
Exploration of Polynomials Through the Use of Code Word Solutions

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Abstract: The study of polynomials is an important foundational idea in elementary algebra and introductory calculus. This activity will reinforce and introduce ideas about where roots occur, change in slopes over the curve, and determining extremum. This project has students choose a word which has meaning to them, encipher it, and produce a polynomial of their own. From this personalized polynomial they will carry out these studies.

Keywords. polynomials, modeling, precalculus

1 Introduction

The purpose of the following exercise is to build an interest in, and to further students' understanding of, the properties of polynomials. This activity will introduce and reinforce ideas about where roots occur, change in slopes over the curve, and determining extremum. The students will choose a code word they will use to develop their own polynomial. They will gain understanding of where roots occur and ideas about how to estimate the shape of a graphed function. It is expected that students who have made a personal connection with a function by creating their own polynomial will better retain fundamental ideas about polynomial properties.

2 Selecting a Word to Code

The idea behind the code word solution is to choose a word, preferably five letters long or less to aid with calculations, which will have a root corresponding to each letter of their chosen word. Their first names or initials are excellent places to start when choosing a code word with which they will connect. Number the alphabet 1 to 26 from *A* to *Z* to establish a key for enciphering the code word into a set of numbers. For our example, I will encode TIGER with this method, giving the number set 20, 9, 7, 5, 18. At this point the students may be given the freedom to choose an independent variable for their polynomial, or instructors may wish to maintain standard notations of x or n . I like to have the students choose their own to further establish ownership and to demonstrate that a variable is just a symbol and not something established in mathematics as immutable. For our demonstration, we will use d , my initial, as our independent variable.

3 The Enciphering Process

Let d equal each of the numbers from the enciphered set so that we have a series of equalities as follows: $d = 20, d = 9, d = 7, d = 5, d = 18$; these are the zero solutions for our polynomial. Now we will bring each of the numbers to the same side of the equalities as the variable, giving us a series of binomials as shown in line (1) below. We are now able to multiply these together to create our personal polynomial. Review or introduction of the FOIL method and the distributive property to multiply polynomials should be done as needed. This exercise, if done by hand, will give students good practice with multiplying polynomials. Educators can quickly check for accuracy with *Mathematica* or a graphing calculator. *WolframAlpha*, a limited version of *Mathematica*, is free to use online. It is also available, for a small fee, as an app. Educators may wish to use this program to incorporate a technology piece with this exercise, allowing students to check their own calculations and to explore what else may be possible with polynomials and mathematics in general. The following steps use our code word TIGER to create a personalized polynomial.

$$(d - 20)(d - 9)(d - 7)(d - 5)(d - 18) \quad (1)$$

$$(d^2 - 29d + 180)(d - 7)(d - 5)(d - 18) \quad (2)$$

$$(d^3 - 36d^2 + 383d - 1260)(d - 5)(d - 18) \quad (3)$$

$$(d^4 - 41d^3 + 563d^2 - 3175d + 6300)(d - 18) \quad (4)$$

$$d^5 - 59d^4 + 1301d^3 - 13309d^2 + 63450d - 113400 \quad (5)$$

In function notation, (5) can be shown as $f(d) = d^5 - 59d^4 + 1301d^3 - 13309d^2 + 63450d - 113400$. To extend the idea that variables are arbitrary, educators may allow students to use a letter other than f for function notation. They will likely see, or have already seen, g and h used in place of f as multiple related functions are examined in textbooks. The ideas in this lesson should be developed artistically and imaginatively by the students. A student's input into how the polynomial is developed, and the notation used, demonstrates the creativity required in higher level mathematics. We expect calculations to be accurate; however, deeper understandings of concepts develop as students explore mathematics by "playing" with numbers and symbols.

4 Generating a Plot

At this point, students can be asked to attempt to plot their enciphered polynomials on a coordinate axis to see what they look like, as shown in Figure 1.

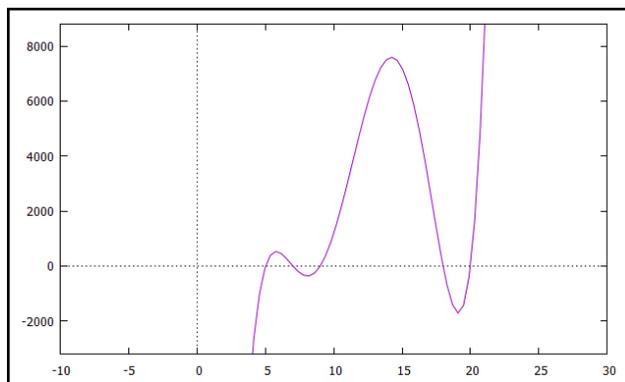


Fig. 1: A plot of (5) interpreted as a function of d .

5 Teaching Ideas and Next Steps

Once students generate graphs, discuss the characteristics of curves and how each function definition is related to its plot. Noting that each letter is a root on the graph, students discuss where roots will appear and how this is determined by various code words. Observing the order of their functions, students also explore how many turns from positive to negative and back again should be expected. The order of the function is determined by the number of letters in the chosen word. As students plot their functions, they will likely use a scaled axis to view the main part of the curve in a reasonable amount of space, an unfamiliar task for many. Students can use maximum and minimum tools on graphing calculators to help with this exercise. Doing so may lead to a discussion about how data displays may be distorted through scaling to promote certain viewpoints in news and research reporting. As Table 1 suggests, using a table of values along with the graphing calculator will be helpful in this part of the project for students to estimate where their graph is increasing and decreasing. Educators who use the activity in an introductory calculus course may wish to present techniques for finding extrema at this time. This method could be discussed with lower level classes as a teaser of things they will learn through further studies.

d	$f(d)$	d	$f(d)$
0	-113400	5.75	531.87
1	-62016	5.8	532.19
5	0	5.9	523.6
5.5	475.78	7	0

Table 1: A table of values, for polynomial (5), on the interval $[0, 7]$.

6 In Summary

At this point, students have invested themselves in the creation of a polynomial, the exploration of its properties, and have created a graph of the respective function. We expect the outcome of this endeavor to have made a personal connection with the student which will continue with them beyond this project and into their further studies. Students should not be discouraged in the future from investing themselves personally in the development of mathematical concepts in which they might include their own variables. This flexibility will help them develop the abstraction needed for algebraic arguments and proof development. It also helps them develop imagination in conjunction with their critical thinking which has the possibility to help them beyond the field of mathematics. Empower your students to learn for themselves.



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