# Integrating Alternative Algorithms: Possibilities and Practices

***Cindy Jong, Haley Dowty, & Bailey Hume,*** *University of Kentucky*
***Maranda Miller,*** *SUNY Maritime College*

***Abstract:*** *This article discusses reasons for learning alternative algorithms and the benefits of exposing preservice teachers to alternative algorithms. It presents two alternative multi-digit subtraction algorithm examples, includes perspectives from preservice teachers, and provides strategies for teachers and teacher educators to integrate alternative algorithms in classrooms.*

***Keywords.*** *subtraction, preservice teachers*

## 1 Introduction

Life is full of alternative approaches to solving problems. Teachers, engineers, doctors, computer scientists, and other professionals are constantly seeking new methods to more effectively and efficiently perform tasks (Ellis & Yeh, 2008). In a classroom context, it is advantageous for students to learn various approaches to solving mathematics problems to think flexibly, enhance decision-making skills, and think critically at an early age (Randolph & Sherman, 2001). To do this, it is important for preservice and inservice teachers to think flexibly themselves. In this article, we discuss reasons for learning alternative algorithms and the benefits of exposing preservice teachers (PSTs) to them. We present two alternative multi-digit subtraction algorithm examples, include the perspectives of PSTs, and suggest possible methods for teacher educators to integrate alternative algorithms in mathematics methods or content courses.

## 2 Why Use Alternative Algorithms?

It is important to integrate the use of alternative algorithms in elementary school classrooms because algorithms have the potential of tailoring to students' unique style of thinking and focus on their individual needs (Randolph & Sherman, 2001). It is also crucial to incorporate multiple methods to solve a problem in classrooms that are full of a variety of learners. From students' perspective, it is helpful to see that there is more than one way to correctly solve a problem. Once an algorithm is mastered, students can determine which is most beneficial and can have the option to use that particular method. This allows students to become more confident in their ability to be successful in the math classroom. Alternative algorithms "allow students to look at math as a process" and provide them with an opportunity to think about applying multiple strategies for solving a problem. (Randolph & Sherman, 2001, p. 480). Standard algorithms can be efficient but do not necessarily allow the students to see why they really work. Students might view it as the only way to complete a problem rather than one way to do it (Ellis & Yeh, 2008).

Preservice teachers can also benefit from being exposed to alternative algorithms to learn it themselves and potentially integrate them into their teaching. Not only do PSTs need to know how to perform the algorithms, but "they need to be able to explain the mathematics underlying the algorithms in a way that will help children understand" (Thanheiser, 2009, p. 277). This type of experience will better prepare them to teach multiple approaches to solving problems in ways that are more meaningful to students.

Steele & Widman (1997) found that when PSTs become more comfortable with alternative algorithms, they start to think of themselves as mathematical thinkers and of mathematics as a human discipline. It is a valuable skill to be able to understand and accept that mathematical problems can be solved many different ways (Simonsen & Teppo, 1999). Teacher educators need to understand the PSTs' background knowledge about conceptions of numbers to help them develop an understanding of the mathematics underlying the algorithms (Thanheiser, 2010).

## 3 Multi-digit Subtraction Algorithms

According to the Common Core State Standards for Mathematics (CCSSI, 2010), students in third grade are expected to "fluently add subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction" (3.NBT.2). However, elementary students often have difficulty with multi-digit subtraction problems that require regrouping. It is also the case that up until this point, students may still be completing mathematics problems from left to right whereas the standard subtraction algorithm (see Figure 1) requires them to solve the problem from right to left, beginning with the ones place. This method necessitates the use of regrouping to subtract quantities when the subtrahend is larger than the minuend within a place value. Students find it difficult to use the standard algorithm when the problem contains zeros. We recommend the use of base-ten blocks to concretely model regrouping and to help facilitate an understanding of place value when using this algorithm. As alternatives or additions to the standard approach, we present the Subtract in Each Place and European algorithms for multi-digit subtraction problems.

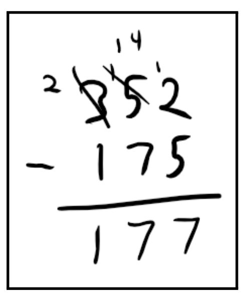


**Fig. 1:** *Standard subtraction algorithm for* $352 - 175$.

The subtract in each place algorithm allows students to solve the problem from right to left or left to right (see Figure 2). Students find the difference within each place value when solving the problem (Huinker, 2003). For example, in the problem $352 - 175$, students find the difference between $300$ and $100$, and so forth. While it is not the case that we refer to differences as positive or negative, it helps to do so and record it in such a way with this algorithm. Thus, the difference between $300$ and $100$ is a positive $200$, the difference between $50$ and $70$ is a negative $20$ (or subtract 20), and the difference between $2$ and $5$ is a negative $3$ (or subtract 3). All the differences are then combined

$(+200-20-3=177)$ to solve the problem. Since an understanding of negative numbers is needed, it is important to connect the concept to real life examples such as temperatures and elevators (Gregg, 2007). However, the concept of negative numbers might be abstract and challenging for students to grasp prior to third grade. Thus, the language "subtract 20" and "subtract 3" can also be appropriate with second and third grade students.

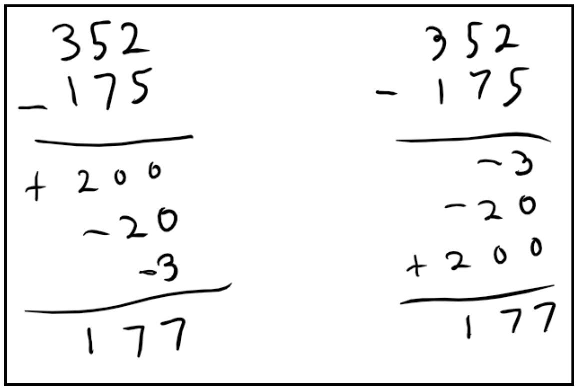


**Fig. 2:** *Two implementations of the Subtract in Each Place Algorithm for* $352-175$.

The European or Equal Additions subtraction algorithm implements a "friendly numbers" strategy in a systematic way (Randolph & Sherman, 2001). When regrouping is required, the same number (ten or hundred) is added to both the minuend and subtrahend. Mathematically, this approach works because you are adding zero to the difference. In other words, when the same quantity is added to both the minuend and subtrahend, the difference stays the same. Take the simpler subtraction problem, $25-18=7$, as an example. If we add 2 to both the minuend and subtrahend to create the new problem, $27-20$, the difference is still 7. Thus, it is critical to explain that the notation in Figure 3, for example, indicates that ten ones and ten tens are added to the minuend, and one hundred and one ten are added to the subtrahend, increasing both by 110. By using the European subtraction algorithm and changing the numbers as an initial step, the problem can then be solved from right to left or left to right.

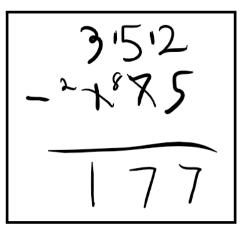

**Fig. 3:** *European or Equal Additions Algorithm for* $352-175$.

Encouraging students to select the algorithm that is most appropriate for the particular mathematics problem is one way to implement the Standard for Mathematical Practice 7: Look for and make use of structure (CCSSI, 2010). For example, students might come to the conclusion (after a classroom discussion about patterns in problem-types) that multidigit subtraction problems with multiple zeros in the minuend might be solved more efficiently using the *Subtract in Each Place* algorithm. In contrast, the standard algorithm might be the most appropriate for a problem that does not require any regrouping. The study of alternative algorithms can provide students with an opportunity to look closely at the structure and properties of operations. Most importantly, students need to make sense of the solutions, be conscientious when solving multi-digit mathematics problems, and learn that they can check their own solutions using addition.

## 4 Preservice Teachers' Perspectives

We interviewed four elementary preservice teachers and analyzed 15 open-ended responses to understand their perspectives on alternative algorithms. The PSTs who were interviewed had completed a mathematics methods course where two alternative algorithms for each multi-digit operation were taught. Below we discuss some of our findings.

Overall, the PSTs we interviewed had a generally positive outlook on alternative algorithms and the use of them in the classroom, even though they may not have had a positive attitude toward mathematics in general. They mentioned the fact that alternative algorithms tend to emphasize number sense and place value more than the standard algorithms. "Friendly numbers" was also a strategy brought up several times in the interviews. PSTs thought that some of the alternative algorithms were also useful because they break up the problem into manageable parts, or friendlier numbers, which can be helpful for students. Yet, there was concern that some of the algorithms could take some students longer to complete when used with large numbers, such as the partial products algorithm for multiplication. However, the PSTs saw various benefits to using multiple methods in the classroom.

All of the PSTs who were interviewed intended to use alternative algorithms in their future classrooms. One said that she would only introduce partial sums (for teaching addition) and partial products (for teaching multiplication) to the whole class and show other algorithms to only the higher achieving students in the class, not realizing that some methods are often used with students who have special needs. Another PST mentioned that she would definitely use the algorithms we discussed and allow her students to invent their own algorithms. The other two PSTs said that they would use alternative algorithms more often if students did not understand the standard algorithm. Thus, we received multiple perspectives on how and when the alternative methods would be introduced in their future classrooms.

For the most part, the PSTs were comfortable using alternative algorithms; however, they found some of them confusing. The European subtraction algorithm was commonly disliked by the PSTs. As one PST said, "it was confusing because you still have to regroup numbers like in the standard algorithm, but you are adding it to the bottom number." This confusion of the European subtraction algorithm led the PSTs to say they would not want to teach it in their future classrooms. Understandably, they would only be willing to use the algorithms that they had fully mastered and felt confident using.

The PSTs provided insights on possible resources that would make them feel more comfortable about using alternative algorithms in their future classrooms. Since many of the PSTs had never been exposed to particular alternative algorithms prior to their mathematics methods course, the material was still very new; thus, they expressed an interest in more practice with the algorithms. One PST said that it would be valuable to have a special training specifically focusing on how to teach the alternative algorithms along with the standard algorithms. They said that it would be helpful to have a book or packet that listed step-by-step procedures and the mathematical reasoning behind each of the alternative algorithms to use before they teach it. One said that even though she is learning them now, in her math methods course, she is probably going to have to go back and remind herself how to perform the algorithms because they are not something she is using on a daily basis.

When open-ended responses were reviewed, PSTs echoed similar thoughts about alternative algorithms. They thought they were beneficial because they emphasized the use of place value knowledge, minimized the need to regroup, and gave students the option of starting from the left (with numbers in the largest place value).

## 5 Strategies for Teachers and Teacher Educators

Teachers can integrate alternative algorithms into the classroom in many different ways. First, it is always helpful to use what students already know about numbers and algorithms to build on that knowledge and development (Thanheiser, 2009). Their pre-existing knowledge and skills can point you toward creating appropriate problems and introducing specific algorithms. Next, when you have determined which problems to solve, it is critical to avoid giving too much guidance so that the students can begin to think about the problem, using manipulatives, pictures, words, or methods to clarify their thoughts (Ellis & Yeh, 2008).

It is also beneficial to have students come up with their own algorithms. "When students are allowed to struggle from time to time and invent their own algorithms for solving problems, they learn mathematics at a much deeper level than students in traditional classrooms" (Groth, 2007, p. 23). Student-invented algorithms are especially effective when students present their own strategies to one another. When doing so, teachers need to be sure that students can explain why it works and highlight critical steps (Huinker, 2003). Students can learn a tremendous amount by explaining to others and, conversely, can learn from listening to their peers explain a problem.

Teacher educators can also support the use of alternative algorithms by introducing their preservice teachers to various algorithms. Presenting preservice teachers with alternative algorithms in problem solving situations and having PSTs explain why they work helps deepen their understanding (Simonsen, 1999). PSTs might have a similar mindset that many K-12 students have that mathematics is a product rather than a process. Having PSTs engage in activities that challenge them as mathematics students and future educators will allow them to better facilitate learning experiences that are meaningful for students.

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**Cindy Jong**, Cindy.jong@uky.edu, Cindy Jong, is an assistant professor of mathematics education in the STEM Education Department at the University of Kentucky. Her research aims to understand methods to improve preservice elementary mathematics teacher education and measure affective factors in mathematics education. Jong served on program committees for the 2013 NCTM Annual Meeting in Denver and consistently presents her research at NCTM, AMTE, and PME-NA.

**Haley Dowty**, Haley.dowty@uky.edu, was an undergraduate research fellow in 2014-2015 for the STEM Education Department at the University of Kentucky. She is an Elementary Education major and President of the Equestrian Team. Dowty also co-presented research on Alternative Algorithms at the 2015 Kentucky Center for Mathematics Conference.

**Bailey Hume**, Bailey.hume@uky.edu was an undergraduate research fellow in 2014-2015 for the STEM Education Department at the University of Kentucky. She is a STEM Plus and Mathematics major. Hume also co-presented research on Alternative Algorithms at the 2015 Kentucky Center for Mathematics Conference.

**Maranda Miller**, mmiller@sunymaritime.edu, is the Learning Center Coordinator for SUNY Maritime College, and a doctoral student in the University of Kentucky's STEM Education program. She is interested in how mathematics self-efficacy and mathematics anxiety influence adult learners ability to master mathematics for the GED.