservation and measurement of area, Piaget used two procedures. We will look at the first procedure.

The child is then shown two rectangular arrangements of blocks. This arrangement was then altered for one rectangle by removing the two blocks at one end and placing them on top or below the others as follows:



The children were then asked if the rectangles were still "the same size" or had the same amount of room.

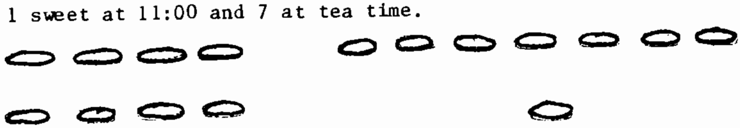
Another variation is to perform the same procedure with the resulting figures being these.

IX. Fractions

Piaget used a conventional activity here. The child is shown a circular slab of clay and two dolls. He is told that the clay is a cake and the dolls are going to eat all of it and that each must have the same amount. He is then given a wooden knife with which to cut the cake. The experiment is repeated with another cake and three dolls.

X. The Relation of the Parts to the Whole in Addition of Numbers

To find out when children learn that the whole remains invarient, regardless of the way its parts are rearranged, Piaget used the number 8 in the form of 4 + 4 and 1 + 7. The child is told that he can have 4 sweets at 11:00 and 4 at tea time. The next day he will be given the same number, but since he is less hungry in the morning, he will be given 1 sweet at 11:00 and 7 at tea time.

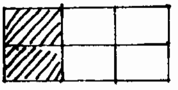
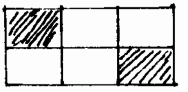


I II

The child is asked if there is the same amount to eat on both days.

Question 1. Suppose you tried the experiment for fractions. How would you revise the language?

Question 2.

How much is shaded? Write a fraction.

We hope that these activities will provide you with enough information about Piaget's work so that you may experiment with your classroom children.

References

1. Glennon, V.J. and Callahan, L.G. Elementary School Mathematics: A Guide to Current Research. NEA: Association for Curriculum and Development, Washington, D.C., 1975.

2. Almy, Millie Corinne. The Impact of Piagetian Theory on Education, Philosophy, Psychiatry, and Psychology. Baltimore, University Park Press, 1979.

3. Copeland, Richard W. How Children Learn Mathematics: Teaching Implications of Piaget's Research. New York, The Macmillan Company, 1974.

Two ladders are leaned against buildings on opposite sides of an alley as shown in the accompanying figure. The ladders are 30 feet and 40 feet long. They cross the pavement at a point 10 feet above the level pavement in the alley. What is the distance between the buildings ?